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INTRODUCTION

TO

PRACTICAL ASTRONOMY,

WITH

A COLLECTION OF

ASTRONOMICAL TABLES.

BY

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OF A "COURSE OF MATHEMATICS."

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RoE

Practical Astronomy has claims upon our attention equal to those of either of the preceding sciences, whether we regard it as a means of mental discipline, or in its bearings upon other branches of study. An acquaintance with the grand principles of astronomy has from time immemorial been regarded as an essential part of a finished education; but no one can feel a rational confidence in the results announced by astronomers without some distinct notion of the methods by which these results are attained. When the student is told that the sun is ninety-five millions of miles distant from us, and that light requires several years to reach us from the nearest fixed star, he may receive these doctrines without dispute on the basis of authority, but he can feel no adequate conviction of their truth without a knowledge of the instruments with which the requisite observations are made, as well as the principles upon which the computations are conducted.

It is believed, therefore, that Practical Astronomy is destined to occupy a more prominent place in our institutions of education than it now holds, and it is hoped that the present volume may contribute something to so desirable a result.

The preparation of this treatise has been attended with serious labor. No considerable portion of it has been exclusively derived from any single work. I have sought for materials from every source within my reach—not only from the standard authorities upon this subject, but also from Astronomical Journals and the Annals of Observatories. The works which I have most frequently consulted with success are, Pearson's Practical Astronomy, and Baily's Astronomical Tables; Delambre's Astronomie, and Francoeur's Astronomie Pratique; Brünnow's Sphärischen Astronomie; Sawitsch's Practischen Astronomie, and Bessel's Astronomische Untersuchungen.

The Tables which accompany this volume have cost me considerable labor. Table XVI. is entirely original. Doubtless similar tables have been heretofore computed, but I have been unable to find such an one in any of the works to which I have had access. Several of the tables have been computed entirely anew, although similar tables are to be found in other works. Of this description are Nos. XVII., XVIII., XXII., and XXVII. Others have been partially recomputed, extended, and modified to suit

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THE following Alphabet is given in order to facilitate, to the student who is u
acquainted with it, the reading of those parts in which the Greek letters are use

Letters.		Names.	Letters.		Names.
Α	α	Alpha.	Ν	ν	Nu.
Β	β	Bēta.	Ξ	ξ	Xi.
Γ	γ	Gamma.	Ο	ο	Omicron.
Δ	δ	Delta.	Π	ϖ π	Pi.
Ε	ε	Epsilon.	Ρ	ρ	Rho.
Ζ	ζ	Zēta.	Σ	σ ς	Sigma.
Η	η	Eta.	Τ	τ	Tau.
Θ	ϑ θ	Thēta.	Υ	υ	Upsilon.
Ι	ι	Iōta.	Φ	φ	Phi.
Κ	κ	Kappa.	Χ	χ	Chi.
Λ	λ	Lambda.	Ψ	ψ	Psi.
Μ	μ	Mu.	Ω	ω	Omega.

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the ground as can conveniently be done. It is a matter of the first importance that the horizon be unobstructed in the direction of the meridian.

The atmospheric obstructions which astronomers aim to avoid as far as possible are fogs—which are uncommonly prevalent in certain places, especially on low, swampy grounds—the smoke and heated air arising from chimneys, factories, etc., as also the dust and noise of public streets. Certain localities are much more subject to clouds and high winds than other places, and these are specially unfavorable to the operations of an observatory.

(3.) A transit instrument and a good clock are indispensable to the furniture of every observatory. The former requires an opening in the roof and down the walls of the building, so as to afford a view of the meridian from the north to the south horizon. This opening should not be less than eighteen inches wide, and should be covered by doors which may be easily thrown open, and which, when closed, shall effectually exclude the rain and snow. A complete observatory must also be furnished with a graduated circle for measuring altitudes or polar distances, which will require a second opening across the roof, similar to the one already described, unless a meridian circle be used for both purposes, in which case one opening may suffice.

(4.) An altitude and azimuth circle, or an equatorial instrument, requires a revolving roof, with an opening from the zenith to the horizon, to enable the observer to follow a heavenly body in any part of its diurnal course. This roof should not be larger than is necessary for giving room to the observer and to the instrument under it, lest its bulk and consequent weight should impede its easy motion. It should be made to turn round on a circular bed, placed in a horizontal position. The dome may revolve on small brass wheels, set in a ring of wood of proper dimensions, or on cast-iron balls, turned in a lathe so as to be of exactly equal diameter.

The figure on the opposite page represents a section of a rotatory dome suitable for a small observatory. The letters AA represent an opening 18 or 20 inches in width, extending from the top of the dome down one side to the horizon, and closed by three doors, of which each upper one overlaps the next

AN INTRODUCTION

TO

PRACTICAL ASTRONOMY.

CHAPTER I.

STRUCTURE OF AN OBSERVATORY.—THE TELESCOPE.

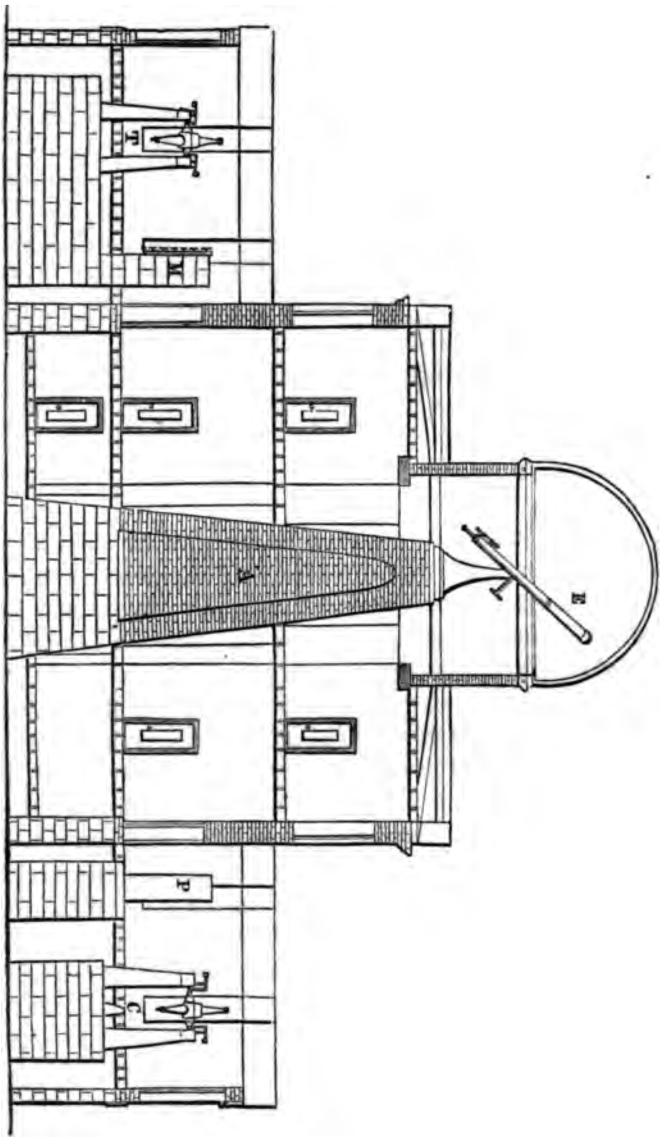
ARTICLE 1. In selecting a site for an astronomical observatory, we should aim to secure the following advantages:

1. Stability in the position of the instruments.
2. A good horizon.
3. Freedom from atmospheric obstructions.

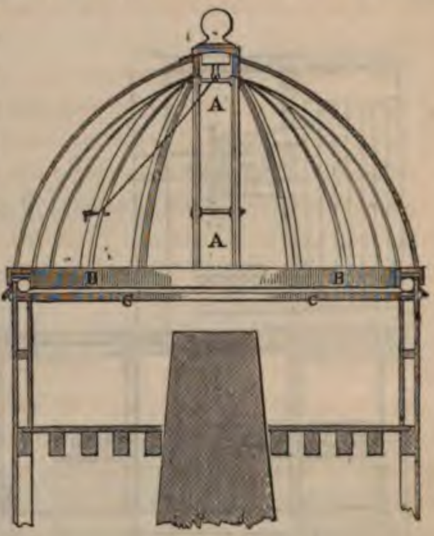
In order to secure the first advantage, we should select a spot which affords a solid foundation for building. The instruments should rest upon stone piers whose foundations are either rock, gravel, or hard clay, for which purpose it is sometimes necessary to excavate the earth to the depth of 20 or 25 feet. To prevent the transmission of tremors from the surface of the ground to the instruments, the earth should not be filled in about the piers, but the latter should be left completely insulated. It is found that ordinary tremors are but little felt a few feet below the surface of the earth.

Proximity to a large city or to great thoroughfares is most undesirable; but, if this should prove unavoidable, it is especially important to attend to the insulation of the piers.

(2.) In order to secure a good horizon, it was formerly customary to build an observatory of great height, but, for the purpose of securing greater stability of the instruments, astronomers now select an eminence of moderate elevation, from which the ground descends on all sides, and place their instruments as near


T
M
A
E
P
C

lower one, so as to exclude the rain and snow. The wooden plate, BB, which appears a straight line, is a circular ring, which forms the base of the dome; and CC is a similar ring, forming the wall plate on which the dome rests and revolves.

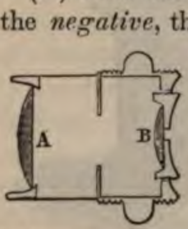


(5.) A modern observatory generally consists of a central building, of moderate elevation, surmounted by a revolving dome covering an equatorial telescope, and having small wings, running east and west, in which are placed the instruments which are designed for observations in the meridian. The sketch on page 16 represents a section of the Washington Observatory. A is a pier of solid masonry, whose foundations are nine feet below the surface of the ground. It runs through the centre of the main building, and on the top rests the equatorial, E, surmounted by a revolving dome. Both on the east and west sides of the central building is a wing, each of which has two openings 20 inches wide, extending through the roof and along the sides of the building, so as to allow an unobstructed view of the meridian. C represents the meridian circle, and T the transit instrument. The mural circle was formerly attached to the pier, M, in the west wing, but it has since been removed to the pier, P, in the east wing.

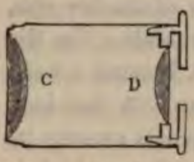
(6.) It is desirable to have access to some distant field, both north and south, where it may be permitted to erect a pillar on which to fix a meridian mark. This mark should be at such a distance that it may be distinctly seen with the solar focus of the transit instrument, which, for a small instrument, may be a distance of half a mile, but for a large instrument, may be a mile or several miles. The Royal Observatory at Edinburgh has two meridian marks, the northern one distant about 8000

This image is magnified by a microscope called an *eye-piece*, consisting of two or more lenses, and several of them are furnished with every telescope, in order to afford a variety of magnifying powers. The eye-piece is set in a sliding tube, and is moved by a milled head, connected with a rack and pinion, to enable the observer to adjust the eye-piece exactly to the image.

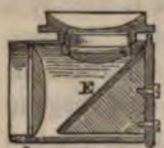
(9.) Two varieties of eye-pieces are in common use, one called the *negative*, the other the *positive* eye-piece. The negative eye-piece is formed of two plano-convex lenses, A, B, fixed with their curved faces toward the object-glass, at a distance from each other something less than half the sum of their focal lengths. It is called a negative eye-piece, because the image viewed by the eye is formed behind the inner lens, and this is the form generally used when distinct vision is the sole object.



(10.) The positive eye-piece is formed of two plano-convex lenses, C, D, having their curved faces turned toward each other, and placed at a distance from each other less than the focal distance of the lens next the eye, so that the image of the object viewed is beyond both the lenses; and this is the form adopted for the transit instrument where spider lines are placed in the focus of the object-glass, and also for telescopes with micrometers, for the piece containing the two lenses can be taken out without disturbing the lines, and is adjustable for distinct vision. As the image formed at the focus of the object-glass lies parallel to the flat face of the contiguous lens, every part of the field of view is distinct at the same adjustment, or, as opticians say, there is a *flat field*.

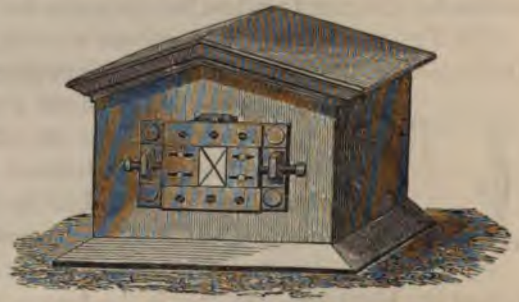


(11.) In looking through a telescope at objects in high altitudes, the head of the observer is brought into a very inconvenient position; to obviate which inconvenience, the diagonal eye-piece was invented, and is commonly applied to the transit instrument. A flat piece of polished speculum metal, E, is usually applied between the two lenses of the eye-piece, at an angle of 45°, which changes the direction of the rays of light, and forms an image which becomes erect with



feet, and the southern about 18,000. These distant marks, however, are not indispensable, and at Greenwich their use has been abandoned.

(7.) By applying to the object end of the telescope a cap, with a lens of long focus, we may employ a near meridian mark, which, in some respects, is more convenient than a distant one. The annexed figure represents a meridian mark used by Captain Smyth, of Bedford, England. A brass plate, five inches long and three inches wide, is secured by screws to a stone which has a firm foundation sunk into the ground. On this plate there slides another of smaller size, adjustable by two screws pressing against its ends. On the sliding plate is soldered a square piece of silver, bearing a well-defined black cross as a mark for the meridian. A four-inch lens, ground to a focal length of 49½ feet, which is exactly its distance from the cross, is attached to an iron plate, which is let into the south wall of the observatory, in a line with the transit instrument. The rays of light from the meridian mark consequently become parallel after passing through the lens, and the mark can be viewed through a telescope adjusted to its solar focus.



THE TELESCOPE.

(8.) The object-glass of a refracting telescope must be achromatic, consisting of two lenses so combined as to destroy the injurious effects of color and aberration. The available diameter of the object-glass is called its *aperture*, and is usually a little less than that of the tube in which it is inserted. It forms the image of an object toward which it may be directed near the eye end of the telescope. The distance from this image to the object-glass is called the focal length of the telescope, and is commonly a little greater than the length of the main tube.

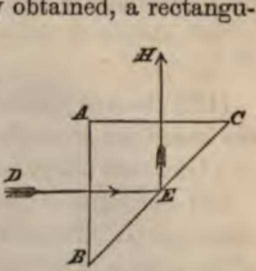
where F denotes the solar focal length of the inner, f that of the outer lens, d the distance between them, and E the focal length of the equivalent lens. Then, if we put S for the solar focal distance of the object-glass, the magnifying power will be $\frac{S}{E}$.

(15.) As it is difficult to measure exactly the focal length of the lenses, other methods of determining the magnifying power of a telescope are generally preferred.

Let the focus of the telescope be accurately adjusted to distant objects. Then, if we direct the telescope toward the light of the sky, a small bright circle will be formed near the eye-piece, which is nothing else than the image of the aperture of the telescope. If, then, we measure the diameter of this circle by means of a scale divided into very small equal parts, and likewise the aperture of the telescope, the diameter of the aperture thus determined, divided by the diameter of the bright image, will express the magnifying power of the telescope. For example, let the clear diameter of the object-glass be 10 inches, and the diameter of the small bright circle be one tenth of an inch, then will 100 represent the magnifying power of the telescope. Various contrivances have been employed for measuring the diameter of this small circle of light, but the best method is by means of *Ramsden's Dynameter*.

(16.) The following is Gauss's method of determining the magnifying power of a telescope: If we invert the telescope, and direct the eye-piece toward some distant object, then, on looking through the object-glass, the image of this object will appear as many times reduced in size as it would be magnified by the telescope if we observed through the eye-piece. We therefore direct the telescope so that two objects can be distinctly seen through the object-glass in the middle of the field of view, or at equal distances on the two sides of the optical axis. We then point a theodolite toward this telescope, so that its optical axis shall coincide nearly with the optical axis of the telescope, and measure the angle a, included between the images of the above-mentioned objects as they appear in the inverted position of the telescope. We then remove the telescope, and measure with the theodolite the angle A, which is comprehended respect to altitude, but is still reversed with respect to azimuth.

(12.) Instead of a piece of reflecting metal, that requires a surface perfectly flat, which is not easily obtained, a rectangular prism of glass is sometimes substituted. A section of the prism ABC, perpendicular to its edge, must be an isosceles right-angled triangle. If, therefore, a ray of light, DE, from the object-glass fall upon the surface, AB, of the prism perpendicularly, it will proceed without change of direction to E, will there suffer total reflection, and will pass through the side AC without deviation. The prism has this advantage over the plane speculum, that much less light is lost in the reflection.



(13.) Reflecting telescopes are of various kinds, but the two chiefly employed at present are the Newtonian and Herschelian. In the Newtonian form, the rays reflected from the large mirror at the lower end of the tube are again reflected at right angles by an inclined plane mirror, and viewed by an eye-piece on the side of the tube. The observer, accordingly, in using this instrument, looks in a direction at right angles with the tube of the telescope.

In the Herschelian construction, the large mirror is slightly inclined, so as to form the image close to one side of the tube, where the eye-piece is placed, and the observer looks down the tube with his back turned toward the object under examination. Some portion of the light from the object is necessarily intercepted by the head of the observer; but in a large instrument this loss is not very serious.

(14.) If the solar focal distance of the object-glass of the telescope be divided by the focal distance of its eye-piece, considered as a single lens, the quotient will express the magnifying power of the telescope. An ordinary celestial eye-piece consists of two lenses; so that, before we can determine the magnifying power of the telescope, we must know what single lens is equivalent to the two lenses of the eye-piece. The focal length of the equivalent lens is given by the formula $\mathrm{E}=\frac{\mathrm{F}f}{\mathrm{F}+f-d}$, between the objects themselves; the required magnifying power $= \frac{\text{tang. } \frac{1}{2} A}{\text{tang. } \frac{1}{2} a}$; and if the angles A and a are small, the magnifying power $= \frac{A}{a}$ nearly.

(17.) The magnifying power of a telescope may also be obtained in the following manner: If a disk of white paper, one inch in diameter, be placed on a black ground at 30 or 40 yards distance from the telescope, and a staff, painted white and divided into inches and parts by strong black lines, be placed vertically near the disk, the eye that is directed through the telescope, when adjusted for vision, will see the magnified disk, and the other eye, looking along the outside of the telescope, will observe the number of inches and parts that the disk projected on it will just cover; and the number of inches thus covered will indicate the magnifying power of the telescope at the distance for which it is adjusted to distinct vision.

For example, a disk of paper, one inch in diameter, was placed at a distance of 101½ feet, contiguous to a graduated vertical staff, and, when the adjustment for vision was made with a 42-inch telescope, the left eye of an observer viewed the disk projected on the staff, while the right eye observed that the enlarged image of the disk covered just 58½ inches on the staff; which number was the measure of the magnifying power P′, at the distance answering to the focal distance F′, which in this case exceeds the solar focal length F by an inch and a half. The solar power P may be obtained from the terrestrial or measured power P′ by the following proportion:

$$F' : F :: P' : P.$$

In the present case we have

$$43.5 : 42 :: 58.5 : 56.5 \text{ nearly.}$$

Hence the magnifying power due to the solar focal length of the telescope is 56.5.

(18.) Every telescope of considerable magnifying power should be furnished with a *finder;* that is, a small telescope of a low power and a large field of view, attached to the side of the larger, with their axes parallel to each other. In the common focus of the object-glass and eye-glass is a pair of coarse wires, intersecting each other in the middle of the field. A telescope with a high magnifying power has a very small field of view, and therefore an observer may have great difficulty in finding a small object for which he is searching. This inconvenience is obviated by the finder. The telescope is pointed approximately toward a star by glancing the eye along the tube, when the star will be seen in the finder, because its field of view is very large. The object is then brought into the middle of the field of the finder, which is indicated by the intersection of the wires, when it will be somewhere in the field of the larger telescope.

(19.) In order to judge of the excellence of a telescope, we should examine the quality of the glass, and also the accuracy with which the chromatic and spherical aberrations are corrected. We may judge of the achromatism by directing the telescope to the moon or to Jupiter, and alternately pushing in and drawing out the eye-piece from the place of distinct vision. In the former case, a ring of purple will be formed round the edges; and in the latter, a ring of light green, which is the central color of the prismatic spectrum; for these appearances show that the extreme colors, red and violet, are corrected.

(20.) We may test the figure of the object-glass by covering its centre by a circular piece of paper, about one half of its diameter, and adjusting it for distinct vision of a given object, and then trying if the focal length remains unaltered when the paper is taken away, and a cap with an aperture of the same size is applied, so that the extreme rays may in their turn be cut off. If the vision is distinct in both cases, without any new adjustment for focal distance, the spherical aberration is corrected.

(21.) If one part of the object-glass have a different refractive power from another part, a star of the first magnitude will point out the defect by the exhibition of an irradiation, or what opticians call a *wing*, at one side, which no perfection of figure or of adjustment will banish; and the greater the aperture, the more liable is the evil to happen. Hence caps with different apertures are usually supplied with large telescopes, that the extreme parts of the glass may be cut off in observations requiring a well-defined image. In case one half of the glass be faulty and the other good, a semicircular aperture, by being turned gradually round, will detect the semicircle which contains the defective portion of the glass; and if such portion should be covered, the only inconvenience that would ensue would be the loss of the light which is thus excluded.

(22). The most precise mode of estimating the capacity of a telescope is by observations of a series of test objects in the heavens. These objects should be selected with reference both to illuminating power and defining power, which qualities are quite distinct from each other. The usual tests of illuminating power are stars of such a degree of faintness as barely to come within the range of the telescope; and the tests of defining power are double stars, as close to each other as can be distinctly seen separated. The following list of close double stars will afford a considerable range of tests for defining power:

Star.	R. A., 1860.			Dec., 1860.			Magnitudes.		Distance.
	h.	m.	s.	°	′				″
86 Andromedæ	0	48	32	22	58.8	N.	6.2	6.8	1.3
42 Ceti	1	13	41	1	8.3	S.	6.2	7.2	1.2
α Piscium		55	50	2	11.1	N.	2.8	3.9	3.1
γ² Andromedæ		56	32	41	45.2	N.	5.5	6.0	0.5
ι Cassiopeæ	2	19	10	66	51.7	N.	4.2	7.1	2.0
γ Ceti		37	5	2	43.8	N.	3.0	6.8	2.7
ε Arietis		52	21	20	51.6	N.	5.7	6.0	1.3
7 Tauri	3	27	20	24	2.7	N.	7.0	7.1	0.4
η Orionis	5	18	27	2	30.4	S.	4.0	5.0	1.1
32 Orionis		24	22	5	51.4	N.	5.0	7.0	0.4
11 Monocerotis	6	23	0	6	57.3	S.	5.5	6.0	2.5
12 Lyncis		35	38	59	33.7	N.	5.2	6.1	1.5
170 P. Canis Minoris	7	33	45	5	30.4	N.	7.0	7.3	1.6
ζ Cancri	8	5	19	18	0.6	N.	5.0	5.7	0.7
ω Leonis	9	22	2	9	34.7	N.	6.0	7.0	0.4
γ Leonis	10	13	20	20	26.9	N.	2.0	3.5	3.2
ξ Ursæ Majoris	11	11	48	32	12.3	N.	4.0	4.9	1.1
35 Comæ Berenicis	12	47	23	21	53.9	N.	5.0	7.8	1.4
42 Comæ Berenicis	13	4	9	18	9.9	N.	6.0	6 0	0.2
127 P. Virginis		28	10	0	17.9	N.	8.0	9.0	2.3
ζ Bootis	14	35	25	14	14.6	N.	3.5	3.9	1.0
ε Bootis		39	45	27	34.8	N.	3.0	6.3	2.6
η Coronæ Borealis	15	18	15	30	43.3	N.	5.5	6.6	0.8
μ² Bootis		19	58	37	47.9	N.	6.7	7.8	0.7
γ Coronæ Borealis		37	42	26	40.7	N.	4.0	7.0	0.2
ξ Libræ		57	46	11	2.5	S.	4.9	5.2	1.1
λ Ophiuchi	16	24	52	2	14.9	N.	4 0	6.0	1.6
ζ Herculis		36	46	31	49.3	N.	3.0	6.0	1.2
20 Draconis		55	49	65	13.3	N.	6.5	7.0	0.2
τ Ophiuchi	17	56	38	8	10.7	S.	5.0	6.0	1.9
73 Ophiuchi	18	3	36	3	58.5	N.	6.0	7.2	1.2
ε¹ Lyræ		40	22	39	32.7	N.	4.6	6.3	3.0
ε² Lyræ		40	24	39	29.3	N.	4.9	5.2	2.6
ε Draconis	19	48	34	69	57.7	N.	4.0	7.6	2.8
178 P. Delphini	20	25	26	10	51.4	N.	8.0	9.0	0.5
4 Aquarii		45	4	6	4.5	S.	6.0	7.0	0.4
ε Equulei		53	5	3	50.0	N.	5.2	6.2	0.1
37 Pegasi	22	23	54	3	50.4	N.	6.0	7.5	0.5

(23.) As tests of illuminating power may be mentioned the satellite of Neptune, which is equal to a star of the fourteenth magnitude, and is not known to have been seen by more than three or four telescopes in any part of the world; the satellites of Uranus, which are about equally difficult; and the three smaller satellites of Saturn. The new satellite of Saturn, discovered by Mr. Bond, is estimated to be equal to a star of the seventeenth magnitude.

The following table of faint and unequal stars will also afford a good measure of illuminating power:

Star.	R. A., 1880.			Dec., 1880.			Magnitudes.		Distance.
	h.	m.	s.	°	′				″
34 Piscium..............	0	8	52	10	28	N.	6.0	13.5	7.0
42 Piscium..............		16	13	12	49	N.	6.8	10.7	29.7
51 Piscium..............		26	12	6	18	N.	5.0	9.0	27.4
η Cassiopeæ.............		41	43	57	11	N.	4.0	7.6	5.3
φ Piscium...............	1	7	14	23	57	N.	4.7	10.1	8.0
Polaris.................		13	45	88	40	N.	2.0	9.0	18.3
ν Ceti..................	2	29	34	5	4	N.	4.5	15.0	6.0
84 Ceti.................		35	4	1	12	S.	6.0	9.2	4.8
θ Persei................		35	59	48	43	N.	4.0	10.0	16.6
41 Arietis..............		42	55	26	46	N.	3.5	11.5	18.8
ζ Persei................	3	46	35	31	32	N.	2.7	9.3	12.5
7 Camelopardi...........	4	47	40	53	34	N.	4.2	11.3	26.0
ρ Orionis...............	5	7	1	2	43	N.	4.7	8.5	7.0
β Orionis...............		8	46	8	20	S.	1.0	8.0	9.1
τ Orionis...............		11	47	7	8	S.	4.0	11.0	36.0
λ Geminorum.............	7	11	12	16	45	N.	4.5	12.0	10.3
δ Geminorum.............		12	57	22	12	N.	3.2	8.2	7.1
κ Geminorum.............		37	15	24	41	N.	4.0	9.0	6.4
γ Crateris..............	11	18	53	17	1	S.	4.0	14.0	3.0
φ Virginis..............	14	22	1	1	41	S.	5.2	10.0	4.3
Antares.................	16	22	2	26	10	S.	1.0	8.0	3.2
α Lyræ..................	18	32	52	38	41	N.	1.0	8.8	48.1
17 Lyræ.................	19	2	53	32	18	N.	5.7	9.8	3.9
δ Cygni.................		41	13	44	50	N.	3.0	8.0	1.6
α^2 Capricorni...	20	11	24	12	55	S.	3.0	12.0	5.0
δ Equulei...............	21	8	38	9	32	N.	4.5	10.0	37.6
41 Aquarii..............	22	7	40	21	40	S.	6.0	8.5	4.1
ξ Pegasi................		40	42	11	34	N.	5.0	15.0	15.0
σ Cassiopeæ.............	23	52	56	55	5	N.	5.4	7.5	3.0

(24.) Some of the nebulæ afford excellent tests of the performance of a telescope. Certain nebulæ, which, to an ordinary telescope, appear merely as a dim patch of light, by more powerful instruments are resolved wholly into stars, and others are partially resolved.

The following are some of the most interesting objects of this class:

	R. A., 1850.			Dec., 1850.		
	h.	m.	s.	°	′	
Nebula of Andromeda	0	34	37	40	26.9 N.	Large and irresolvable.
33 H. Persei	2	8	39	56	27.1 N.	Glorious mass of stars.
19 H. Andromedæ	2	13	12	41	38.9 N.	Irresolvable.
60 H. Persei	3	58	55	49	6.2 N.	Compressed oval group.
Crab nebula	5	25	27	21	54.7 N.	Large oval nebula.
Nebula in Orion	5	27	54	5	29.5 S.	Great irresolvable nebula.
2 H. Geminorum	6	46	31	18	9.8 N.	A compressed cluster.
67 M. Cancri	8	42	58	12	21.4 N.	Loose cluster.
27 H. Hydræ	10	17	29	17	53.6 S.	Fine planetary nebula.
97 M. Ursæ Majoris	11	5	59	55	49.7 N.	Planetary nebula.
60 M. Virginis	12	36	3	12	22.8 N.	A double nebula.
64 M. Comæ Berenicis	12	49	21	22	30.0 N.	Not resolved.
53 M. Comæ Berenicis	13	5	32	18	58.1 N.	A globular cluster.
Spiral nebula	13	23	31	47	58.6 N.	Irresolvable pair of nebulæ.
3 M. Canum Venaticorum	13	35	12	29	7.3 N.	A globular cluster.
70 H. Virginis	14	21	44	5	17.8 S.	Resolved in largest telescopes.
5 M. Libræ	15	10	56	2	39.0 N.	A mass of stars.
13 M. Herculis	16	36	19	36	44.6 N.	A splendid cluster.
10 M. Ophiuchi	16	49	16	3	52.8 S.	A rich round mass.
17 M. Clypei Sobieskii	18	11	57	16	15.8 S.	Horse-shoe nebula.
28 M. Sagittarii	18	15	17	24	56.7 S.	Compact globular cluster.
Annular nebula in Lyra	18	47	59	32	50.8 N.	Annular and irresolvable.
56 M. Lyræ	19	10	42	29	55.2 N.	Globular cluster.
Dumb-bell nebula	19	53	4	22	18.7 N.	Irresolvable.
103 H. Delphini	20	26	50	6	55.2 N.	A mass of minute stars.
15 M. Pegasi	21	22	42	11	30.0 N.	Globular cluster.
2 M. Aquarii	21	25	40	1	29.5 S.	A ball of stars.
30 M. Capricorni	21	31	50	23	49.8 S.	A bright cluster.

Many of the preceding nebulæ are quite conspicuous, even with a small telescope; but the visible boundaries of these objects, and the number of stars which they exhibit, depend upon the power of the instrument.

(25.) The following statement, by Captain Smyth, will afford to beginners a tolerable idea of what kind of performance they ought to expect from their telescopes. Captain Smyth's telescope was an achromatic refractor of $8\frac{1}{2}$ feet focal length, with an object-glass of $5\frac{9}{10}$ inches clear aperture. The cap which covered the object-glass was pierced with two circular holes, of two inches and four inches diameter. With the two-inch aperture, and magnifying powers of from 60 to 100, he saw Polaris and its companion distinctly, and clearly perceived double

α Piscium,
γ Leonis,
μ Draconis,
η Cassiopeæ.

With the four-inch aperture, and powers varying from 80 to 120, and upward, he readily saw

β Orionis,
α Lyræ,
δ Geminorum,
ξ Ursæ Majoris,
σ Cassiopeæ,
γ Ceti,
ε Draconis,
ι Leonis.

But it required the full aperture, and powers of from 240 to 300, with favorable circumstances, to scrutinize satisfactorily the following test objects:

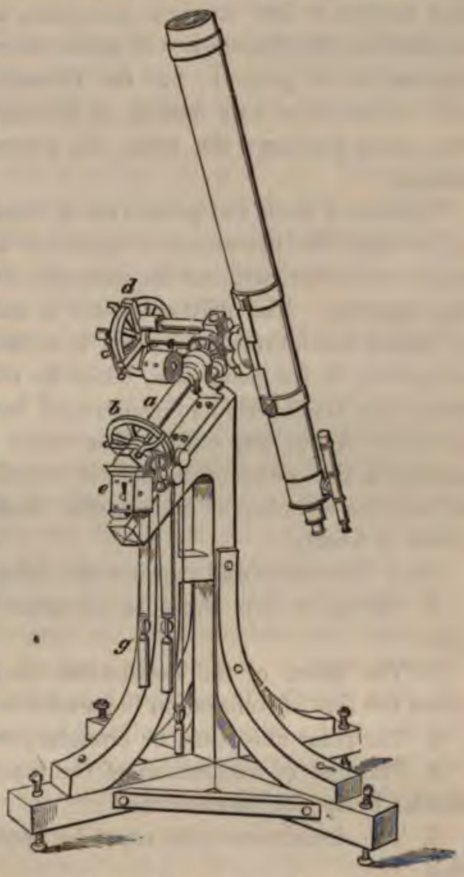
α Arietis,
λ Ophiuchi,
ε Equulei,
δ Cygni,
20 Draconis,
ζ Herculis,
32 Orionis,
κ Geminorum.

EQUATORIAL TELESCOPE.

(26.) An equatorial telescope consists of a telescope so mounted as to have two axes of motion at right angles to each other, and also a graduated circle connected with each axis, at right angles to its length. When the instrument is adjusted for use, one axis is parallel to the axis of the earth, and is called the polar axis; the other is parallel to the plane of the equator, and is called the declination axis. When a telescope is mounted with an altitude and azimuth movement (as is the case with common portable instruments), it requires a motion in altitude as well as in azimuth to follow a star in its diurnal course; and these movements are sufficient to occupy both hands of the observer. But with a telescope mounted equatorially, only one motion is required to follow a star; that is, a motion parallel to the plane of the equator; and this motion being perfectly uniform, can easily be effected by clock-work; by which means the observer has both his hands at liberty to use a micrometer, or for any other purpose. Such an instrument, therefore, affords great advantages in measuring the relative position of two contiguous bodies, in measuring the diameters of the planets, etc. The circle which is connected with the polar axis is graduated into hours, minutes, and seconds of time, to indicate the right ascension of the object under examination; while the circle connected with the declination axis is graduated into degrees, minutes, and seconds of arc, to indicate declination or polar distance.

(27.) The mode of mounting now generally preferred is that employed by Fraunhofer, and is represented in the annexed cut.

a represents the polar axis parallel to the axis of the earth; *b* is the right ascension circle attached to the supporting frame, while two verniers, attached to the polar axis, and revolving with the telescope, point out the right ascension of a star upon this circle. The axis *c* is the declination axis, at right angles with the polar axis, and is mounted, so as to revolve in its supports; while the declination circle, *d*, indicates the declination of the object under examination. When the right ascension circle is clamped, the declination axis may be made to revolve through 360°, by which means the telescope will be pointed successively to every degree of declination. When the declination circle is clamped, the polar axis may be made to revolve through 360°, by which means the telescope will describe a complete circle of diurnal motion. Thus, by means of these two motions, which are at right angles to each other, the telescope may be turned toward any part of the heavens. Indeed, there are always two positions of the instrument, with reference to the polar axis, in which the telescope may be pointed upon any star. If we suppose the tele-



scope to be in the position represented in the preceding cut, and revolve it 180° in right ascension, and also about 180° in declination, the telescope will point toward the same part of the heavens as at present; but the telescope will be on the west side of the polar axis instead of the east side. One is called the direct position; the other, the reversed position of the telescope.

Connected with the polar axis is clock-work, represented at *e*, by which the instrument is turned, so as to follow the diurnal motion of a star, without the necessity of any interference from the observer. The driving power is the descent of a weight, *g*, which communicates motion to a train of wheel-work, and ultimately to the polar axis, while its too swift descent is regulated by the friction of centrifugal balls. This contrivance serves to retain any object upon which the telescope may be pointed in the centre of the field of view for hours in succession, leaving the attention of the observer undistracted, and both his hands at liberty.

(28.) The equatorial requires the following adjustments:

1. The polar axis must be elevated to the altitude of the pole.

2. The index of the declination circle must point to zero when the line of collimation is parallel to the equator.

3. The polar axis must be brought into the meridian.

4. The line of collimation of the telescope must be perpendicular to the declination axis.

5. The declination axis must be perpendicular to the polar axis.

6. The index of the hour circle must point to zero when the telescope is in the meridian of the place.

(29.) *First Adjustment.*—Observe the polar distance of any known star when near the meridian, and then, turning the polar axis half round, observe the same star again. Take the mean of the two observations, which is the distance of the star from the pole of the instrument; correct it for refraction, and compare the result with the true north polar distance given by the Nautical Almanac. If the star is above the pole, and the instrumental exceeds the true polar distance, the pole of the instrument is below the pole of the heavens, and *vice versâ*.

Correct this error by the proper screws for raising or depressing the polar axis.

Example. When ε Ursæ Minoris was near the meridian, its north polar distance was observed to be 7° 44′ 7″, the face of the declination circle being west; and 7° 44′ 40″ when the face of the circle was east.

The mean of these two observations is 7° 44′ 23″.5; the refraction was 52″.8; making the corrected polar distance 7° 43′ 30″.7. The polar distance by the Nautical Almanac was 7° 42′ 40″.7. Hence the polar axis was 50″ too low. The refraction is derived from Table VIII., page 364.

(30.) *Second Adjustment.*—Take half the difference of the above two observations; this will be the index error of the declination verniers, and they must be moved accordingly by their adjusting screws. Several pairs of observations should be taken, in order to ascertain these errors with great accuracy.

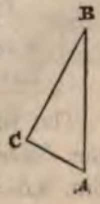
Example. According to the observations above given, the index error was 16″.5, to be added to observations when the circle is west. See Art. 61 respecting line of collimation.

(31.) *Third Adjustment.*—Observe the polar distance of a star which is six hours from the meridian, the star being not very near the pole, nor yet near the horizon. Correct this for refraction in polar distance, and compare the result with the true polar distance from the Nautical Almanac. If the star is to the east of the meridian, and the instrumental exceeds the apparent polar distance, the north pole of the instrument is to the west of the celestial pole.

Example. The polar distance of α Ursæ Majoris, when six hours west from the meridian, was observed to be 27° 23′ 49″.0, the face of the circle being west. Correcting this for index error found above, 16″.5, and for refraction 30″.8, the result is 27° 24′ 36″.3. The polar distance by the Nautical Almanac is 27° 24′ 43″.7. Hence the pole of the instrument was 7″.4 west.

(32.) The influence of refraction upon the right ascension and declination of a star may be computed as follows:

Let A represent the true position of a star, B its apparent position affected by refraction; then AB represents the refraction in altitude. Let BC represent



a portion of an hour circle passing through B, and let AC be an arc perpendicular to BC. Then ABC may be regarded as a plane right-angled triangle, in which AC represents the effect of refraction in right ascension, and BC the effect in declination.

Now $AC = AB\ \sin. ABC$; and $BC = AB\ \cos. ABC$.

The angle ABC, which we will represent by p, is called the parallactic angle, and the mode of computing it is shown in Art. 145. Hence the refraction in right ascension is equal to the refraction in altitude, multiplied by the sine of the parallactic angle; and the refraction in declination is equal to the refraction in altitude, multiplied by the cosine of the parallactic angle. The refraction in right ascension is here expressed in parts of a great circle; if we wish to reduce it to arc of right ascension, we must divide this result by the cosine of the star's declination, as shown in Art. 72. Hence we have

$$\text{Refraction in R. A.} = \frac{\text{ref. in alt.} \times \text{sin. } p}{\text{cos. dec.}}.$$

$$\text{Refraction in Dec.} = \text{ref. in alt.} \times \text{cos. } p.$$

(33.) *Fourth Adjustment.*—Observe the transit of an equatorial star over the middle vertical wire, or mean of the wires; note the time, and read off the verniers of the hour circle. Turn the polar axis half round, and observe the same star a second time exactly as before. Now the interval between the two observations should correspond exactly to the difference between the two readings of the hour circle. If they do not correspond, it is evident that one of the transits has been observed too early, and the other too late, on account of the erroneous position of the wires. One half the difference between the interval as measured by the clock, and that by the hour circle, will be the error of collimation.

Example. The following observations were made upon δ Ophiuchi:

Face of Circle.	Sidereal Time.	Hour Circle.
	h. m. s.	h. m. s.
West . . .	16 12 58.8	0 6 51.0
East . . .	16 19 9.7	0 13 0.5

The interval between the two observations is 6m. 10.9s.; the difference between the two readings of the hour circle is 6m. 9.5s. One half the difference is 0.7s., which is the error of collimation to be added to the readings of the hour circle when the circle is east.

(34.) *Fifth Adjustment.* — The declination axis should be placed by the maker perpendicular to the polar axis, and, having been once accurately adjusted, is not liable to subsequent derangement. The accuracy of this adjustment may be tested as follows: Bring the declination axis into a horizontal position by means of a spirit-level, whose legs rest upon the extremities of the declination axis, and read the hour circle. Turn the polar axis half round; bring the declination axis into a horizontal position by means of the level, as before, and again read the hour circle. If the readings agree in both positions, or differ by 12h. (when the graduation is to 24h.), the declination axis is adjusted. If the readings do not agree, the declination axis is not perpendicular to the polar axis. If the declination axis is furnished with adjusting screws, place the hour circle half way between the position it actually has and that which it ought to occupy, in order that the readings may differ by exactly twelve hours, and make the declination axis horizontal by raising or depressing the proper screws.

(35.) This adjustment may be tested astronomically as follows: Observe the transit of a star not less than 45° from the equator, in both positions of the polar axis, as directed for the fourth adjustment. Since an elevation of the west end of the declination axis causes the line of sight to describe a circle to the east of the pole, all the transits observed in that position will be too early; and, *vice versâ*, all will be too late when the east end is high. Again, if the west end is too high before reversing, the east end is too high after reversing, so that an error of inclination has a different effect upon observations in reversed positions, and thus the interval is increased or diminished by twice the error of a single observation. The law of the error is, that it varies as the tangent of the star's declination. If we represent the interval between the observations, as measured by the clock, by c, and the interval, as measured by the hour circle, by h, then

$$\frac{c-h}{2 \text{ tang. dec.}}$$

will be the error in the position of the declination axis.

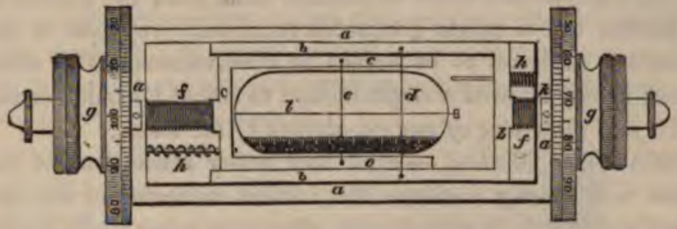
(36.) *Sixth Adjustment.*—Set the declination axis horizontal by means of the level, when, if the previous adjustments have been properly performed, the instrument will be in the meridian, and the verniers may be set to zero. Or, clamp the instrument approximately in the meridian, observe the transit of one or more known stars not far from the equator, and correct the time of observation for the error of the clock. Then, since the right ascension of the star = the true sidereal time of observation, ± the true hour angle from the meridian, the true hour angle is known, and the verniers may be set to mark it.

If it is proposed to determine the absolute place of a heavenly body by means of the equatorial, it is necessary to determine its errors with great accuracy; but this instrument is chiefly employed for determining *differences* of right ascension and declination of objects very near each other, in which case entire accuracy in all the adjustments becomes of less importance.

THE MICROMETER.

(37.) The object of the micrometer is to measure small celestial arcs in the field of view of a telescope. It appears under a great variety of forms; but the one now most commonly employed is called the spider-line micrometer, or filar micrometer. It consists essentially of two parallel spider lines inserted in the common focus of the object-glass and eye-glass, in such a manner that they may be made to coincide or be separated by the slow motion of a screw. The number of revolutions and parts of a revolution of the screw are indicated by a scale outside of the tube, and this affords a measure of the distance of the spider lines, or of any two celestial objects with which they may be made to coincide.

(38.) The figure on the opposite page represents the spider-line micrometer, as made by Troughton. It consists of a rectangular box, three or four inches long, about one inch broad, and a quarter of an inch in thickness, with a graduated screw-head at each extremity; *aaaa* are the sides of the box seen edgewise; *bbb*, *ccc* are two forks of brass, which slide one within the other, in opposite directions, and across them are stretched the spider lines *d* and *e*; *ff* are fine screws attached to the forks, and, passing through the ends of the box, enter the milled heads *gg*, with each of which is connected a small graduated circle. Whenever the heads *gg* are turned in the direction of the numbers upon the circles, the forks *bc* are drawn outward; and when they are turned in the contrary direction, the springs *hh* push the forks inward, and thus prevent any loss of motion in the screw. The screws have about 100 threads to the inch, so that one revolution of the head *g* carries the line *d* over the hundredth part of an inch. The circumference of the circle attached to the head *g* is divided into a hundred equal parts, so that the motion of the head *g* through one of these divisions advances the line *d* through one ten thousandth part of an inch. The long line *l*, running at right angles to the small ones, should be placed parallel to the two objects whose distance is to be measured.



On one side of the field of view is a notched scale of teeth, corresponding in size to the threads of the screw. Every fifth one is cut deeper than the rest, and they are numbered from zero, at the centre, by tens, in each direction. The spider lines may be made to coincide at zero, which is represented by the small circular hole made near the middle of the scale, and they may be made to glide by each other a short distance, *e* passing very close under *d*.

(39.) In order to measure any distance, as, for example, the sun's diameter, turn one of the heads until the attached line is drawn 15 or 20 notches to the left of zero, and the other head in the contrary direction, until *d* may be made to touch one limb while *e* touches the other. Read off in the field of view how many notches have been passed over by each wire, and the fractional part of a revolution, on the divided heads. The sum of the two quantities will give the whole number of revolutions and parts indicated. If the distance to be measured is small, it will only be necessary to move one of the lines while the other remains at zero.

The micrometers made at Munich differ from the preceding in having but a single graduated head, and only one of the threads is designed to be moved in observations. The other thread is only allowed a slight motion to adjust for index error.

To find the value of one revolution of the screw.

(40.) *First Method.*—Find how many revolutions of the screw will exactly measure the vertical diameter of the sun when his altitude is considerable, allowance being made for the difference of refraction of the upper and lower limbs. The whole diameter, as given by the Nautical Almanac, reduced to seconds, and divided by the number of revolutions and decimal parts observed on the head of the screw, will give the value of a single revolution.

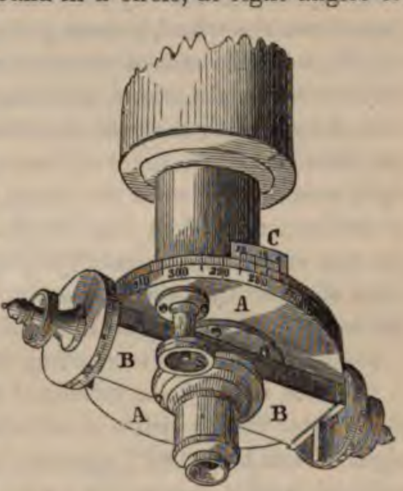
Example. When the sun's altitude was 40° 30′, his vertical diameter was measured by 40.98 revolutions of the micrometer screw. The sun's diameter, according to the Nautical Almanac, was 31′ 31″.2. The refraction of the lower limb was computed to be 1″.2 greater than that of the upper limb; hence the apparent vertical diameter of the sun was 1890″. Therefore, the value of one revolution of the screw was

$$\frac{1890}{40.98}=46''.12.$$

(41.) *Second Method.*—Separate the two lines by any convenient number of revolutions, and observe the time required by an equatorial star to pass from one line to the other. We thus obtain the interval between the two lines in seconds of time, which, divided by the number of revolutions, gives the value of one revolution of the screw. This result will be the more reliable, the greater the distance between the lines, because the unavoidable error in estimating a fraction of a second of time is reduced by a larger divisor; and the observation should be repeated until a satisfactory result is obtained. The same result may be obtained from a star situated out of the equator, by reducing the observed interval, in the manner described in Art. 72. This method will be illustrated by an example with the transit instrument, Art. 73.

POSITION MICROMETER.

(42.) It is often required to measure the angle which the line joining the centres of two stars makes with the meridian. This is effected by means of rack-work and a screw, carrying the spider-line micrometer round in a circle, at right angles to the axis of the telescope, and the motion is measured by means of a graduated circle.



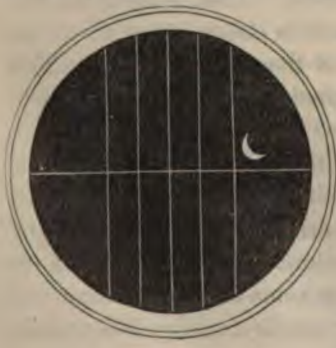
The annexed figure represents the spider-line micrometer attached to the end of a telescope, and having, besides, a graduated circle, AA, with a milled head, S, acting upon a concealed rack, by which the micrometer, BB, may be made to revolve entirely round the axis of the telescope. This motion is measured upon the circle AA by means of the fixed index C.

In order to measure the angle of position of two stars, point the telescope upon one of the stars, and turn the micrometer until the line *l* (see figure, page 33) is made to bisect the star during the whole time of its crossing the field of view; this line will then be parallel with the equator. Let the index of the position circle now be put to zero. Then revolve the micrometer until the line *l* can be made to bisect both stars at the same time, and note the reading of the position circle. This will be the angle of position measured from a parallel of declination, as practiced by Sir William Herschel.

(43.) An equatorial, furnished with a micrometer, affords the most convenient means of determining the position of a comet, by comparing it with some neighboring star. The method of observation is as follows: The equatorial having been previously adjusted, point the telescope upon some convenient star, and make the wire *l* of the micrometer (see figure, page 33) parallel to the equator, which is known to be the case when a star will travel along the wire during its passage through the field of the telescope. Then turn the circle 90°, and the line *l* will be perpendicular to the equator. Now point the telescope upon the comet, and, having clamped both the hour and declination circles very firmly, note by the clock the time when the comet passes over the wire *l*, bisecting it at the same time by the wire *e*. Wait till the star of comparison passes over the field, note its transit over the wire *l*, and bisect it in declination with the wire *d*, by turning the head, *g*, of the micrometer screw. Then the difference of the times of observation gives the difference of right ascension between the comet and star, and the difference of declination is taken from the micrometer. If the place of the star of comparison is not already known, it must be afterward observed by meridian instruments, and then the place of the comet is deduced with the greatest accuracy.

(44.) *Method of illuminating the Lines.*

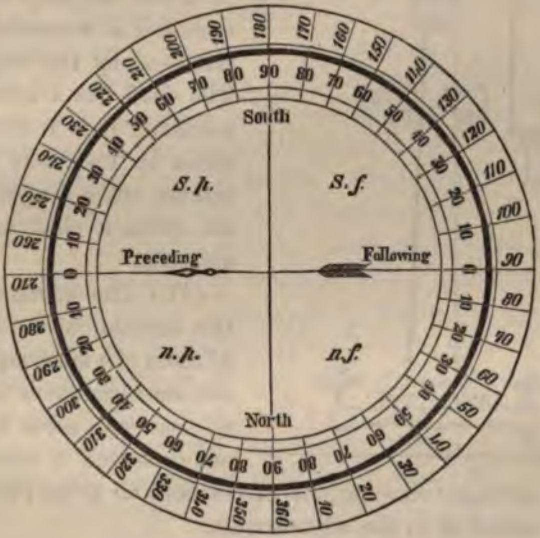
Hitherto it has been presumed that the spider lines in the micrometer will be visible in all celestial observations, made by night as well as by day; whereas, in all nocturnal observations, artificial light is required to render the lines of the micrometer visible. This light is supplied by opening a circular hole in the side of the main tube, before which a lamp is suspended, and placing an oval ring of gilt metal, deadened so as to reflect a mitigated light up the telescope, at a proper angle of inclination within the tube. In order to regulate the quantity of light, so as not to conceal faint stars from view, variable diaphragms may be interposed, or darkening glasses of different shades of color.



By means of a small lamp, fitted to an aperture in the tube next to the eye-piece, the wires may be illuminated while the field remains dark, thus enabling the observer to have bright lines and a dark field, or a bright field and dark lines, at pleasure.

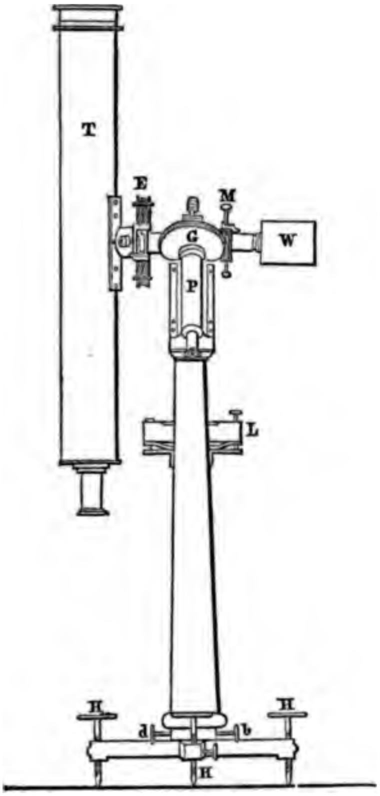
The annexed diagram represents the illuminated lines as they are usually arranged in a small transit instrument, with a star just going off on the left side, and the planet Venus approaching to the first wire.

(45.) Sir William Herschel was accustomed to distinguish angles of position by the terms *north*, *following*, *south*, *preceding*; which words were designated by the initials *n*, *f*, *s*, *p*; and he measured angles only up to 90°; beginning at the preceding and following points, and reading each way, north and south, up to 90°. It is now more common to measure angles of position from the north point by the east, round to 360°. The following diagram shows both forms, as used in the reversed field of a telescope.



COMET-SEEKER.

(46.) The comet-seeker is a telescope having an object-glass of large aperture and short focal length, with which low magnifying powers are used, that it may have a large field of view, and collect the greatest possible amount of light. The figure on the next page represents such an instrument mounted equatorially. It rests upon a tripod, with foot screws, H, H, H, for leveling. From the tripod rises a vertical shaft, whose upper extremity is enlarged for the support of an axis, P, parallel to the axis of the earth. To this is attached an hour circle, G, graduated to hours, minutes, and seconds; and upon its edge are cut threads, to receive an endless screw, M, which communicates a slow motion about the axis. At right angles to the polar axis is the declination axis, with its circle, E, divided into degrees and minutes, and having also a tangent screw for slow motion. The telescope tube, T, is of the ordinary construction, and is accurately counterpoised by the weight W. The shaft has a level, L, for adjusting it to a vertical position by means of the foot screws, and a tangent screw, *bd*, gives a slow motion in azimuth.



(47.) The adjustments of this instrument are the same as those of a large equatorial; but inasmuch as it is designed merely to scour the heavens in search of comets, and not for accurate observations of position, no great precision is usually aimed at in the adjustments.

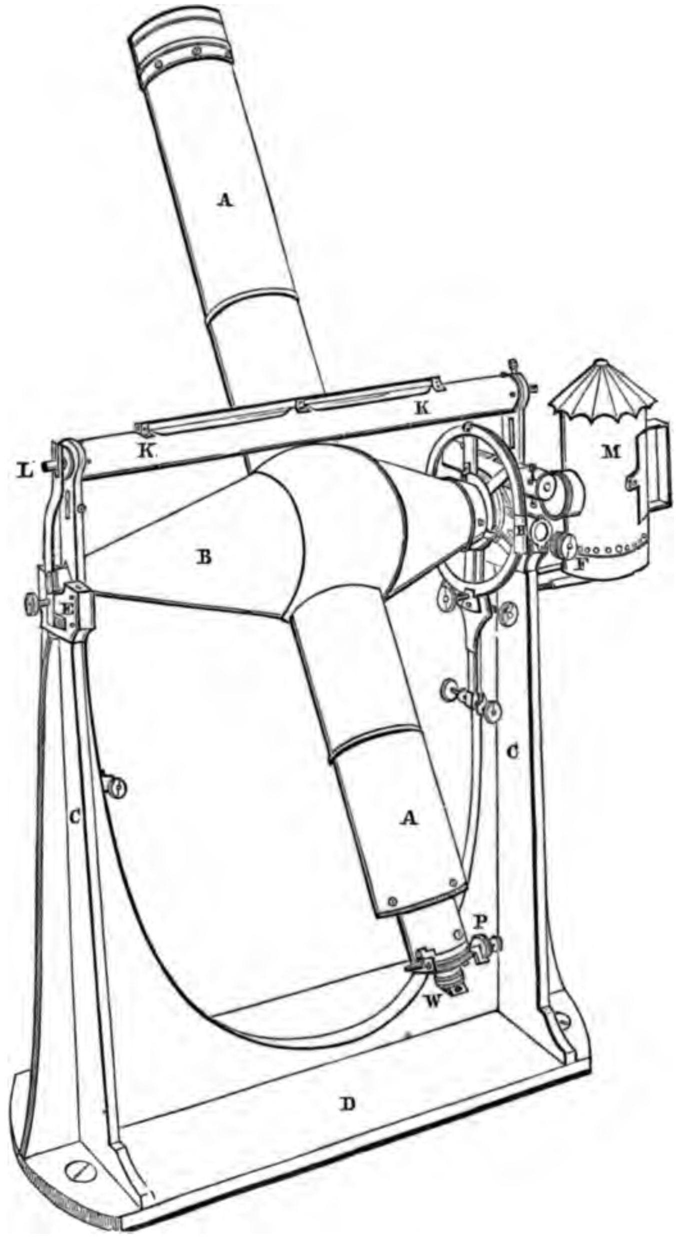
CHAPTER II.

THE TRANSIT INSTRUMENT.

(48.) The transit is a meridional instrument, employed, in connection with a clock or chronometer, for observing the passage of celestial objects across the meridian, either for obtaining correct time or determining their right ascensions.

(49.) The figure on the next page represents a portable transit instrument. The telescope tube, AA, is supported upon an axis, BB, placed at right angles to the direction of the telescope. This axis terminates in two cylindrical pivots which rest in Y's, which are strongly united to the two uprights CC. The stand CDC, carrying the Y's, is made of cast-iron, and should be made of a single piece, in order to secure a steady and permanent position. At the left end of the axis there is a screw, E, by which the Y of that extremity may be raised or lowered a little, in order that the axis may be made perfectly horizontal. At the right end of the axis is a screw, F, by which the Y of that extremity may be moved backward or forward, in order to enable us to bring the telescope into the plane of the meridian. Near the right end of the axis is fixed a circle, G, which turns with the axis, while the vernier, H, remains stationary in a horizontal position, and shows the altitude to which the telescope is elevated. The vernier is set horizontal by means of an attached spirit level, I. The level KK rides on the pivots of the axis. There is a pin at each end, which drops into a fork at L, to hold the level safely and upright. At the left end is the adjustment for setting the level tube parallel with the axis. At the other end is an adjustment for raising or depressing the extremity of the level.

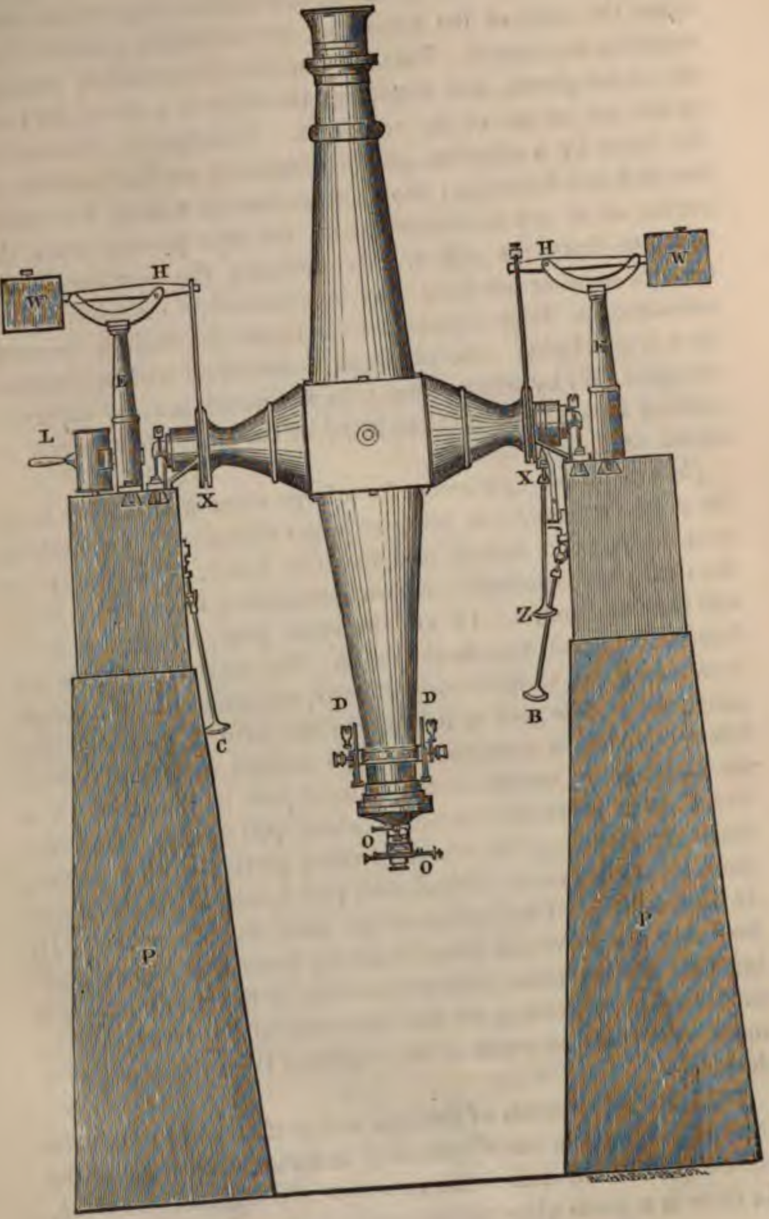
(50.) Near the eye end, and in the principal focus of the telescope, is placed the diaphragm or wire plate, carrying five or seven vertical wires, and two horizontal ones, between which the star is observed. The central vertical wire ought to be fixed in the optical axis of the telescope, and perpendicular


A
K
K
L
M
B
E
F
C
C
A
P
W
D

with respect to the pivots of the axis. These wires are rendered visible in the day time by the diffuse light of day, which penetrates the tube of the telescope; but at night, artificial illumination is required. This illumination is effected by piercing one of the pivots, and admitting the light of a lamp, M, fixed on the top of one of the standards. This light is directed to the wires by a reflector, placed diagonally at the junction of the axis and telescope; the reflector having a large hole in its centre, so as not to interfere with the rays passing down the telescope from the object. By inclining the opening of the lantern, more or less light may be admitted to the telescope, to accommodate faint objects, which might be entirely eclipsed by a bright light. The telescope is furnished with a diagonal eye-piece, W, by which stars near the zenith may be observed without inconvenience. The head of the micrometer screw is shown at P.

(51.) A transit instrument for a large observatory differs from the preceding chiefly in being of larger dimensions, and resting upon stone piers instead of a movable frame. The figure on the next page represents such an instrument, as made by Ertel and Son, of Munich. PP are the stone piers, which rest upon foundations sunk deep in the earth. The axis of the telescope is made strong, so as to resist flexure, and was cast in a single piece, the middle part of it being in the form of a cube. The telescope tube is composed of two conical frustums, which are fastened by screws to the cubical part of the axis. The weight of a seven-feet transit is about 200 pounds. In order that the pivots may be relieved from a portion of this weight, there is raised upon the top of each pier a brass pillar, E, about 18 inches high. On the top of the pillar there is a lever, H, from one end of which hangs a strong brass hook, X, supporting two friction rollers under the ends of the great axis. A counterpoise, W, sliding on the other end of the lever, may be made to support as much of the weight of the instrument as is desired.

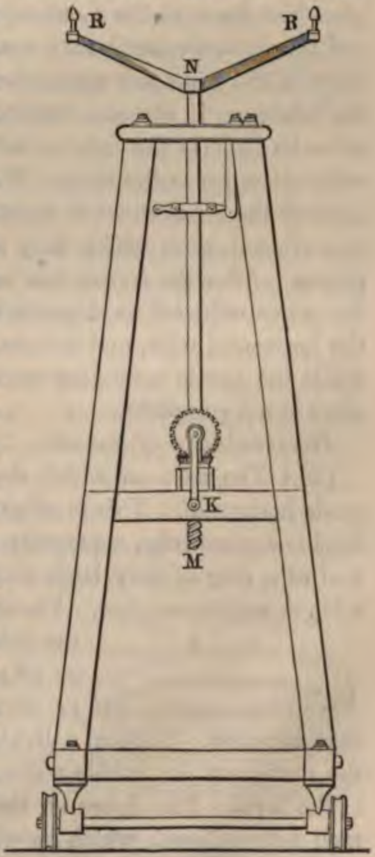
(52.) Both the pivots of the axis are perforated to admit the light of a lamp, on an elliptic-ring reflector placed inside the square part of the axis. To moderate the light of the lamp, L, there is a green glass wedge, movable up and down between


H
H
W
W
E
E
L
X
X
Z
D
D
C
B
O
O
P
P

the lamp and pivot. The thinnest part of the wedge transmits nearly all the light of the lamp, while the thickest part transmits only as much light as is barely sufficient to show the spider lines. On each side of the eye end of the telescope is a finding circle, D, with a level, by which the telescope can be set to any zenith distance.

There are also in the eye-tube, at the back of the plate carrying the spider lines, two oblong openings covered with glass. A lighted taper held near either of these apertures illumines the lines without enlightening the field of view, by which means very small stars can be observed, which could not be seen by the ordinary illumination. Z represents the handle of the screw which fastens the clamp arm to the axis of the instrument; B and C are handles, by which a slow motion in altitude is given to the telescope; OO is the eye-piece, with spider lines, micrometer, etc.

(53.) It is frequently necessary to reverse the axis of the transit instrument, and large telescopes need some special contrivance by which this may be readily accomplished. The most convenient apparatus for this purpose is a reversing stand, represented in the annexed figure. The stand, being on rollers, is brought under the axis of the transit, when, by turning the handle K, the rod MN is elevated by means of a screw, and the instrument lifted from the piers by means of the forks RR. The stand is then rolled away from between the piers, and the instrument turned half round. The stand is



again rolled between the piers, and the axis returned to its place.

ADJUSTMENTS.

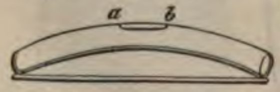
(54.) When the instrument is set up, it should be so placed that the telescope, if turned down to the horizon, may point north and south as near as can possibly be ascertained. This, of course, can only be done approximately, as the meridian can not be accurately determined until the other adjustments have been completed.

Distinctness of vision and parallax.

(55.) The system of wires or spider lines should be in the common focus of the object-glass and eye-glass. In order to place the lines in the focus of the eye-glass, push in or draw out the eye-tube until they are seen with perfect distinctness. Now, if the wires are not in the focus of the object-glass when the telescope is directed toward a distant mark, if the eye be moved a little to the right or left, the mark will appear to move with reference to the lines. When this is the case, the object-glass or the wires must be moved in the tube until the parallax is corrected, after which they must be secured firmly to their places. After the transit has been placed in the meridian, and the wires adjusted as described hereafter, let a star run along the horizontal wire, and if it does not remain perfectly bisected while the eye is moved up and down, the adjustment for parallax is not complete.

Horizontality of the axis.

(56.) The axis on which the telescope turns must next be made horizontal. This is effected by means of the level. The level is a glass tube, apparently cylindrical, but in reality a portion of a ring of very large radius, nearly filled with spirit of wine or sulphuric ether. The convex side being placed upward, the bubble will occupy the higher part, as *ab;* and if either end of the level be elevated, the bubble will move in that direction. If, then, a divided scale be attached to the level, the motion of the bubble will measure the elevation of the end of the level. The figure on the opposite page shows the common form of level, which should be made of such dimensions,



that the legs may extend from one pivot of the transit instrument to the other.

Fig. 1

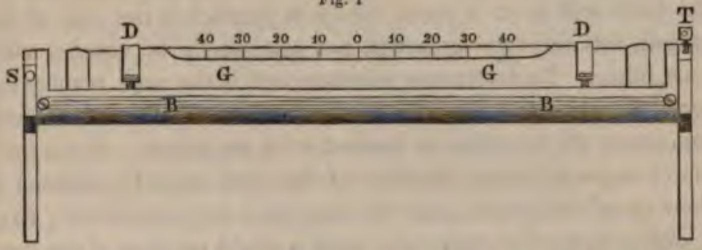


Fig. 1 represents a front view of the level, *Figs.* 2 and 3 represent end views of the legs. The level consists of a glass tube, GG, which lies in a semi-cylinder of brass, BB, and is secured to it by two thin brass straps, DD. The cylinder is connected with one leg of the level by the two screws, SS′, seen in *Fig.* 2, and with the other leg by the screws TT′, seen in *Fig.* 3. The screws SS′ serve to move the cylinder BB in a horizontal direction; the screws TT′ serve to move it in a vertical direction. Each foot of the level has two planes, inclined at an angle of 60 to 90 degrees, which are designed to rest upon the pivots of the transit.

Fig. 2.

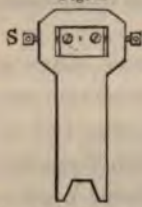
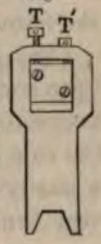


Fig. 3.



(57.) The level should be so adjusted that its axis may be parallel with the axis of the transit. For this purpose, place the level upon the pivots of the axis, and bring the air-bubble to the centre of the glass tube by turning the screw which raises or lowers the end of the axis. Then reverse the level so that the end which before rested on the right pivot may rest on the left. If the bubble settles in the same position as before, we may conclude that the axis of the transit is horizontal; but if the bubble moves from its former position, the amount of this motion will be equal to twice the inclination of the axis to the horizon. Turn the screw at the end of the axis so as to move the bubble over half this distance; then loosen one of the screws, TT′, *Fig.* 3, and tighten the other, until the bubble is brought back to the middle of the tube.

Since it is difficult to make this adjustment perfect at a single trial, we must repeat the same series of operations until the bubble occupies the same place in both the direct and reversed positions of the level. When this is accomplished, the axis of the level will be in a plane which is parallel to the axis of the transit, but the two axes will not necessarily be parallel with each other. To determine whether such is the case, revolve the level slightly upon the axis of the transit, the feet of the level remaining all the while in contact with the pivots. If the bubble changes its place, the axis of the level must be inclined to the axis of the transit, and we must turn the screws SS′, *Fig.* 2, either forward or backward, until a slight rotation of the level about the axis of the transit causes no sensible change in the position of the bubble. When this second correction is completed, the former must be verified anew.

(58.) To discover whether the level is well made, place it upon a rule, having at one end two points, which enter two corresponding cavities upon an iron bar, while at the other end of the rule is a delicate micrometer screw, pressing firmly against a cavity in the iron bar. The whole must be placed upon a very firm support. Then, upon turning the micrometer screw so as to change the inclination of the level to the horizon, it will be easily seen whether equal parts of a rotation of the screw correspond to equal movements of the bubble along the glass tube. When this is the case, the level is good. By means of this arrangement we may easily determine the value of one division of the level, expressed in seconds of arc. Measure the distance of the cavity in which the micrometer screw rests from the middle of the line connecting the two other cavities in the iron bar, and represent this distance, expressed in inches and parts of an inch, by d. Count the number of threads contained in an inch upon the screw, from which we can determine the distance between two threads, expressed in parts of an inch. Represent this distance by b. Then, it is plain that the inclination of the level to the horizon will be changed in one revolution of the screw by an angle equal to $\frac{b}{d \sin. 1''}$. If, therefore, we know how many divisions of the level correspond to one revolution of the screw, we may determine the value of one division of the level, expressed in parts of a second.

(59.) We may also determine the value of one division of the level in the following manner:

Fix the level to the tube of a telescope connected with a vertical divided circle reading to seconds. Move the telescope by means of the tangent screw, so as to carry the bubble successively to one side and the other of the level, and read off the circle in the two positions. The difference of these readings in seconds, divided by the number of divisions of the level that the bubble has moved, will give the value of one division. Delicate levels are generally designed to be divided in such a manner that one division shall represent one second of arc. At one end of the level-tube are small screws, by which that end may be elevated or depressed, so as to bring the bubble into the middle of the tube when the level is placed on a horizontal surface.

Perpendicularity of the wires.

(60.) It is desirable that the central or middle wire should be truly vertical, as we may then observe the transit of a star on any part of it as well as the centre. For this purpose, direct the telescope upon a small, well-defined, and distant object. If, on moving the telescope in altitude, this mark is perfectly bisected by the central wire from top to bottom, the wire is perpendicular to the horizontal axis. If not, the ring or tube containing the wires must be turned round until the mark is bisected by every part of the wire. The other vertical wires are placed by the maker as nearly as possible equidistant from each other, and parallel to the middle one; therefore, when the middle one is adjusted, the others are also adjusted. The transverse wires are also placed at right angles to the vertical middle wire.

Collimation.

(61.) The optical axis of a lens is the line which joins the centres of the spherical surfaces by which the lens is bounded. When a telescope is properly constructed, the axes of the object-glass and eye-glass must lie in the straight line which joins the centres of the object-glass and eye-glass. This straight line is called the *optical axis* of the telescope.

The principal line of sight, or *the line of collimation*, is determined by the direction of the ray of light which passes through the centre of the object-glass, and touches the middle vertical thread midway between the two horizontal threads. In the rotation of the telescope about its axis, the line of collimation should describe a plane perpendicular to this axis. To determine whether such is actually the case, direct the telescope to some small, well-defined, and distant object, and bisect it with the middle vertical wire. Then lift the telescope very carefully from its supports, and replace it with the axis reversed. Point the telescope again to the same object, and if it be still bisected, the collimation adjustment is correct; if not, move the wires one half the error by turning the small screws which hold the diaphragm, near the eye end of the telescope. But as half the deviation may not be correctly estimated in moving the wires, it becomes necessary to verify the adjustment by moving the telescope the other half, which is done by turning the screw F (see figure, page 40). Having again bisected the object, reverse the axis as before, and, if half the error was correctly estimated, the object will be bisected when the telescope is directed to it. If this is found not to be the case, half the remaining error must be corrected as before, and these operations must be continued until the object is found to be bisected in both positions of the axis. The adjustment will then be complete.

POSITION IN THE MERIDIAN.

(62.) This adjustment is effected with the assistance of a clock, which, for convenience, should be regulated to sidereal time, so that the time of each star's passing the meridian will be indicated by its right ascension.

By the pole star.

(63.) Direct the telescope to the pole star at the instant of its crossing the meridian, as near as the time can be ascertained. The transit will then be nearly in the plane of the meridian. Having leveled the axis, turn the telescope to a star about to cross the meridian, near the zenith. Since every vertical circle intersects the meridian at the zenith, a zenith star will cross the field of the telescope at the same time, whether the plane of the transit coincide with the meridian or not. At the moment the star crosses the central wire, set the clock to its right ascension, as given by the catalogue, and the clock will henceforth indicate nearly sidereal time. The approximate times of the upper and lower culmination of the pole star are then known. Observe the pole star at one of its culminations, following its motion until the clock indicates its right ascension, or its right ascension plus 12 hours. Move the whole frame of the transit, so that the central wire shall coincide nearly with the star, and complete the adjustment by means of the azimuth screw. The central wire will now coincide almost precisely with the meridian of the place.

(64.) The axis being supposed perfectly horizontal, if the middle wire of the telescope is exactly in the meridian, it will bisect the circle which the pole star describes, in 24 sidereal hours, round the polar point. If, then, the interval between the upper and lower culminations is exactly equal to the interval between the lower and upper, the adjustment is complete. But if the time elapsed while the star is traversing the eastern semicircle is greater than that of traversing the western, the plane in which the telescope moves is westward of the true meridian on the north horizon; and *vice versâ*, if the western interval is greatest. This error must be corrected by turning the screw F (page 40). The adjustment must then be verified by further observations, until, by continued approximations, the instrument is fixed correctly in the meridian.

By a pair of circumpolar stars.

(65.) Take two well-known circumpolar stars, the nearer the pole the better, differing about twelve hours in right ascension, and observe one above and the other below the pole. Now it is evident that any deviation of the instrument from the meridian will produce *contrary* effects upon the observed times of transit. Hence the time which elapses between the two observations will differ from the time which should elapse according to the catalogue, by the *sum* of the effects of the deviation upon the two stars. The stars 51 Cephei and δ Ursæ Minoris are well suited to this purpose. The right ascension of the former on the 1st of January, 1881, was 6h. 44m. 55s.; of the latter, 18h. 10m. 13s. The difference between the times at which one should make its upper and the other its lower transit is 34m. 42s. If the observed interval differs from this, the error must be corrected by the azimuth screw, and the observations repeated until the adjustment is perfect.

By the pole star, combined with any star distant from the pole.

(66.) If the transit moves in the plane of the meridian, the error of the clock, as determined by the culmination of the pole star, will be exactly the same as from any other star situated, for example, near the equator. But if the transit describes a vertical circle which differs from the meridian, the pole star will be longer in crossing from the transit plane to the true meridian than the equatorial star. If, then, the two stars do not indicate the same clock error, the azimuth screw must be moved until the adjustment is perfect.

By a high and low star.

(67.) This method may be practiced in situations which do not permit an observation of the pole star. Choose two stars differing but little in right ascension, one of them passing the meridian as near as possible to the zenith, and the other as near as convenient to the south horizon. Make the axis of the transit perfectly horizontal, so that the transit shall describe a vertical circle. This circle will coincide with the meridian at the zenith, however much it may depart from it at the horizon. A star near the zenith will pass the middle wire of the telescope at about the same time as if the transit was in the meridian; but this will not be the case with a star near the south horizon. If the low star passes the central wire too early, the plane of the instrument deviates to the east; if it passes too late, the plane deviates to the west. In either case the error must be corrected by the azimuth screw, until stars at all altitudes indicate the same error of the clock.

MERIDIAN MARK.

(68.) When the transit instrument has once been brought to the meridian, a mark may be placed, either to the north or south, for verification, in case the instrument should at any time be disturbed. Instead of placing the mark at a great distance, it is now common to place it comparatively near to the instrument, and in the focus of a lens which renders the rays from the mark parallel, by which means the mark can be viewed through a telescope adjusted to its solar focus as stated on page 17. At Pulkova the lens for this purpose is placed on a pier within the transit room, and has a focal length of 556 feet, which is therefore the distance of the mark from the pier. A method more convenient will be described on page 81.

THE CLOCK—ITS RATE AND ERROR.

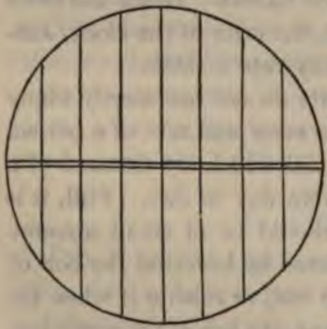
(69.) A clock designed to be used for astronomical purposes should be of the best workmanship. The pendulum should be compensated, so as to be free from the effects of heat and cold. The two forms chiefly used for astronomical purposes are the mercurial and the gridiron. The former is generally used in England, the latter in France and Germany.

It is most convenient to have the clock regulated to sidereal time, and it is desirable that it should keep exact pace with the stars, so as always to indicate the exact right ascension of the star then passing the meridian. But every clock has both an *error* and *rate*. The *error* of the clock at any time is its difference from true sidereal time. The *rate* of the clock is the *change* of its error in 24 hours. Thus, on the 8th of January, 1851, Aldebaran was observed to pass the meridian of Greenwich at 4h. 26m. 52.02s. The true right ascension of the star was 4h. 27m. 22.86s.; hence the clock was slow 30.84s. Again, on the 9th of January, the same star passed the meridian at 4h. 26m. 51.22s., and the clock was slow 31.64s. Hence the clock lost 0.80s. per day. In other words, the error of the clock, January 9th, was −31.64s., and its daily rate −0.80s.

(70.) The preceding error and rate do not necessarily imply any imperfection of the clock. The error and rate of a perfect clock may be of any magnitude. All which we demand of a clock is that its rate be uniform from day to day. Still, it is convenient in practice that both should be of small amount. The rate of the clock may be corrected by lowering the bob of the pendulum, if the clock runs too fast, or raising it when the clock runs too slow. For this purpose, the bob of the pendulum is furnished with a fine adjusting screw. The clock may be made to indicate true sidereal time by setting it to the right ascension of any known star, and starting the pendulum at the moment when the star crosses the middle transit wire. After the rate has been reduced to a small quantity, it is better to let the error accumulate than to stop the clock. When the error amounts to a whole minute, the minute-hand may be moved one division without disturbing the motion of the pendulum. The transit clock at Greenwich Observatory generally loses about half a second a day, and when this error amounts to an entire minute (which happens about every three months), the clock is put forward one minute.

METHOD OF OBSERVING AND REGISTERING TRANSITS.

(71.) For a night observation, the field of view must be illumined by the lamp M (see figure, page 40), so that the wires may be distinctly visible; and the telescope must be set to the proper altitude by means of the attached circle. This circle is sometimes designed to indicate altitudes or zenith distances, and sometimes declinations or polar distances. In either case, the zero of the circle may require adjustment. If the circle indicates altitudes, the index should point to zero when the bubble of the attached level stands in the middle of the tube. If the circle indicates declinations, the index should point to zero when the telescope is directed toward an equatorial star. Since the telescope inverts the position of objects, a star for an upper culmination will appear to enter the field of view on the west side, and pass out on the east; but for a lower culmination, it will cross the field from east to west. The telescope contains five or seven vertical, and two horizontal wires, placed a short distance from each other. The star should be made to cross the field between the two horizontal wires, in order that the transits may always be observed on the same part of the vertical wires. It is the business of the observer to note the times of the star's passage over the several wires with the utmost accuracy; and as it will seldom happen that a star will cross a wire at the exact instant of the beat of the clock, he must estimate the fractions of a second as well as he is able. This is done by comparing the distance of the star from the wire at the beat preceding the transit, with its distance on the other side at



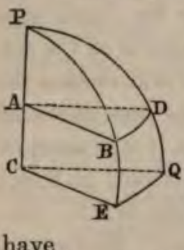
the beat succeeding the transit. The clock should be so placed, and its face illumined, that the observer, seated at the transit, can readily follow the seconds' hand. A little before the star is expected to cross the first wire, the observer takes a second from the clock—suppose 5s.—and, listening to the beats, goes on silently counting 6, 7, 8, 9, etc., while his eye is at the telescope following the motion of the star. If the star crossed the first wire between the beats 9 and 10, and if the star appeared as far beyond the wire at the succeeding beat as it was short of it at the preceding beat, the time of the transit would be 9.5s.; but if the distances were unequal, it would be 9.3s. or 9.7s., etc., according to its apparent distance from the wire. Having recorded the passage over the first wire, the same observation must be made at each of the other wires, and a mean of the whole taken, which will represent the time of the star's passage over the mean or meridional wire. Five or seven wires are more valuable than a single one, since the chances are that an error which may have been committed at one wire will be compensated by an opposite error at another. Thus the mean result of several observations is deserving of more confidence than a single one. The following is an observation of Arcturus, made at Greenwich Observatory, November 13th, 1850:

First wire,	14h.	7m.	7.7s.	Mean, 14h. 7m. 48.53s.
Second wire,		7	21.3	
Third wire,		7	34.9	
Fourth wire,		7	48.6	
Fifth wire,		8	2.1	
Sixth wire,		8	15.7	
Seventh wire,		8	29.4	

It will be perceived that the observation at the middle wire differs 0.07s. from the mean of the seven wires. If the observations were perfect, and the wires equidistant, these two numbers should agree exactly.

EQUATORIAL INTERVAL OF THE WIRES.

(72.) By comparing the transits of different stars, it will be seen that the time occupied by a star in traversing the interval between the wires is different on different points of the meridian; being least at the equator, and increasing with the distance from that circle. The time occupied by a star on the equator, in passing between any two of the wires, is called their *equatorial interval;* and when this interval is known, the interval for any parallel of declination may be computed. Thus, let P be the pole of the heavens, EQ a portion of the equator, and BD a portion of any parallel of declination; PBE and PDQ two meridians, but slightly inclined to each other. A star at B moves over the arc BD in the same time that one at E moves over EQ. But we have



Geom., B. VI., Prop. 13, Cor. 1,

arc EQ : *arc* BD : : CQ : AD : : 1 : cos. Dec.

Therefore, BD = EQ cos. Dec.

Now the time in which a star on the parallel BD would move over a constant space, EQ, must be, to the time in which an equatorial star moves over the same, inversely as their rates of motion, or as

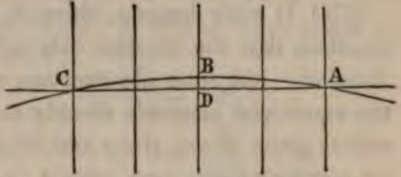
EQ : BD : : 1 : cos. Dec. : : sec. Dec. : 1.

(73.) If, then, x represent the equatorial interval of the wires, x sec. Dec. will be the interval for any star. The equatorial interval may therefore be computed from observations made upon any star whose declination is known, by multiplying the observed interval by the cosine of the star's declination. Thus, in the preceding observation of Arcturus, the difference between each observation and the mean of the seven wires is as follows:

	Observed intervals.	Equatorial intervals.
First wire,	40.83s.	38.376s.
Second wire,	27.23	25.594
Third wire,	13.63	12.811
Fourth wire,	0.07	0.066
Fifth wire,	13.57	12.755
Sixth wire,	27.17	25.537
Seventh wire,	40.87	38.414

And if we multiply these numbers by .939908, the cosine of the star's declination (19° 57′ 50″), we shall obtain the equatorial intervals, as given in the last column above.

(74.) The equatorial interval may, however, be obtained more accurately by observations of a star near the pole—the pole star, for example; but in this case a slight modification of the preceding rule becomes necessary, for the pole star does not pass perpendicularly from wire to wire, but describes a considerable arc of the small circle ABC. Now AD is the sine of the arc AB. In order, therefore, to obtain the equatorial intervals from the pole star, we must multiply the *sine* of the observed interval by the cosine of the declination, and we shall obtain the sine of the equatorial interval.



The following observations of Polaris, Dec. 88° 30′ 27″.0, were made at Greenwich Observatory, April 26th, 1850:

Wires.	Observations.	Observed Intervals.	Equatorial Intervals.
	h. m. s.	m. s.	s.
A	0 39 48.0	−24 43.29	−38.559
B	48 3.0	−16 28.29	−25.719
C	56 19.0	− 8 12.29	−12.820
D	1 4 32.0	+ 0 0.71	+ 0.018
E	12 44.0	+ 8 12.71	+12.830
F	20 57.0	+16 25.71	+25.652
G	29 16.0	+24 44.71	+38.595
Mean	1 4 31.29		

The letters in column first are used to distinguish the wires of the transit. The wires at Greenwich are designated by the letters of the alphabet in such a manner that, when the illumined end of the axis is east, the order of the wires for stars above the pole is A, B, C, D, E, F, G; but when the illumined end of the axis is west, the order is G, F, E, D, C, B, A. Column third shows the difference between each observation and the mean of the seven wires. Column fourth shows the equatorial interval thence deduced. The fourth column is computed as follows:

24m. 43.29s. = 6° 10′ 49″.35 sine = 9.0320497;
cos. Dec. 88° 30′ 27″.0 = 8.4157426;
38.559s. = 9′ 38.″39 sine = $\overline{7.4477923}$;

and in the same manner for the other wires.

It will be perceived that the middle wire differs slightly from the mean of the seven wires, which may be called the *mean* *wire.* It is customary, at Greenwich, to reduce all observations to the standard of the mean wire, and not of the middle wire.

To reduce an observation when all the wires are not observed.

(75.) It may happen, through inadvertence or unfavorable weather, that the transits over only a portion of the wires are observed; but such observations may be reduced by means of the equatorial intervals already determined. According to the values given above, if we add 38.559s. to the time of transit of an equatorial star over wire A, it will give the time of transit over the mean wire; and, in the same manner, the observation of an equatorial star at each wire may be reduced to the mean, by adding or subtracting, as the case may be, to the time of observation, the equatorial interval between that wire and the mean wire. For a star out of the equator, these intervals must each be multiplied by the secant of the star's declination. Or the following rule is more convenient in practice, and evidently gives the same result:

Add together the equatorial numbers from the table on page 55 *for the wires observed, regard being had to their signs; divide by the number of wires, and multiply by the secant of the star's declination. The product will be the correction to be applied to the mean of the wires observed.*

The corrections to transits of an equatorial star over wires A, B, C, D, E, F, G, for 1851, at Greenwich, were +41.443s.; +27.646s.; +13.816s.; −0.002s.; −13.811s.; −27.654s.; −41.438s.; and these are the intervals to be used in reducing the subsequent observations.

Ex. 1. The following observations of Capella were made at Greenwich, January 27th, 1851:

A	5h. 4m.	—
B		20.2s.
C		40.2
D		59.8
E	5	19.7
F		39.5
G		59.4
Mean of wires observed,	5h. 5m.	9.8s.

The sum of the equatorial numbers for wires B, C, D, E, F, G is −41.443s., which, divided by 6, gives −6.907s., and, multiplied by the secant of the declination, 45° 50′ 26″, gives −9.91s.; which, being applied to the above mean, gives 5h. 4m. 59.89s. as the time of transit over the mean of the seven wires.

Ex. 2. The following observations of Sirius were made at Greenwich, February 13th, 1851:

A	6h. 37m.	—
B		—
C		—
D		43.7s.
E		58.2
F	38	12.6
G		26.9
Mean of wires observed,	6h. 38m.	5.35s.

The sum of the equatorial numbers for wires D, E, F, G is −82.905s., which, divided by 4, gives −20.726s., and, multiplied by the secant of the star's declination, 16° 31′ 12″, gives −21.62s.; which, applied to the above mean, gives for the time of transit over the mean wire, 6h. 37m. 43.73s.

Ex. 3. The following observations of Spica, Dec. 10° 22′ 56″, were made at Greenwich, February 21st, 1851:

A	13h. 15m.	—
B		—
C	16	9.1s.
D		23.1
E		37.0
F		51.1
G	17	5.0

Required the time of transit over the mean wire.

Ans. 13h. 16m. 23.01s.

(76.) In the case of a star near the pole, we must multiply the *sine* of the equatorial interval by the secant of the star's declination, and we shall obtain the *sine* of the reduction to the mean wire according to Art. 74.

Example. The following observations of Polaris, at its upper culmination, were made at Greenwich, May 30th, 1851:

A	0h. 38m.	—
B	47	—
C	56	7.0s.

D	1h.	4m.	59.0s.
E		13	53.0
F		22	47.0
G		31	40.0

To determine the time of transit over the mean of the seven wires, the declination of Polaris being 88° 30′ 38″.4.

The reduction for each wire is computed as follows:

Wire C.

sine 13.816s.=7.0020484
sec. Dec.=1.5851796
sine 8m. 51.70s.=8.5872280

Wire D.

log. 0.002.s=7.301
sec. Dec.=1.585
log. 0.08s.=8.886

Wire E.

sine 13.811s.=7.0018912
sec. Dec.=1.5851796
sine 8m. 51.51s.=8.5870708

Wire F.

sine 27.654s.=7.3034239
sec. Dec.=1.5851796
sine 17m. 45.05s.=8.8886035

Wire G.

sine 41.438s.=7.4790643
sec. Dec.=1.5851796
sine 26m. 37.92s.=9.0642439

The sum of these corrections is −44m. 22.86s., which, divided by 5, gives −8m. 52.57s., which is the correction to be applied to the mean of the wires observed to obtain the mean of the seven wires. The mean of the wires observed is 1h. 13m. 53.2s. Hence the concluded transit over the mean of the seven wires is 1h. 5m. 0.63s. As the time required for the pole star to pass from one wire to another is nearly the same for every day of the year, and only varies in consequence of a small change in the star's declination, it is customary, in regular observatories, to compute the intervals for an assumed value of the declination, and the variation caused by a change of one second in the declination. All the reductions are then made with great facility.

(77.) In observing the sun, the times of passage of both the first and second limbs over the wires are observed and set down as distinct observations, the mean of which gives the time of passage of the centre across the meridian. The wires of the instrument are generally placed by the maker at such a distance from each other that the first limb of the sun shall have passed all of them before the second limb arrives at the first wire.

If only one limb is observed, the passage of the centre may be inferred by adding or subtracting the sidereal time of semidiameter passing the meridian, as given on page first of each month in the Nautical Almanac.

Only one limb of the moon can be observed, except when her transit happens to be within an hour or two of her opposition; and, in observing the larger planets, the first and second limbs may be observed alternately over the seven wires. If only one limb of a planet is observed, the ephemeris must be consulted for the time of passage of its semidiameter.

(78.) In correcting imperfect transits of the sun and planets, the value of the intervals found, as for a star of the same declination, must be increased by a small quantity. For if a fixed star and the sun's first limb were together at the first wire, the sun would be behind the star when it passed the second wire, on account of the sun's apparent motion among the stars. For the sun or a planet, therefore, the interval found for a star must be multiplied by the factor

$$\frac{3600+\mathrm{I}}{3600},$$

where I represents the hourly increase of right ascension in seconds of time taken from the Nautical Almanac.

Example. The following observations of the sun's second limb were made at Greenwich, February 22d, 1851:

A	22h. 20m.	—
B		—
C		54.5s.
D	21	8.8
E		22.9
F		36.8
G		51.0
Mean of wires observed,	22h. 21m.	22.8s.

The sum of the equatorial numbers for wires C, D, E, F, G is −69.089s., which, divided by 5, gives −13.82s., and, multiplied by the secant of the sun's declination, 10° 17′ 41″, gives −14.04s., which is the correction for a star of the same declination. The sun's hourly increase of right ascension, February 22d, according to the Nautical Almanac, was 9.52s. Hence

$$3600 : 3609.52 :: 14.04 : 14.08,$$

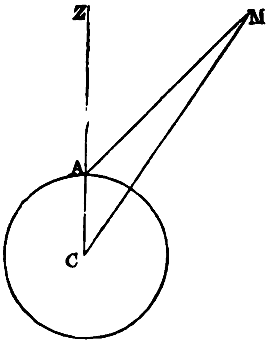
which is the correction to the mean of the wires observed.

Hence the concluded transit over the mean of the seven wires is

22h. 21m. 8.72s.

In order to facilitate these reductions, it is convenient to have a table showing the logarithm of the factor $\frac{3600+\mathrm{I}}{3600}$ for every value of I from 1s. up to 30s.

(79.) The reduction of an imperfect transit of the moon's limb requires a peculiar method, on account of the moon's proximity to the earth.



Let C represent the centre of the earth, A any place on the earth's surface, and M the centre of the moon; then, since the angular value of any small line at different distances is inversely as those distances, the angular value of the moon's hourly motion in its orbit from a place, A, is to the angular value of the same from C as CM to AM. But CM is to AM as the sine of CAM, or sine of ZAM, is to the sine of ZCM; that is, as the sine of the apparent zenith distance of the moon is to the sine of the geocentric zenith distance. In order, therefore, to reduce an observation at any wire to the mean of the wires, the interval found for the sun or a planet must be multiplied by the factor,

$$\frac{\text{sine of moon's geocentric Z.D}}{\text{sine of moon's apparent Z.D}};$$

or the entire factor for the moon will be

$$\frac{3600+\mathrm{I}}{3600} \times \frac{\text{sine of moon's geocentric Z.D}}{\text{sine of moon's apparent Z.D}} \times \text{secant of moon's geocentric declination,}$$

where I is the hourly increase of the moon's right ascension in seconds of time.

Example. The following observations of the moon's second limb were made at Greenwich, February 21st, 1851:

A	15h., 34m.	—
B		—
C	35	9.5s.
D		24.0
E		38.7
F		53.2
G	36	8.0
Mean of wires observed,	15h., 35m., 38.68s.	

The sum of the equatorial numbers for the wires observed, divided by 5, gives -13.8178s.

The moon's declination $=14° \; 13' \; 12''$ S.
Moon's geocentric zenith distance $=65 \;\; 41 \;\; 50$
Moon's apparent zenith distance $=66 \;\; 34 \;\; 10$
Moon's hourly increase of R. A. $=135.24$s.

The correction to the mean of the wires observed is then computed as follows:

$$\begin{aligned} 13.8178\text{s.} &= 1.14044 \\ 3600+\text{I}=3735.24\text{s.} &= 3.57232 \\ 3600 \text{ comp.} &= 6.44370 \\ \sin.\ 65° \; 41' \; 50'' &= 9.95970 \\ \text{cosec.}\ 66 \;\; 34 \;\; 10 &= 0.03737 \\ \sec.\ 14 \;\; 13 \;\; 12 &= 0.01351 \\ 14.69\text{s.} &= \overline{1.16704} \end{aligned}$$

Subtracting 14.69s. from the mean of the wires observed, we obtain the time of transit over the mean of the seven wires,

15h. 35m. 23.99s.

(80.) We have hitherto supposed the transit instrument to be perfectly adjusted—that there is no error of collimation—that the axis is perfectly horizontal—and that the middle wire of the transit describes the plane of the meridian. In practice, these adjustments can never be perfectly made; but we make the adjustments as complete as we are able. We then compute the amount of each error, and apply a correction to the observations.

PROBLEM.

To determine the inclination of the axis of the transit.

The spirit-level which rests on the pivots of the axis determines the inclination of the axis. Above the glass tube, and parallel to its length, is placed a fine graduated scale, which indicates any deviation from horizontality by the air-bubble receding from the centre toward that pivot which is the highest; but as the legs of the level may not be of exactly equal length, it is necessary to reverse the level on the axis, and read the scale at each extremity of the air-bubble in both its positions; that is, with the same end of the level on both the east and west pivots alternately. Half the difference of the means of the two readings will be the amount of deviation. It is customary to make several observations in each position of the level, in order to diminish the effect of incidental errors. The following example will illustrate this method:

Readings of the Scale.

East end.		West end.
32.3		30.0
32.4		30.0
32.4		30.0
	Level reversed.	
32.6		29.6
32.6		29.5
32.5		29.6
194.8	sums	178.7
32.47	means	29.78
	Difference,	2.69.

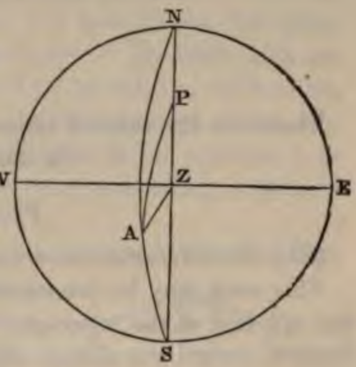
Half the difference is 1.34; and, since the value of one division of the level is 1″.25, the east end of the axis is too high by 1″.67, for the mean of the eastern readings is greater than the mean of the western. This quantity, divided by 15, will give the inclination expressed in seconds of time.

(81.) Having determined the inclination of the axis, the correction to be applied to the time of observation of any star may be computed by the following method:

PROBLEM.

To compute the correction to the time of transit for inclination of the axis.

Let P represent the pole of the earth, Z the zenith, N and S the north and south points of the horizon. Suppose the transit tel-



escope is in the meridian, at the north and south points of the horizon, N and S, but the axis is inclined to the horizon by a small angle; the telescope, instead of describing the meridian, NZS, will describe an oblique circle, NAS; and the star, A, when it passes through the telescope, will be distant from the meridian by the angle APS. Now, in the triangle, APS, we have $\sin. PA : \sin. S :: \sin. SA : \sin. P$;
or, putting b to represent the angle S, and Z the zenith distance of the star (b being supposed to be a small angle),

$$\cos. \text{Dec.} : b :: \cos. Z : P = \frac{b \cos. Z}{\cos. \text{Dec.}},$$

which must be subtracted from the observed time of passage to have the true time, when the telescope is inclined to the west. When the eastern pivot is too high, the level error is considered negative; when the western pivot is too high, the level error is positive.

(82.) The expression for the zenith distance of a star, in terms of its declination and of the latitude of the place, will vary according as the observations are made to the south of the zenith or to the north of the zenith; and, in the latter case, according as the observations are made above or below the pole. These several values will be as follow, representing the latitude by ϕ, and the declination by δ (see page 139):

$Z = \phi - \delta$ - - if the observations be made to the south;
$Z = \delta - \phi$ - - if to the north, *above* the pole;
$Z = 180° - (\phi + \delta)$ if to the north, *below* the pole.

Example. Castor was observed to pass the meridian of Greenwich, February 22d, 1851, at 7h. 24m. 6.52s., its declination being 32° 12′ 32″ N., and the error of level $-3''.92$; required the corrected time of transit.

The latitude of Greenwich is 51° 28′ 39″; therefore Z = 19° 16′ 7″.

$b = -3''.92 = 0.261\text{s}.\quad \log. = 9.4166$

$\cos. Z = 9.9749$

$\sec. \text{Dec.} = 0.0726$

$-0.29\text{s}. = \overline{9.4641}$

Therefore, the time of transit, corrected for error of level, is 7h. 24m. 6.23s.

PROBLEM.

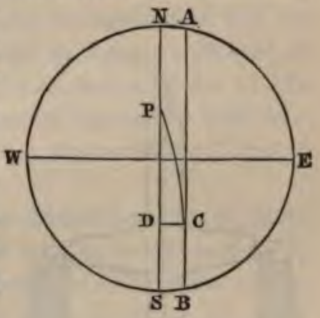
(83.) *To determine the error of collimation.*

This error may be determined by a micrometer attached to the eye end of the telescope, by which, when the telescope is directed toward any distant object, the angular distance of that object from the central wire is measured. The instrument is then reversed, and the distance of the same object from the central wire again measured. Half the difference of these measures is the error of collimation for the middle wire.

(84.) At many observatories the error of collimation is determined, not by observations of a distant mark, but by means of a small transit instrument, mounted at a short distance from the large transit, and in the same meridian, and having in its focus a cross in the form of an acute X. A reflector is attached, for the purpose of throwing the light of the sky upon the wires, and, when the telescopes are pointed toward each other, the cross in the small transit is distinctly seen by looking through the large telescope. The following are the results of a set of observations at Greenwich: When the illuminated end of the axis was east, and the micrometer was made to coincide with the cross, the reading was 10.888r.; when the axis was reversed, the reading was 9.461r.; hence the reading of the micrometer for the true line of collimation was 10.174r. When the micrometer was made to coincide with the middle wire, the reading was 10.191r.; hence the error of collimation for the middle wire was 0.017 revolutions of the screw, which is equal to 0''.28 in seconds of arc. Correcting this for the distance of the middle wire from the mean of the seven wires, we obtain the error of collimation for the mean of the seven wires.

(85.) This error may also be determined by observing the transit of Polaris, or any other close circumpolar star, over the first three wires; and then, reversing the axis, observing the same intervals in a reversed order. The wires which were the first three in the former position, will now be the last three. Let each of the observations be reduced to the mean wire, according to Art. 76; then, if there were no error of collimation, the mean of the observations in the first position of the telescope ought to be the same as the mean in the reversed position. But if the two results differ from each other, it must be owing to error of collimation.

(86.) Suppose the telescope does not move in the meridian, NS, but in a small circle, AB, parallel to the meridian, and every where a certain number of seconds (c) east of it. Let P be the pole, and C the place of the star. Draw CD perpendicular to NS. Then, when the star passes the telescope, its angular distance from the meridian will be CPD. Now, in the triangle CPD, we have



Trig., Art. 211,

$$\text{R. sin. CD} = \text{sin. PC sin. CPS.}$$

Whence $$\text{CPS} = \frac{\text{CD}}{\text{sin. PC}} = \frac{c}{\text{cos. Dec.}} \quad . \; . \; . \; (1)$$

and $$c = \text{CPS cos. Dec.} \quad . \; . \; . \; . \; . \; (2)$$

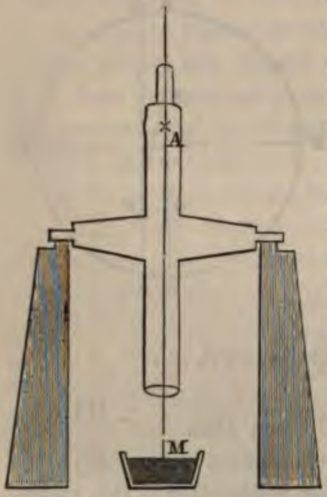
The following example will show the application of this method. At Edinburgh Observatory the transit of Polaris was observed over two wires; the instrument was then reversed, and the transit observed over the same wires, as follows:

Times observed.		Reduction.	Times reduced.	
	h. m. s.	m. s.	h. m. s.	
Wire I. . . .	0 44 33.5	+16 23.59	1 0 57.09	57.68s.
Wire II. . .	0 52 46.0	+ 8 12.28	1 0 58.28	
Instrument reversed.				
Wire II. . .	1 8 56.0	− 8 12.28	1 0 43.72	44.31s.
Wire I. . . .	1 17 8.5	−16 23.59	1 0 44.91	

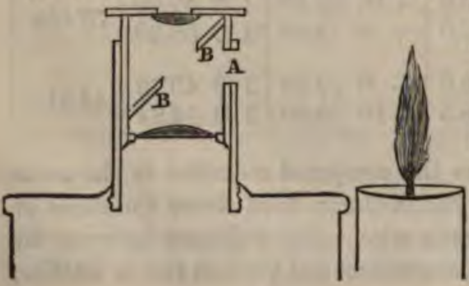
Column second shows the computed reduction to the mean wire, according to Art. 76. Column third shows the times of transit reduced to the mean wire. The difference between the mean of the first two observations and the last two is 13.37s.;

half of which, being 6.685s., represents the angle CPS, which, multiplied by the cosine of the star's declination, gives the error of collimation, 0.181s., expressed in seconds of time, which is minus for the first position of the instrument, and plus for the second position.

(87.) There is another method of determining the error of collimation, which is exceedingly convenient and accurate. It consists in pointing the telescope vertically downward toward a vessel of mercury, and observing the spider lines of the telescope as reflected from the surface of the mercury, a strong illumination being thrown upon the system of wires by a lateral lamp. The rays diverging from the wires at A issue in parallel lines from the object-glass, fall upon the mercury, M, and are thence reflected back in parallel lines to the object-glass, which is enabled to collect them again in its focus. Thus is formed a reflected image of the system of spider lines; and if the axis is perfectly horizontal, and there is no error of collimation, the reflected system ought to coincide exactly with the real system, as seen in the eye-piece of the instrument. If, however, when the axis has been leveled, the two systems of lines do not coincide, the difference is twice the error of collimation, and may be measured by the micrometer.



(88.) The preceding observation requires a peculiar eye-piece, called the collimating eye-piece, first suggested by Bohnenberger in 1825. The collimating eye-piece consists of an ordinary positive eye-piece, with an aperture, A, cut in its side, and a plane



perforated speculum, BB, inserted between the two lenses, at an angle of 45° with the optical axis of the telescope, as represented in the preceding figure. A lamp being held so as to throw a strong light upon the speculum, the reflected images of the wires may be seen with great distinctness. Instead of a perforated opaque speculum, a piece of plane glass, with parallel faces, without any perforation, is sometimes used. The observer looks through the plane glass without difficulty, while sufficient light is reflected from the lower surface to render the lines visible.

(89.) If the axis of the telescope be not horizontal, half the distance between the middle wire and its image, corrected for error of level, will give the error of collimation of the middle wire. Correcting this for the distance of the middle wire from the mean of the seven wires, we obtain the error of collimation for the mean of the seven wires.

(90.) By reversing the axis of the transit upon its supports, we may obtain the error of level, as well as of collimation, by means of the micrometer. If the errors of collimation and inclination of the axis are both in the same direction, the deviation of the middle wire from its reflected image will represent twice the sum of the errors of collimation and level; but if the axis be reversed, the deviation will be twice the difference of these quantities. Knowing the sum and difference of these quantities, their values are readily determined.

PROBLEM.

(91.) *To determine the correction to the time of transit for error of collimation.*

This correction is readily computed by equation (1), Art. 86. We have only to multiply the error of collimation by the secant of the declination of the star.

Ex. The transit of Castor, Dec. 32° 12′ 32″ N., was observed at Greenwich, February 22d, 1851, at 7h. 24m. 6.52s., the error of collimation being $-0''.93$; required the corrected time of transit.

$$\begin{aligned} -0''.93 = 0.062\text{s.}\quad \text{log.} &= 8.792 \\ \text{sec. Dec.} &= \underline{0.073} \\ -0.07\text{s.} &= 8.865 \end{aligned}$$

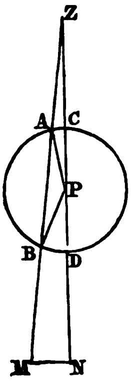
Therefore, the corrected time of transit is 7h. 24m. 6.45s.

PROBLEM.

(92.) *To determine the deviation of the transit instrument from the meridian.*

First Method.—By the pole star, or any close circumpolar star.

If the transit telescope revolves on a horizontal axis in the plane of the meridian, the intervals of time between two successive passages of the pole star over the central wire must be exactly 12 hours. If this interval differs from 12 hours, then the instrument deviates from the true meridian, and the amount of deviation, as measured on the horizon, may be computed as follows:



Let ZPN be a meridian, and ZAM the vertical circle described by the telescope; let ABDC be the small circle described by the star about the pole, P. This star will be observed with the transit telescope at the points A and B instead of C and D.

Let ϕ = the latitude of the place;
p = the polar distance of the star;
a = the angle MZN = the deviation of the telescope from the meridian;
Δ = the interval between two successive transits, minus 12 hours.

In the triangle APZ, we have

$$\sin. \mathrm{PA} : \sin. \mathrm{ZA} :: \sin. \mathrm{AZP} : \sin. \mathrm{APZ};$$

or, since small angles are nearly proportional to their sines,

$$\sin. p : \cos. (\phi+p) :: a : \mathrm{APZ} = \frac{a \cos. (\phi+p)}{\sin. p}$$

$$= \frac{a(\cos. \phi \cos. p - \sin. \phi \sin. p)}{\sin. p}$$

$$= a \cos. \phi \cot. p - a \sin. \phi.$$

(See Trig., Art. 72)

Also, in the triangle BPZ, we have

$$\sin. \mathrm{BP} : \sin. \mathrm{BZ} :: \sin. \mathrm{BZP} : \sin. \mathrm{BPZ},$$

or

$$\sin. p : \cos. (\phi - p) :: a : \text{BPN} = \frac{a \cos. (\phi - p)}{\sin. p}$$

$$= \frac{a\,(\cos. \phi \cos. p + \sin. \phi \sin. p)}{\sin. p}$$

$$= a \cos. \phi \cot. p + a \sin. \phi.$$

Therefore, $\Delta = \text{APC} + \text{BPD} \quad = 2a \cos. \phi \cot. p,$

and $$a = \frac{\Delta}{2 \cos. \phi \cot. p} = \frac{\Delta}{2} \sec. \phi \cot. \text{Dec.};$$

that is, the azimuthal deviation of the transit is equal to half the difference between the observed interval and 12 hours, in seconds, multiplied by the secant of the latitude and the cotangent of the star's declination.

Ex. 1. January 6th, 1850, the transit of Polaris, Dec. 88° 30′ 50″, was observed, sub polo, at Greenwich, at 13h. 4m. 39.40s.; and January 7th, at 1h. 4m. 57.62s.; the observations being corrected for collimation, level, rate of clock, and change of right ascension. Required the azimuthal error.

$$\frac{\Delta}{2} = 9.11\text{s.} = 136''.65 \quad \log. = 2.1356$$

$$\cot. 88^\circ\ 30'\ 50'' = 8.4140$$

$$\sec. \phi = 0.2056$$

$$a = +5''.69 = \overline{0.7552}$$

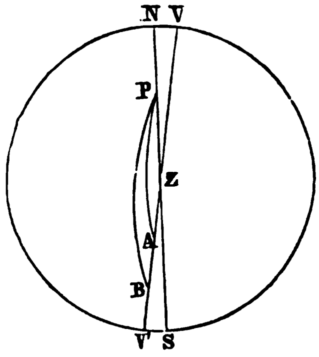
Since the time elapsed in traversing the eastern semicircle is more than 12 hours, the plane of the telescope falls to the west of the true meridian on the north horizon.

Ex. 2. April 23d, 1850, the transit of Polaris, Dec. 88° 30′ 27″, was observed at Greenwich at 1h. 3m. 34.99s.; and April 24th, sub polo, at 13h. 3m. 22.55s. Required the azimuthal error. *Ans.* +3″.90

The factor sec. ϕ cot. Dec. is sensibly constant for Polaris at each observatory, through an entire year or more; hence it is well to prepare this factor once for all. At Greenwich, in 1850, the rule was to divide Δ, in seconds of time, by 3.206, to obtain the azimuthal deviation in seconds of space. Thus, in Ex. 1, 18.22s., divided by 3.206, gives $5''.69 = a$.

(93.) *Second Method.—By two stars differing considerably in declination.*

If we take the difference between the observed passages of two stars over the meridian, and also the difference of their com-



puted right ascensions, and we fi
these differences to be precis
equal, the instrument will be e
actly in the meridian; if they a
not equal, this inequality shows
deviation from the meridian.

Let NZS be a meridian, VZ
the vertical circle described by t
telescope, P the pole, Z the zeni
and A and B two stars observed
the transit telescope.

Let ϕ = the latitude of the place;

δ and δ' = the declinations of the two stars;

Δ = the difference of the observed times, *minus* t
difference of right ascensions;

a = the azimuthal deviation of the transit.

In the triangle ZPA, we have

$$\sin. PA : \sin. ZA :: \sin. PZA (= \sin\ AZS) : \sin. ZPA,$$

or

$$\cos. \delta : \sin. Z :: a : ZPA = \frac{a \sin. Z}{\cos. \delta} \ldots\ldots\ldots (1)$$

$$= \frac{a \sin. (\phi - \delta)}{\cos. \delta}$$

$$= \frac{a(\sin. \phi \cos. \delta - \cos. \phi \sin. \delta)}{\cos. \delta}$$

$$= a(\sin. \phi - \cos. \phi \operatorname{tang}. \delta).$$

In the same manner, we find

$$ZPB = a(\sin. \phi - \cos. \phi \operatorname{tang}. \delta').$$

Therefore, $APB = a \cos. \phi(\operatorname{tang}. \delta - \operatorname{tang}. \delta')$,

or

$$a = \frac{APB}{\cos. \phi(\operatorname{tang}. \delta - \operatorname{tang}. \delta')}.$$

But, by Trig., Art. 76,

$$\operatorname{tang}. A - \operatorname{tang}. B = \frac{\sin.(A - B)}{\cos. A \cos. B}.$$

Hence .

$$a = \frac{\Delta \cos. \delta \cos. \delta'}{\cos. \phi \sin. (\delta' - \delta)}.$$

The sign of Δ is determined from the equation

$$\Delta = (T^n - T) - (R^n - R),$$

where T^n and R^n represent the observed time and right asce
sion of the most northern star.

Example. Aug. 18, 1850, the transit of θ Ceti (Dec. 8° 57′ S.) was observed at Greenwich at 1h. 16m. 0.95s., and that of Polaris (Dec. 88° 30′ N.) at 1h. 5m. 17.63s., the difference of the tabular right ascensions of the stars being 10m. 40.39s. Required the azimuthal error.

The difference between the observed passages is 10m. 43.32s.; hence $\Delta = -2.93$s.

$$\begin{aligned} 2.93\text{s.} = 43''.95 \text{ log.} &= 1.6430 \\ \cos.\ \delta &= 8.4158 \\ \cos.\ \delta' &= 9.9947 \\ \sec.\ \phi &= 0.2056 \\ \text{cosec. } 97^\circ\ 27' &= 0.0037 \\ a = -1''.83 &= \overline{0.2628} \end{aligned}$$

The azimuthal deviation may, in like manner, be found by comparing any two stars differing considerably in declination, and whose places are known; but it is always best to employ Polaris, or some close circumpolar star, for one of the stars.

(94.) *Third Method.—By two circumpolar stars at opposite culminations.*

There are two stars near the north pole which culminate nearly at the same time, one above and the other below the pole. These stars are 51 Cephei and δ Ursæ Minoris. The observation of these stars affords one of the best methods of determining the deviation of the transit from the meridian. The method of reduction is the same as in Art. 93, except that instead of $\delta' - \delta$ we must put $\delta' + \delta$, since one star is below the pole. The observed transits must first be corrected for the errors of level and collimation. Then put $\Delta =$ the difference of the observed times, *minus* the difference of the right ascensions, neglecting the 12 hours. The error in azimuth will be

$$a = \frac{\Delta \cos.\ \delta' \cos.\ \delta}{\cos.\ \phi \sin.(\delta' + \delta)}.$$

Example. On the 9th of February, 1850, the transit of δ Ursæ Minoris (Dec. 86° 35′ 43″), sub polo, was observed at Greenwich at 6h. 19m. 29.74s., and that of 51 Cephei (Dec. 87° 15′ 26″) at 6h. 28m. 1.58s.; the difference of the tabular right ascension of the stars being 12h. 8m. 20.99s. Required the error in azimuth.

$$\Delta=10.85\text{s.}=162''.75 \quad \log.=2.2115$$
$$\cos.\ \delta'=8.6799$$
$$\cos.\ \delta=8.7737$$
$$\sec.\ \phi=0.2056$$
$$\text{cosec.}\ 6^\circ\ 8'\ 51''=0.9703$$
$$a=+6''.93=\overline{0.8410}$$

Hence the error in azimuth is +6″.93.

Since the observed interval between the stars was too great, it is plain that the telescope pointed to the west of the true meridian on the north horizon.

PROBLEM.

(95.) *To compute the correction to the time of transit for the error of azimuth.*

According to equation (1), Art. 93,

$$\text{ZPA}=\frac{a\ \sin.\ Z}{\cos.\ \delta};$$

that is, the numerical correction, in seconds of time, to each transit is equal to the azimuthal error, expressed in seconds of time, multiplied by the sine of the zenith distance and the secant of the star's declination.

Ex. 1. The transit of Castor, Dec. 32° 12′ 32″ N., was observed at Greenwich, February 22d, 1851, at 7h. 24m. 6.52s., the azimuthal error being −8″.32. Required the corrected time of transit.

The zenith distance of the star was 19° 16′ 7″.

$$\sin.\ Z=9.5185$$
$$\sec.\ \delta=0.0726$$
$$8''.32=0.5546\text{s.}=9.7440$$
$$-0.22\text{s.}=\overline{9.3351}$$

Hence the time of transit corrected for error of azimuth is 7h. 24m. 6.30s. By combining the results of pages 64 and 67, we find the time of transit corrected for errors of level, collimation, and azimuth to be

7h. 24m. 6.52s.−0.29s.−0.07s.−0.22s.=7h. 24m. 5.94s.

Ex. 2. It is required to compute the corrections for the following observations made at Greenwich, February 22d, 1851:

Star	Declination.	Observed Transit.	Error of			Seconds of Transit Corrected.
			Collim., $-0''.93$.	Level, $-3''.92$.	Azimuth, $-8''.32$.	
	°	h. m. s.	s.	s.	s.	s.
β Tauri	28 28 N.	5 15 53.56	−0.07	−0.27	−0.25	52.97
δ Ursæ Minoris S. P...	86 36 N.	6 19 16.31	+1.05	+3.27	−6.24	14.39
Sirius	16 31 S.	6 37 36.32	−0.06	−0.10	−0.54	35.62
Antares	26 5 S.	16 19 17.27	−0.07	−0.06	−0.60	16.54

The errors of collimation, level, and azimuth being taken, as given at the head of columns 4, 5, and 6, the computed corrections are as given above, and the corrected seconds of transit are given in the last column above.

(96.) The preceding results require to be still further corrected for the error of the clock, and we shall obtain the apparent right ascension of the object observed. Hence we have

$$\text{R.A.} = (\text{T} + dt) + a\frac{\sin.(\phi - \delta)}{\cos.\ \delta} + b\frac{\cos.(\phi - \delta)}{\cos.\ \delta} + \frac{c}{\cos.\ \delta}, \text{ where}$$

R.A. = the apparent right ascension required.

T = the observed time of transit, as shown by the clock.

dt = the correction for error of the clock; plus when the clock is too slow.

a = the deviation of the telescope in azimuth; plus when the eastern pivot deviates to the north of east.

b = the inclination of the axis of the telescope; plus when the west end of the axis is too high.

c = the error in collimation; plus when the mean of the wires falls on the east side of the optical axis.

ϕ = the latitude of the place.

δ = the declination of the star.

(97.) The coefficients of a, b, and c, being of daily use in the reduction of observations, should be computed for each observatory. Table IX. furnishes their values for Washington Observatory, by means of which the reductions are made with great facility.

Example. It is required to compute the corrections for the following observations made at Washington Observatory, December 30th, 1845:

Star.	Declination.	Observed Transit.	Error of			Seconds of Transit Corrected.
			Azimuth, −0.301s.	Level, +0.249s.	Collim., −0.085s.	
	° ′	h. m. s.	s.	s.	s.	s.
α Persei.......	+49 18	3 13 55.67	+0.083	+0.376	−0.130	56.00
γ Eridani......	−13 57	3 51 24.14	−0.247	+0.155	−0.088	23.96
α Tauri	+16 12	4 27 39.13	−0.121	+0.239	−0.088	39.16
α Aurigæ......	+45 50	5 5 53.76	+0.052	+0.355	−0.122	54.04
β Tauri	+28 28	5 17 7.60	−0.062	+0.279	−0.097	7.72

The errors of azimuth, level, and collimation being taken, as given at the head of columns 4, 5, and 6, the corrections given above are readily found by employing the coefficients of table IX.; and the corrected times of transit are given in the last column.

(98.) *Of the figure and unequal size of the pivots of the transit instrument.*

The pivots of the horizontal axis of the transit instrument ought to be perfectly cylindrical. By the assistance of the level we can easily determine whether such is the case. For this purpose we place the level upon the pivots, and point the object end of the telescope downward, as low as possible; we then direct the telescope upward, as far as the level will permit. If the bubble of the level remains stationary during this rotation, we may, with great probability, assume that the pivots are cylindrical. This conclusion, however, is not necessarily exact; for if the sections of the pivots were perfectly equal curves, of whatever kind, symmetrically placed with respect to the axis of rotation, the bubble would not be disturbed during the revolution of the telescope. As, however, the coincidence of all these conditions is not to be expected, we may safely assume the pivots to be cylindrical when they will stand the preceding test.

(99.) If the pivots are made perfectly cylindrical, but of unequal diameters, when the level is placed upon the pivots, and the telescope revolved, the bubble will not change its position, but the inclination of the axis, shown by the readings of the level, will be erroneous; for if the axis of rotation were perfectly horizontal while the pivots were unequal, the level would indicate an inclination, and the thickest end of the axis would appear to be higher than the other. Let this inclination be accurately measured by means of the level. Now reverse the axis of the telescope. If the largest pivot was before on the east side of the telescope, it will now be on the west side, and the inclination of the axis will be changed. Let the inclination be again accurately measured by means of the level. One half the difference between the level errors in the two positions of the axis gives the effect of the difference in the diameter of the pivots; and one fourth the difference gives the effect of the difference in the radii of the pivots, which is a correction to be always subtracted from the larger end.

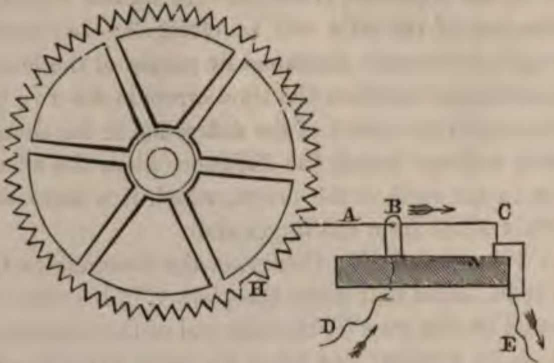
(100.) For example, Mr. Curley, at the Georgetown Observatory, in 1846, found that when the pivot C of his transit instrument rested on the west Y, the east end of the axis appeared to be too high by 1″.046; but when the same pivot rested on the east Y, the east end of the axis appeared to be too high by 1″.660. Hence the pivot C is the largest, and the correction to the level error on account of the difference in the diameter of the pivots is $\frac{1.660-1.046}{4}=0''.153$. This correction is to be applied to all level readings with this instrument, inasmuch as the level determines only the inclination of the tops of the pivots, while in transit observations we require to know the inclination of the *axis* of the pivots.

TRANSIT OBSERVATIONS RECORDED BY MEANS OF ELECTRO-MAGNETISM.

(101.) Quite recently there has been introduced a new method of recording transit observations by means of electro-magnetism. This application involves two contrivances entirely distinct from each other. The first is a method by which an astronomical clock may be made to break the electric circuit at the end of every second; and the other is the register, for recording not only the beats of the clock, but also any other arbitrary signals at the pleasure of the operator.

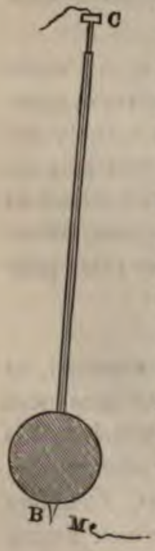
1st. *The electric clock.*

(102.) The electric circuit may be broken every second, by means of a clock, in a variety of ways. Dr. Locke introduced into the astronomical clock a wheel with 60 teeth, which makes one revolution per minute. Each tooth, in succession, strikes against the handle of a platinum tilt-hammer, AC, weighing about two grains, and knocks up the hammer, which almost



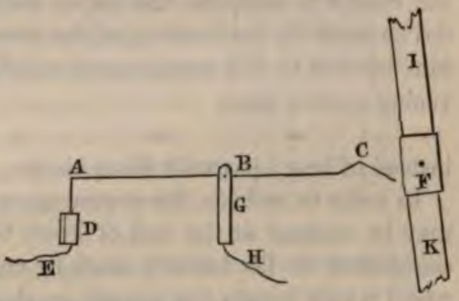
immediately falls to a state of rest on a bed of platinum. The fulcrum, B, of the tilt-hammer and the platinum bed rest, severally, on a small block of wood. Each is connected, by wires D and E, with a pole of the galvanic battery, and the circuit is alternately broken and completed by the rising and falling of the hammer. The circuit is open about one tenth of a second, and closed the remaining nine tenths of each second.

(103.) Professor Bond insulates the axis of the escapement wheel, and also the axis of the steel pallets, by a ring of shellac. Wires from the two poles of the battery are connected with each axis, so that when either pallet comes in contact with an escapement tooth, the galvanic circuit is closed; and when the contact is broken (as it must be at every oscillation of the pendulum), the galvanic circuit is opened.



(104.) At the Washington Observatory the same object was accomplished in the following manner: A small piece of metal, M, is attached to the back of the clock, near the lower extremity of the pendulum, and upon it is placed a small globule of mercury, so that the index, B, attached to the lower extremity of the pendulum may pass through the globule of mercury once every vibration. A wire from one pole of the battery is connected with the supports of the pendulum, C, and another wire from the other pole of the battery connects with the metallic support of the mercury globule. If, now, the pendulum were at rest, with the pointer, B, in the mercury, it is evident that the electric circuit would be complete through the pendulum. If, then, the pendulum be set in motion, it will break the circuit whenever it passes out of the mercury, and restore it again as soon as it touches the mercury.

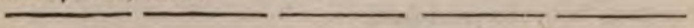
(105.) Mr. Saxton employed a small tilt-hammer, like Dr. Locke, but he broke the circuit by means of a small glass pin projecting from the pendulum.



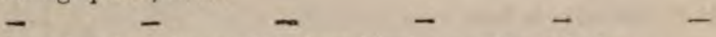
ABC represents a fine platinum wire, mounted upon a pivot at B, the end A being somewhat heavier than the other, and resting upon a metallic bed, D. At C, the wire is bent so as to form an obtuse angle. The wire E goes from D to one pole of the battery, while the wire H, from the other pole of the battery, communicates with the metallic support, G, and thence with the wire AB. When the end A of the platinum wire rests upon the support D, it is evident that the electric circuit is complete. This apparatus is placed near the middle of the pendulum (a portion of which, IK, is represented in the cut), and just in front of it, so that the pendulum may swing behind it without obstruction. A small glass pin, F, about half an inch in length, is attached to the pendulum in such a position that, at every vibration of the pendulum, the pin shall slightly impinge upon the angle C of the platinum wire, and force up the end A. As soon as the pin has passed the point C, the end A falls back again upon its support, D. Thus, at every vibration of the pendulum, the end A of the platinum wire is lifted about a tenth of a second, and rests upon D during the remaining nine tenths of the second. At present (1881), at the Washington Observatory, the electric circuit is closed by a small platinum point which projects from the middle of the pendulum rod, and at each vibration touches the surface of a small globule of mercury.

2d. *The Register.*

(106.) The most obvious mode of registering the beats of the clock is upon a long fillet of paper, after the ordinary method of telegraphic communications. If the paper be allowed to run through an ordinary Morse registering apparatus, and the circuit be broken every second by the clock, the graver will trace upon the paper a series of lines of equal length, separated by short interruptions, thus:



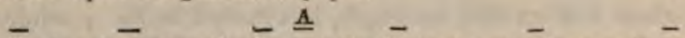
It is easy to reverse the action of the graver, so that, when the circuit is complete, the paper shall be entirely free, and a dot be made by the breaking of the circuit. A paper graduated into seconds by this arrangement exhibits dots with long intervening spaces, thus:



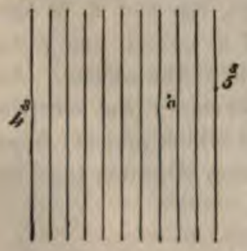
instead of long lines with short blanks, as shown before.

In order to indicate the commencement of the minute, a dot may be omitted at the end of every 60 seconds. This is accomplished in Dr. Locke's clock by omitting one tooth in the wheel which breaks the circuit, as shown at H, in the figure, page 76.

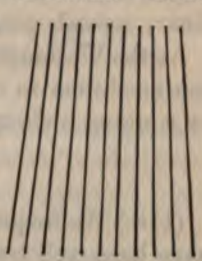
(107.) The mode of using the register for marking the date of any event, is to tap on a break-circuit key simultaneously with the event. The beginning of the short line thus printed upon the graduated scale of the register, fixes, by a permanent record, the date of the event. Thus A represents such a record printed upon the graduated paper.



By tapping upon the key at the instant a star is seen to pass each of the wires of a transit instrument, the observation is instantly and permanently recorded. The usual rate of progress of the fillet under the pen is about one inch per second, and the observations are read off by means of a graduated transparent scale, about an inch square, as represented in the annexed cut, consisting of equidistant and parallel lines, ruled upon a piece of glass by means of a diamond, or etched with fluoric acid. If the interval between the second dots be greater than the breadth of the scale,



the scale is turned obliquely across the fillet, until the first and last divisions exactly comprehend the space between the two second dots. Let the distance from 4s. to 5s., on the above scale, be the distance on the fillet between the fourth and fifth seconds, and let the dot, *a*, between them represent the observation. It appears, by inspection, that the observation was recorded between 4.7 and 4.8 seconds. The distance of *a* from the nearest scale division may be estimated to tenths. Thus time is accurately measured to tenths, and may be estimated to hundredths of a second. On some accounts, it is more convenient to employ a scale consisting of diverging lines, as represented in the annexed cut, so that the breadth of the scale may always exactly comprehend the interval between the second dots, which intervals must necessarily vary somewhat in length.



(108.) This method of recording transits not only possesses the advantage of precision, but also of performing vastly more work in a given time. Fifteen seconds is the ordinary equatorial interval for the wires of a transit instrument; but when the transits are printed on paper, in the manner now described, this interval may easily be reduced to two or three seconds. The value of a night's work with the transit instrument is thus increased many fold.

To obviate the inconvenience of a long fillet of paper, Mr. Saxton has substituted a cylinder, about eight inches in diameter and two feet long, enveloped with paper, which may be removed at pleasure. This cylinder is made to revolve, with a uniform motion, upon a screw axis, so that the recording dots are made upon a perpetual spiral. One sheet, filled in this manner, will contain about two hours' work with a transit instrument.

(109.) In order to secure the full advantage of the preceding method, it is important that the paper which contains the register be made to advance with entire uniformity. The Messrs. Bond have invented for this purpose a machine which they call the Spring Governor, consisting of a train of clock-work connected with the axis of a fly-wheel. It has an escapement-wheel, into the teeth of which pallets are operated by the oscillations of a pendulum, as in ordinary clocks, the wheel being so connected with its axis by a spring as to allow the axis to move while the wheel is detained by the pallets. The register is made upon a sheet of paper wrapped round a cylinder.

Professor Airy, in order to impart a uniform motion to the paper, employs a large conical pendulum, revolving in a circle, whose diameter is about equal to the arc of vibration of an ordinary seconds pendulum.

At the Washington Observatory the recording cylinder makes one revolution in a minute, and its uniform motion is secured by a spring making 132 vibrations in a second.

PERSONAL EQUATION.

(110.) We frequently find that two individuals, both of whom have been well trained in transit observations, will differ by a large and nearly constant quantity in observing the exact moment at which a star passes a transit wire. This difference is called their *personal equation;* and an allowance should always be made for it whenever observations, which have been made by two individuals for the determination of absolute time, are to be combined. This equation may be determined by either of the following methods:

(111.) *First Method.*—Let one observer note the passage of a star over the first three or four wires of the transit instrument, and the other observer note the same star over the remaining wires. Let each set of observations be reduced to the mean wire by employing the equatorial interval previously determined. The difference between the two mean results thus obtained is the personal equation of the observers. A dozen stars observed in the course of an hour, will furnish this equation within a few hundredths of a second.

(112.) *Second Method.*—The same object may be accomplished still more conveniently by employing an equatorial telescope. Place the two threads of the micrometer at a distance from each other equal to about ten seconds of time, and adjust them to the position of an hour circle. Direct the telescope upon a star near the meridian, and let the two astronomers observe the passage of the star over the two wires alternately. By means of the tangent screw belonging to the hour-circle, bring the star back again, and repeat the observation as many times as may be thought necessary; suppose, for example, 20 times. At 10 of these observations, the individual A should have made the observation at the first wire, and the individual B at the second; and *vice versâ* for the other 10 observations. Let M represent the mean of the first set of observations, and M′ the mean of the second set of observations; then will the personal equation be

$$\frac{M-M'}{2}.$$

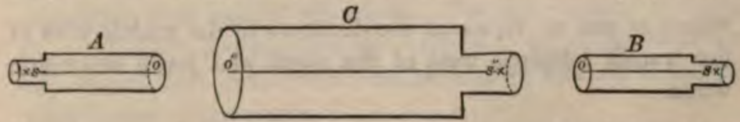
In 1843, Dr. Peterson, at Altona, and M. O. Struve, of the Pulkova Observatory, from a series of observations made $M=7.175$s., and $M'=7.581$s. Hence their personal equation was 0.203s.

In the same manner, the personal equation between Dr. Peterson and M. Sabler, of the Pulkova Observatory, was found to be 0.324s.

The personal equation between the late Professor Henderson, of the Edinburgh Observatory, and Mr. Wallace, his assistant, was 0.42s.

This personal equation is but another name for positive error in the estimation of fractions of a second; and it not only varies with different individuals, but varies with the same individual at different times. The amount of this error in skillful and long-practiced observers is truly surprising. Observations made by different individuals for the determination of absolute time should therefore never be combined without investigating the personal equation of the observers.

(113.) *Collimating telescopes.*—As a substitute for distant meridian marks, the following arrangement is now generally adopted in the large observatories. Two small telescopes, A and B, are mounted on piers in the transit room with their optical axes nearly in the same line with that of the telescope, C, of the transit instrument, one north and the other south of it. The



object-glasses of the telescopes A and B are turned towards each other, and therefore towards the transit instrument.

Let o, o', and o'' be the optical centres of the three object-glasses; let s, s', and s'' be the foci for parallel rays, and suppose each of the collimators to have a single vertical wire in its focus. Rays of light proceeding from a wire at s, s', or s'' will emerge from the object-glass of the telescope as parallel rays. If the telescope C be pointed upon A, an image of the wire at s will be formed at s''; and if the telescope C be pointed upon B, an image of the wire at s' will be formed at s''. In the north and south sides of the central cube of the transit instrument let a circular aperture be made, so that when the axis of the telescope C is placed vertically, an image of the wire of either collimator will be formed at the focus of the other, and either wire may be adjusted so as to coincide exactly with the image of the other. The two sight lines of the collimators will then be in the same straight line, or will be parallel to each other, and their wires when viewed by the transit telescope C will represent two objects infinitely distant, whose difference of azimuth is 180°. Let now the telescope of the transit instrument be directed towards the collimator A. An image of the wire at s will be formed at s'', and we will suppose that it does not coincide exactly with the central wire of the transit telescope, but is seen at a small distance from it upon its west side. Let this distance, m, be carefully measured by a micrometer. Now revolve the telescope C upon its rotation axis and direct it towards the collimator B. An image of the wire at s' will be formed at s'', but it probably will not coincide with the middle wire of the transit telescope. Let this distance, m', be measured by the micrometer. We shall then have the collimation error equal to

$$\tfrac{1}{2}(m+m'),$$

where m and m' represent the distances of the middle wire of the transit telescope west of the north and south collimator wires.

CHAPTER III.

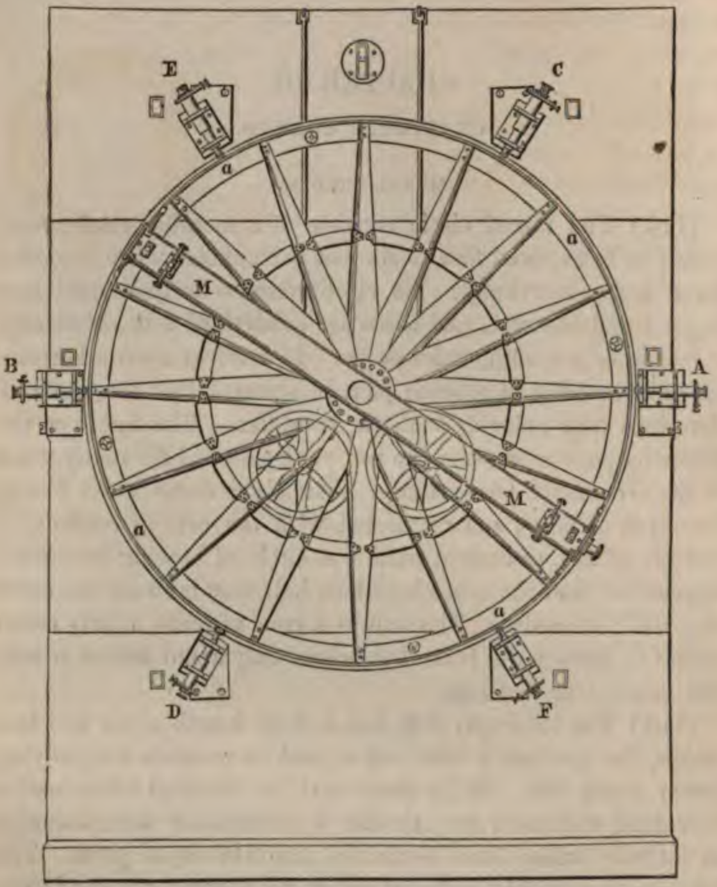
GRADUATED CIRCLES.

MURAL CIRCLE.

(114.) THE mural circle consists of a metallic circle, commonly of brass, from four to six feet in diameter when intended for a large observatory. Its circumference is graduated into degrees and minutes, and these are subdivided into seconds by a vernier or a reading microscope. It revolves upon a horizontal axis, inserted in a stone pier, so situated that the plane of the circle may coincide with the meridian. The figure on the following page represents the mural circle used for many years at the Greenwich Observatory. The circle *aaaa* is six feet in diameter, of brass, and connected with the central nucleus by sixteen spokes, or conical radii. A circle of bracing bars is interposed to bind the cones together, half way between the outer ring and the centre. The axis is a cone of brass, nearly seven inches in diameter in front, but behind only about half as much, and nearly four feet long.

(115.) The telescope, MM, has a focal length of six feet two inches, the aperture is four inches, and its common magnifying power about 150. At its focus are five vertical wires, and a horizontal stationary one, besides a micrometer wire, movable in altitude, whose head is divided into 100 equal parts. The telescope is attached to the circle at the centre by a steel axis, which passes through the proper axis of motion from end to end, so that the motion of the telescope round its own axis is concentric with that of the circle. For the purpose of fixing the telescope in any position with respect to the circle, there are two clamps, one at each end, which may be secured to the exterior border of the circle.

The limb of the circle consists of two rings, the interior one having its plane parallel, and the exterior one perpendicular to the plane of the circle, so that, when united, their section is represented by the letter T. The graduation is made on the broad surface of the exterior ring. The divisions are made upon a narrow ring of white metal, composed of four parts of gold to one of palladium; and the figures which count the degrees are engraved upon a similar ring of platina. Neither of these metals tarnishes in the least degree. The degrees are cut into twelve parts, or 5′ spaces, and are numbered from the pole southward to the same pole again, viz., from 0° to 360°.



(116.) Placed at equal distances round the circle, and firmly attached to the pier, are six reading microscopes, A, B, C, D, E, F, with an acute cross of wires at their foci for measuring angles less than five minutes. *Fig.* 1 represents the appearance of one of these microscopes.

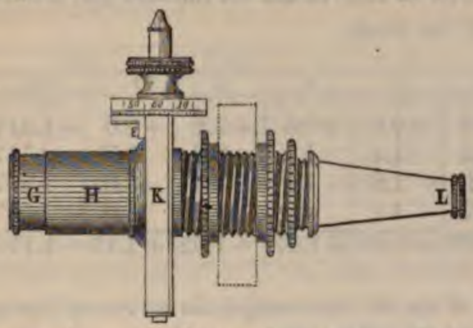
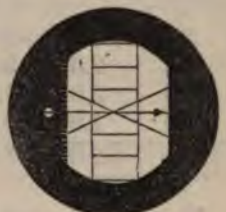


Fig. 2.



It is a compound microscope, consisting of three lenses, one of which is the object lens, at L, and the other two are formed into a positive eye-piece, G H. In the common focus of the object lens and the eye-piece, at K, is placed the spider-line micrometer, similar in principle to that described in Art. 38. It consists of a small rectangular frame, across which are stretched two spider lines, forming an acute cross, and is moved laterally by means of a screw, whose head is divided into 60 equal parts. *Fig.* 2 shows the field of view, with the magnified divisions on the instrument, as seen through the microscope. When the microscope is properly adjusted, the image of the divided limb and the spider lines are distinctly visible together; and, also, five revolutions of the screw must exactly measure one of the 5′ spaces on the limb. If the five revolutions do not include the whole of one space, the object lens must be screwed up toward the image of the limb, and the position of the microscope altered till distinct vision is obtained both of the spider lines and the divisions of the limb. It may require repeated trials before these conditions are completely fulfilled. Moreover, it is found that changes of temperature and other causes produce a continual variation in the value of one division by the microscope. Each of the microscopes must therefore be examined from time to time, and allowance made for *error of runs.* It is usual to measure the value of one division of the circle by each microscope, at several different parts of the circle, and take the mean. The following example is taken from the Washington observations of 1845:

July 15, 1845. Error of runs of the six microscopes determined for four points of the circle.

Pointer.	A.	B.	C.	D.	E.	F.	Mean.
°	″	″	″	″	″	″	″
72	+2.0	+2.3	+2.5	+0.5	+1.2	+0.0	+1.417
112	2.5	3.1	1.4	1.7	2.3	1.7	2.117
240	2.0	1.9	1.8	1.0	2.5	2.0	1.867
360	1.8	2.2	1.7	1.9	1.7	1.0	1.717
Mean.	+2.07	+2.37	+1.85	+1.27	+1.92	+1.17	+1.779

The average error of the six microscopes for an arc of 5 minutes, July 15th, was +1″.779. Consequently, smaller arcs, which are measured by the microscopes, should have a proportional part of this error applied to them; that is, the correction due to an observation for error of runs is

$$-\frac{\text{M}\,.\,\text{R}}{5'},$$

where R represents the observed error of runs, and M is the mean of the microscopes, omitting the largest contained multiple of 5 minutes.

(117.) The axis of the circle is made horizontal by the aid of a plumb line, suspended in front of the circle, and viewed by two microscopes, one near the top, and the other near the bottom of the circle. Or, as this instrument is supposed to be used in conjunction with a transit instrument, we may render the axis horizontal by moving the adjusting screws, so as to make a zenith star pass the middle wire at the instant the star is passing the middle wire of the transit. We may also bring the plane of the circle into the meridian by selecting a star near to the horizon, and moving the proper screws so as to cause it to pass the middle wire at the same instant that it passes the middle wire of the transit.

(118.) To make an observation with the mural circle, the telescope is pointed upon a star just before it passes the meridian, and, by means of the tangent screw, the telescope is moved in altitude until the star appears bisected by the horizontal wire. An index or pointer shows the number of degrees, and the nearest five minutes, while the minutes less than five and are obtained from the microscopes.

The following observations were made at Washington, November 28, 1845:

Stars.	Pointer.		A.		B.	C.	D.	E.	F.
	°	′	′	″	″	″	″	″	″
η Tauri	315	15	0	37.5	65.0	55.3	39.6	75.7	37.7
α Tauri	322	40	1	15.5	44.6	36.0	18.6	56.2	14.7
α Orionis	331	30	0	22.5	52.0	43.0	26.0	63.3	21.0
ε Canis Majoris .	7	35	1	45.6	77.8	66.0	53.2	83.6	43.6

It is required to determine the true circle readings, the error of runs on an arc of 5′ at the time of each of the preceding observations being −1″.51; −1″.76; −1″.81; −1″.83.

The mean of the seconds readings by the six microscopes for η Tauri is 51″.80. The error of runs for 5′ being −1″.51, the error for 51″.8 will be −0″.27, which, subtracted from the preceding mean, gives 52″.07 for the number of seconds. The pointer indicates 315° 15′. Hence the concluded circle reading is 315° 15′ 52″.07.

For α Tauri, the mean of the seconds readings by the six microscopes is 30″.93; correction for runs, +0″.54, making 31″.47. The pointer indicates 322° 40′, and the microscopes give 1′ 31″.47. Hence the concluded circle reading is 322° 41′ 31″.47.

For α Orionis, mean of the six microscopes, 37″.97; correction for runs, +0″.23.

Concluded circle reading, 331° 30′ 38″.20.

For ε Canis Majoris, mean of microscopes, 61″.63; correction for runs, +0″.77.

Concluded circle reading, 7° 37′ 2″.40.

These results require to be still further corrected for refraction, which is furnished by Table VIII.

(119.) *To determine the horizontal point, or the zenith point, on the limb of the circle.*

Point the telescope upon any known star when it crosses the meridian, and record the reading of the circle. On the next night, observe the same star as it crosses the meridian, by pointing the telescope upon the image of the star reflected from the surface of mercury. As the surface of a fluid at rest is horizontal, and as the angle of [illegible] is equal to the angle of incidence, this image will [illegible] much depressed below the horizon as the star its[illegible] arc intercepted on the limb of the circle, between the star and its reflected image, is the double altitude of the star, and its middle point is the horizontal point of the circle, allowing for the difference of refraction at the moments of observation. By skillful management it is possible to observe the star *on the same night*, both by reflection and direct vision, sufficiently near to the meridian to give the horizontal point without risking the change of refraction in 24 hours.

(120.) This may be effected in the following manner: Several minutes before the star in question comes to the meridian, let the telescope be pointed downward upon a basin of mercury, previously placed in the proper position to see the star reflected from its surface. Let the telescope be firmly clamped, and the six microscopes be read and registered. When the star enters the field of the telescope, let it be followed by the micrometer wire which moves in altitude, and let it be accurately bisected at the instant the star passes the first vertical wire. Then unclamp the telescope and point it upward toward the star; and, by means of the tangent screw, let the telescope be moved in altitude until the star is brought upon the fixed horizontal wire, and let it be accurately bisected at the instant of its passing the last vertical wire. The observer may then read the microscopes at his leisure, and also the micrometer of the telescope. Knowing the value of one revolution of the screw, the first observation is easily reduced to the fixed horizontal wire, so that we have secured a reflected observation at the first vertical wire, and a direct observation at the last vertical wire. Both of the observations are to be reduced to the middle wire, as explained in Art. 174. The mean of the two observations thus corrected furnishes the horizontal point on the circle.

(121.) The *nadir* point, and, consequently, the zenith point of the circle, may also be found in the mode described in Art. 87. When the telescope is directed vertically downward upon a basin of mercury, and the reflected image of the horizontal wire is brought to coincide with its direct image, the telescope is directed toward the *nadir*, which is distant 90 degrees from the horizontal point, or 180 degrees from the zenith point. As this observation can be made at any time independently of the weather, it is a most valuable method, and in many observatories is the one exclusively employed.

(122.) The horizontal point, determined by direct and reflected observations, should differ exactly 90° from the zenith point, determined by the collimating eye-piece. By combining the two methods, therefore, we have the means of testing the accuracy of each of them. The following are the results of observations made at Washington in 1845:

	Star.	Horizontal Point.			Zenith Point.			Difference.
		°	′	″	°	′	″	″
Aug. 1	δ Ursæ Minoris .	330	0	1.16	240	0	1.30	−0.14
" 4	δ Ursæ Minoris .	330	0	1.42	240	0	0.65	+0.77
" 18	δ Ursæ Minoris .	30	0	0.01	300	0	1.00	−0.99
" 23	α Ursæ Minoris .	29	57	34.17	299	57	35.23	−1.06
Oct. 30	γ Cephei	100	0	3.02	190	0	3.22	−0.20
Nov. 7	γ Cephei	120	0	8.50	210	0	8.40	+0.10
" 18	γ Cephei	160	0	11.80	250	0	11.69	+0.11

During the interval of these observations, the position of the telescope was repeatedly changed, so that the horizontal point was brought upon different parts of the circle. The last column in the above table shows the errors of the observations, combined with the error in the graduation of the circle; yet the resulting error, it will be seen, is scarcely appreciable.

TRANSIT CIRCLE.

(123.) As the mural circle has a short axis, its position in the meridian is unstable, and therefore it can not be relied upon to give the right ascension of stars with great accuracy. It was formerly thought necessary at Greenwich to have two instruments for determining a star's place; viz., a transit instrument to determine its right ascension, and a mural circle to determine its declination. The German astronomers have, however, combined both instruments in one, under the name of meridian circle, which is essentially the transit instrument already described, with a large graduated circle attached to its axis. Until recently, the English astronomers have generally contended that this combination was only suited to instruments of moderate dimensions; but a large transit circle has lately been constructed for Greenwich Observatory, under the direction of Professor Airy. The telescope has an aperture of eight inches, and a focal length of 11½ feet. The length of the axis between the extremities of the pivots is six feet, and the diameter of each pivot is six inches.

The circle is six feet in diameter, and is made of cast iron. This instrument has been in constant use since the commencement of the year 1851, and the old transit instrument and mural circle have been abandoned.

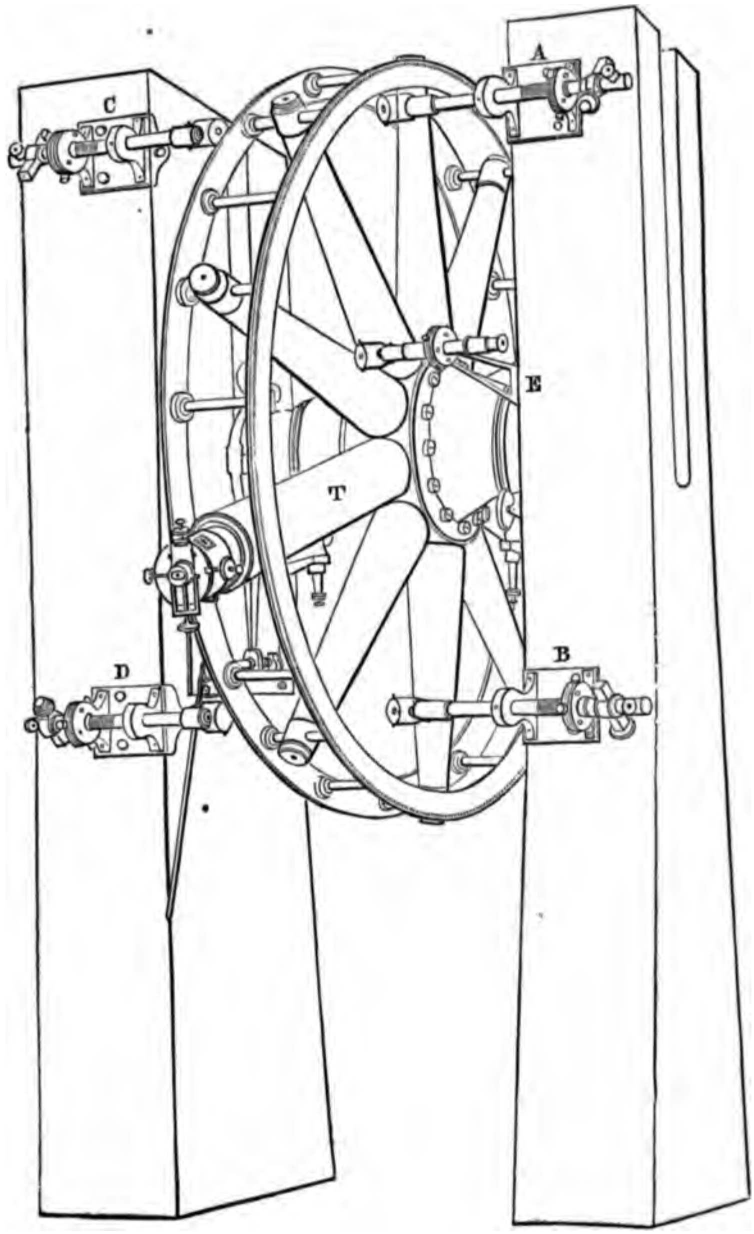
(124.) The figure on the opposite page represents the transit circle belonging to the observatory at Cambridge, Massachusetts. The telescope, T, has an object-glass of four and one eighth inches aperture, and five feet focal length. The length of the axis between the shoulders of the pivots is twenty-six inches; the pivots are of steel, two and a half inches in diameter, and the same in length. The eye-piece is provided with two micrometers, one having a vertical, and the other a horizontal movement. Besides the usual mode of illuminating the field through the axis, there are facilities for illuminating the wires in a dark field. The circles are four feet in diameter, being cast in one piece, and are both graduated on silver, from 0° to 360°, into five-minute spaces. There are eight micrometer reading microscopes, and these are attached immediately to the granite piers being four for each circle. Four of these microscopes are seen at A, B, C, and D, the other four are on the opposite side of the piers. These microscopes serve to bisect diametrically both circles. The five-minute spaces of the limbs are subdivided by the micrometers, a single division of the micrometer head being equal to one second of arc, and may be read, by estimation, to two tenths of a second. The arm, E, attached to the pier, supports an additional microscope, which serves as a pointer to indicate the degrees and minutes approximately. There are friction wheels for relieving the pressure of the axis pivots upon the Y's, supported by plates secured to the piers.

For leveling the axis, a striding level is employed, which, combined with the method of reflection from quicksilver at the nadir point, affords an independent means of ascertaining the amount of collimation of the mid wire without reversal of the pivots. There is, however, apparatus for reversing the instrument.

The object-glass is by Merz, of Munich; the mounting by Simms, of London.

With this instrument one observer can, at the same time, determine both the right ascension and declination of a star with

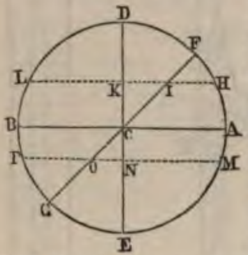
TRANSIT CIRCLE.



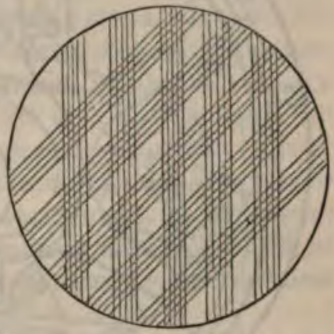
as great precision as it can be done by two observers with an ordinary transit instrument and a mural circle.

Differences of declination recorded by electro-magnetism.

(125.) *Differences* of declination may be recorded by means of electro-magnetism. This is accomplished by inserting in the focus of the meridional telescope two systems of spider lines, one vertical, and the other inclined at an angle of 45°. Let AB represent the horizontal wire of the transit instrument, DE the middle vertical wire, and FG a wire inclined to the latter, at an angle of 45°. Let the telescope be pointed upon a star as it approaches the meridian, and let it be bisected by the wire AB, while the time of passing the vertical wire, DE, is recorded. Let the telescope remain firmly fixed in its position, and suppose a second star enters the field at H, and traverses the path HL. Let the instants of passing FG at I, and DE at K, be recorded. Then, if the angle DCF is 45°, CK (which is the difference of declination of the two stars) will be equal to KI. The line KI is measured by the time required for the star to describe this portion of its path; and, in order to convert the observed time into arc of a great circle, we must multiply it by fifteen times the cosine of the star's declination, according to Art. 72. If a third star enters the field at M, and crosses the wire DE at N, and FG at O, then CN is the difference of declination of the first and third stars; and, in the same manner, by observing the transits of any number of stars over the wires DE and FG, in the same position of the telescope, we shall obtain their differences of declination, as well as of right ascension. In order to diminish the errors of observation, we introduce a large number of inclined wires, at intervals of two or three seconds from each other, as well as a large number of vertical wires; and the times of transit over each system of wires are recorded by electro-magnetism, as explained in Articles 101–109.



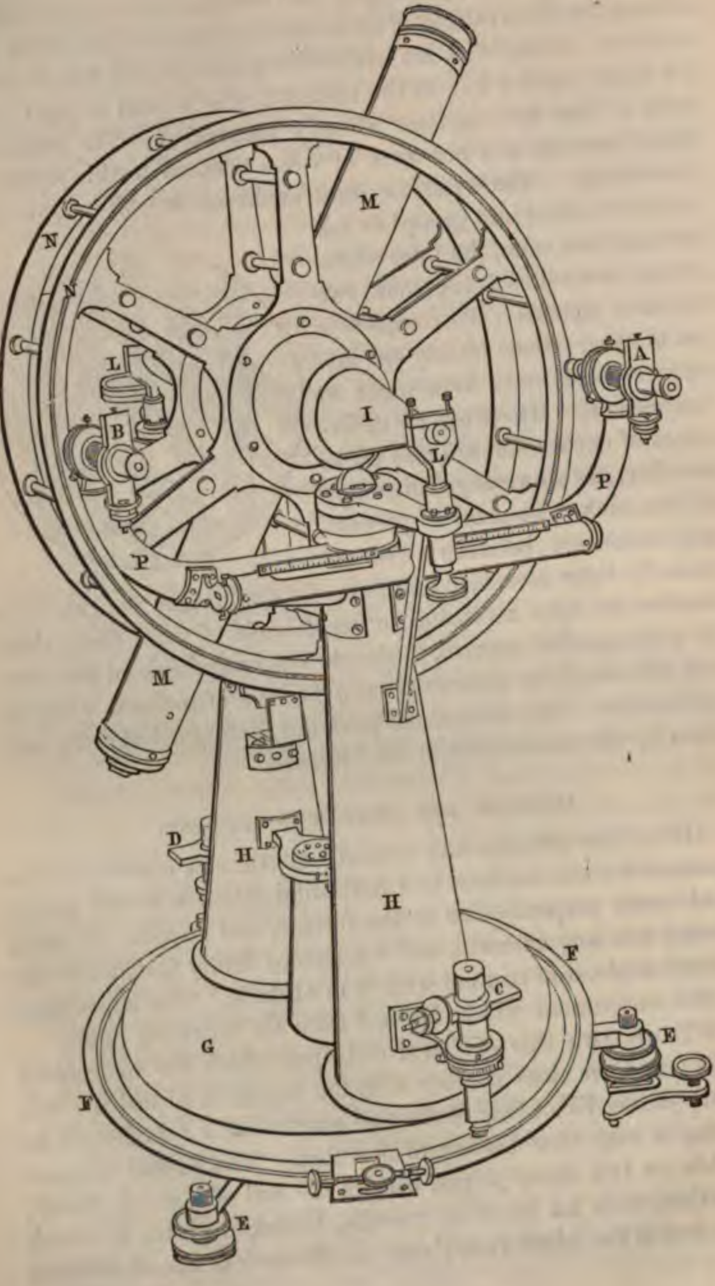
(126.) This method is well adapted to the construction of a catalogue of stars, where it is proposed to record the position of every star within the range of the telescope. For this purpose the telescope is firmly clamped, and remains fixed in its position during the observations of an entire evening or night, while the observer, sitting with his eye at the telescope, has but to press his finger upon a key at the instant a star is seen to pass each wire of the two systems already mentioned. The annexed figure represents a net-work of wires adapted to this mode of observation. The wires for right ascension are 35 in number, and are divided into groups or fascicles of five each, the interval between two wires being from two to three seconds. To complete a set of observations on any one fascicle requires only from eight to ten seconds. The wires for differences of declination are also 35 in number, and are arranged in groups of five each. In order to prevent any confusion between observations for right ascension and those for declination, the rule is, to observe for right ascension on one fascicle of wires first; then, by a telegraphic symbol, to denote the magnitude of the star; and afterward, to observe it on a fascicle of inclined wires for declination. The several fascicles are distinguished from each other by the inequalities of the intervals.



ALTITUDE AND AZIMUTH INSTRUMENT.

(127.) The altitude and azimuth instrument consists of one graduated circle confined to a horizontal plane, a second graduated circle perpendicular to the former, and capable of being turned into any azimuth, and a telescope firmly fastened to the second circle, and turning with it in altitude. The appearance of this instrument will be learned from the following figure.

EE are two legs of the tripod upon which the instrument rests; and, in close contact with the tripod, is placed the azimuth circle, FF. One of the foot screws has a contrivance for giving a very slow motion to this foot. This detached piece stands on two sharp points, besides the end of a screw, which, together, form an isosceles triangle, having a gutter in which one foot of the tripod rests; and the slowness of the adjustment


M
N
N
L
A
I
B
L
P
P
M
D
H
H
F
C
E
G
F
E

depends on the distance of this foot from the two projecting pins. This leg of the tripod is designed to be placed either to the north or south. Above the azimuth circle, and concentric with it, is placed a strong circular plate, G, which carries the whole of the upper works, and also a pointer, to show the degree and nearest five minutes to be read off on the azimuth circle; the remaining minutes and seconds being obtained by means of the two reading microscopes, C and D. The pillars HH support the transit axis, I, by means of the projecting pieces, LL. The telescope, MM, is connected with the horizontal axis in a manner similar to that of the transit instrument. Upon the axis, as a centre, is fixed the double circle, NN, each circle being placed close against the telescope. The circles are fastened together by small brass pillars, and the graduation is made on a narrow ring of silver, inlaid on one of the sides, which is usually termed the *face* of the instrument. The reading microscopes, AB, for the vertical circle are carried by two arms, PP, attached near the top of one of the pillars.

In the principal focus of the telescope are stretched spider lines, as in the transit instrument, and the illumination is effected in a similar manner.

(128.) *Of the adjustments.*

The horizontal circle is first to be leveled, which is to be effected in the same manner as with a theodolite. The axis of the telescope must also be leveled, as in the transit instrument, and the spider lines adjusted for collimation and verticality.

The meridional point on the azimuth circle is its reading when the telescope is pointed north or south, and may be determined by observing a star at equal altitudes east and west of the meridian, and finding the point midway between the two observed azimuths; or the instrument may be adjusted to the meridian, in the same manner as a transit. The horizontal point of the altitude circle is its reading when the axis of the telescope is horizontal, and may be found, as with the mural circle, by alternate observations of a star directly and reflected from the surface of mercury.

(129.) This instrument has the advantage over the transit instrument and mural circle, in its being able to determine the place of a star in any part of the visible heavens; but we ordinarily require the place of a star to be given in right ascension and declination, and not in altitude and azimuth, and to deduce the one from the other requires a laborious computation. Hence the altitude and azimuth instrument is but little used in astronomical observations, except for special purposes, as, for example, to investigate the laws of refraction.

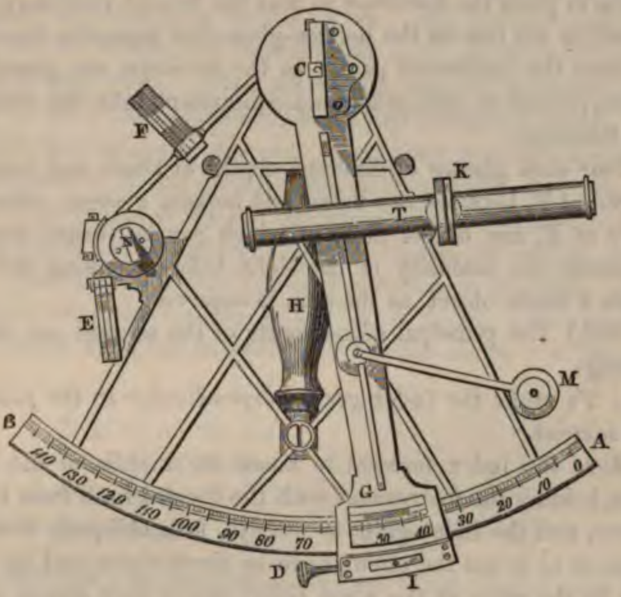
The use of this instrument has, however, been recently revived at the Greenwich Observatory. In the year 1847, an altitude and azimuth instrument was erected, having its horizontal and vertical circles each three feet in diameter. The length of the telescope is 5 feet, and the aperture of its object glass 3¾ inches.

(130.) The leading object in view in the erection of this instrument was to obtain observations of the moon in portions of her orbit where she could not be observed on the meridian. It frequently happens, from the unfavorable state of the weather, that the moon can not be seen when she is on the meridian; and although the sky may be perfectly clear, it is impossible to see the moon on the meridian for several days before and after her conjunction with the sun, on account of the brightness of the solar rays. But with the new altitude and azimuth instrument it is found that the moon may be observed in the morning and evening when she is only an hour distant from the sun. In the year 1851, observations of the moon were obtained with this instrument on 206 days, while with the meridional instruments it was only observed 110 days. Mr. Airy considers these results to be hardly, if at all, inferior in accuracy to those obtained by the use of the mural circle.

SEXTANT.

(131.) The arc of a sextant, as its name implies, contains sixty degrees, but, on account of the double reflection, is divided into 120 degrees. The figure on the opposite page represents a sextant, the frame being generally made of brass or other hard metal; the handle, H, at its back, is made of wood. When observing, the instrument is to be held with one hand by the handle, while the other hand moves the index, G. The arc, AB, is divided into 120 or more degrees, numbered from A toward B, and each degree is divided into six equal parts of 10′

each, while the vernier shows 10″. The divisions are also con-



tinued a short distance on the other side of zero, toward A, forming what is called the arc of excess. The microscope, M, is movable about a centre, and may be adjusted to read off the divisions on the graduated limb. A tangent screw, D, is fixed to the index, for the purpose of making the contacts more accurately than can be done by hand. When the index is to be moved any considerable distance, the screw I must be loosened; and when the index is brought nearly to the required division, the screw I must be tightened, and the index be moved gradually by the tangent screw. The upper end of the index, G, terminates in a circle, across which is fixed the silvered index-glass, C, over the centre of motion, and perpendicular to the plane of the instrument. To the frame, at N, is attached a second glass, called the horizon-glass, the lower half of which only is silvered. This must also be perpendicular to the plane of the instrument, and in such a position that its plane shall be parallel to the plane of the index-glass, C, when the vernier is set to zero on the limb AB.

The telescope, T, is carried by a ring, K, attached to a stem, which can be raised or lowered by turning a milled screw. Its use is to place the telescope so that the field of view may be bisected by the line on the horizon-glass that separates the silvered from the unsilvered part. In the telescope are placed two wires, parallel to each other, and equidistant from the centre of the telescope.

Four dark glasses of different depths of shade and color are placed at F, between the index and horizon glasses; also three more at E, any one or more of which can be turned down, to moderate the intensity of the light before reaching the eye, when a bright object, as the sun, is observed.

(132.) The principal adjustments of the sextant are the following:

1. *To make the index-glass perpendicular to the plane of the sextant.*

Move the index forward to about the middle of the limb; then, holding the instrument with the divided limb from the observer, and the index-glass to the eye, look obliquely down the glass, so as to see the circular arc by direct vision and by reflection in the glass at the same time; and if they appear as one continued arc of a circle, the index-glass is adjusted. If it requires correcting, the arc will appear broken where the reflected and direct parts of the limb meet. As the glass is, in the first instance, set right by the maker, and firmly fixed in its place, its position is not liable to alter, except by violence; and therefore no direct means are supplied for its adjustment.

2. *To set the horizon-glass perpendicular to the plane of the sextant.*

Screw in the telescope, T, and point it toward a star. Move the index arm backward and forward past the zero of the limb, and if the two images of the star do not exactly coincide in passing one another, turn a screw at the top or bottom of the horizon-glass, N, until this coincidence is effected.

3. *To find the index error.*

When the zero on the index is set to zero on the limb, the horizon and index glasses should be parallel; and if the telescope be directed to a star, the two images should exactly coincide. If the two images do not coincide, this deviation constitutes what is called the *index error.* The amount of the index error may be found in the following manner: Clamp the index at about 30 minutes to the left of zero, and, looking toward the sun, the two images will appear either nearly in contact, or overlapping each other. Then perfect the contact by moving the tangent screw, and call the minutes and seconds denoted by the vernier, the reading *on* the arc. Next place the index about the same quantity to the right of zero, or on the arc of excess, and make the contact of the two images perfect, as before, and call the minutes and seconds on the arc of excess, the reading *off* the arc. Half the difference of these numbers is the index error; additive when the reading on the arc of excess is greater than that on the limb, and subtractive when the contrary is the case.

EXAMPLE.

Reading on the arc	31′ 56″
Reading off the arc	31 22
Difference	0′ 34″
Index error	= −0′ 17″

In this case, the reading on the arc being greater than that on the arc of excess, the index error (17″) must be subtracted from all observations taken with the instrument, until it is found, by a similar process, that the index error has changed.

4. *To set the axis of the telescope parallel to the plane of the sextant.*

There are two parallel wires on opposite sides, and equidistant from the centre of the field of the telescope, and these are usually crossed by two others. Turn either pair around until they are parallel to the plane of the instrument. Select two stars distant from each other 90° or more, and bring them into contact just at the wire of the telescope which is nearest the plane of the sextant. Fix the index, and alter the position of the instrument so as to make the objects appear on the other wire. If the contact still remains perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten, to the up-and-down piece, the collar into which the telescope screws. This adjustment is not very liable to be deranged.

(133.) *To measure the altitude of the sun by reflection from mercury.*

Set the index near zero. Hold the instrument with the right hand in the vertical plane of the sun, toward which the telescope should be pointed. Two images will be seen in the field of view, one of which, viz., that formed by reflection, will apparently move downward when the index is pushed forward. Follow the reflected image as it travels downward, until it appears to be as far below the horizon as it was at first above, and the image of the sun, reflected from the mercury, also appears in the field of view. Fasten the index, and, by means of the tangent screw, bring the upper or lower limb of the sun's image, reflected from the index-glass, into contact with the opposite limb of the image reflected from the artificial horizon, taking care that the images shall be midway between the parallel wires. The angle shown on the instrument, when corrected for the index error, will be double the altitude of the sun's limb above the horizontal plane; to the half of which, if the semidiameter, refraction, and parallax be applied, the result will be the true altitude of the centre.

In making this observation, the observer should move the instrument round to the right and left a little, making the axis of the telescope the centre of motion. By this movement, the image reflected from the index-glass may be made to sweep the arc of a circle, and will pass and repass the image seen in the mercury. The altitude of a star can be measured in the same way as the sun, but in this case there will be no correction for parallax or semidiameter to be applied.

(134.) *To take an altitude of the sun by means of the natural horizon.*

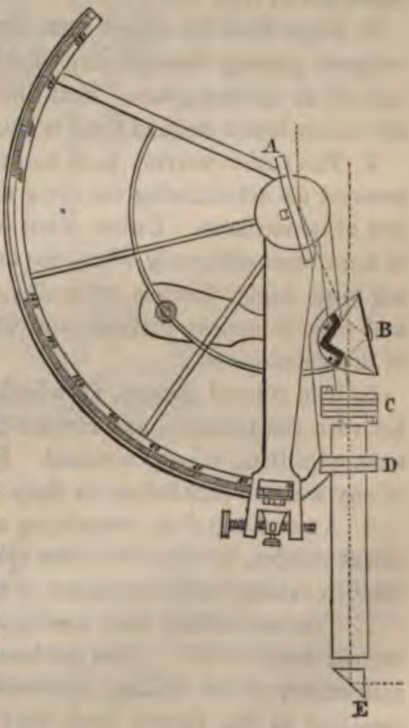
If the observer is at sea, the natural horizon must be employed. Direct the sight to that part of the horizon beneath the sun, and move the index till you bring the image of its lower limb to touch the horizon directly underneath it; but as this point can not be exactly ascertained, the observer should move the instrument round to the right and left a little, making the axis of the telescope the centre of motion. By this means the sun will appear to sweep the horizon, and must be made to touch it at the lowest point of the arc.

(135.) *To find the distance between the moon and sun, or between the moon and a star.*

Hold the sextant so that its plane may pass through the sun and moon. If the sun be to the right hand of the moon, the sextant is to be held with its face upward; if to the left hand, the face is to be held downward. With the instrument in this position, look directly at the moon through the telescope, and move the index forward till the sun's image is brought nearly into contact with the moon's nearest limb. Fix the index by the screw under the sextant, and make the contact perfect by means of the tangent screw. At the same time, move the sextant slowly, making the axis of the telescope the centre of motion; by which means the objects will pass each other, and the contact be more accurately made; observing that the point of contact of the limbs must always be observed in the middle, between the parallel wires. The index will then point out the distance of the nearest limbs of the sun and moon. In a similar manner may we measure the distance between the moon and a star.

PRISMATIC SEXTANT OF PISTOR AND MARTINS.

(136.) A new form of sextant, constructed by Pistor and Martins, Berlin, Prussia, is represented in the annexed figure. It differs in several important particulars from the common sextant.



1. It measures *any angle up to* 180°. Hence double altitudes of objects near the zenith can be taken with it. The common sextant is limited to about 60° as the maximum of altitude. The limb of the instrument is *one third* of a circle, and is graduated from zero, toward the left, up to 140°,

like other sextants. The graduation stops at 140°, because near this point the prism B interferes with the rays which should reach the mirror A from the object; and beyond 140° the object becomes invisible in this position of the sextant.

For angles greater than 140°, the graduation begins again at the left extremity of the limb with 110°, and increases toward the right up to 180°. When an observation is made on this part of the limb, the face of the sextant is turned in the contrary direction from what it had in the former observations, and the prism B no longer interferes with vision. But near 180° the head of the observer obstructs the rays from the object; to obviate which inconvenience, a diagonal eye-prism, E, is adapted to the eye-piece of the telescope, to be used in measuring angles near 180°.

2. In place of the common horizon-glass is substituted a *rectangular prism*, B, the diagonal face of it forming a mirror, as explained in Art. 12.

3. Rays from the object seen directly, come to the telescope without passing through any medium, such as the unsilvered part of an horizon-glass. Both the reflected and direct images are much better defined than is usual in other instruments.

4. The *index mirror*, A, is so attached as to admit of ready reversal for determining the error arising from want of parallelism of its surfaces. Unlike other sextants, it receives the rays of light *most obliquely* when the index is at *zero*. In measuring large angles there is no confusion or multiplicity of images, and objects appear distinct and well-defined in every position of the index-glass.

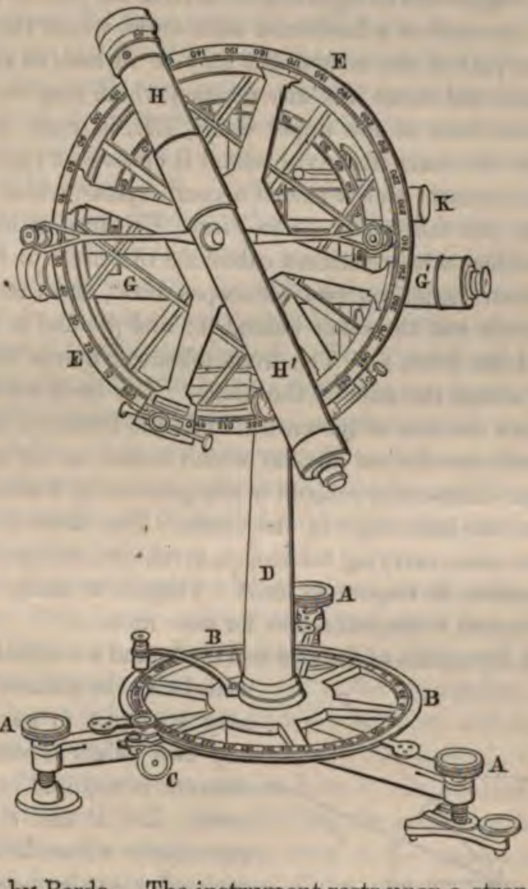
5. The colored glasses, C, which are *semicircles*, are placed between the telescope and horizon-glass, and are attached to an axis, admitting of easy reversal. By this contrivance the effect of any want of parallelism in their surfaces is entirely obviated.

6. A revolving disk, containing several colored glasses of different shades, is adapted to the eye end of the telescope, to be used in taking double altitudes of the sun.

7. The instrument here described is 6 inches radius, and the vernier reads to 10″. The graduation is very clear, and the arrangement of the reading microscope and ground-glass screen (omitted in the figure) such that the divisions are nearly as easily read by lamplight as by daylight.

REPEATING CIRCLE.

(137.) The repeating circle bears some resemblance to the altitude and azimuth instrument described on page 93, but it has some peculiarities of construction, and the mode of using it is peculiar. The following figure represents a repeating circle, as employed by Borda. The instrument rests upon a strong tripod, with feet screws, AAA; and a steel spindle, about 15 inches long, has one end inserted into the middle of the tripod, to which it is perpendicular. A hollow brass pillar, D, turns freely round this spindle, and sustains the weight of the upper circle, with its telescopes.

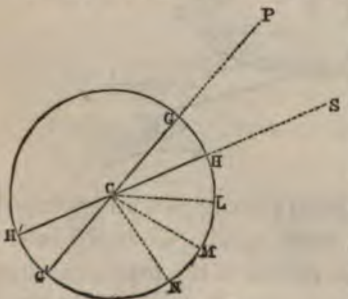


The azimuth circle, BB, is connected with the pillar, and revolves with it, while the divisions are read by a vernier attached to the tripod. A screw apparatus at C, attached to one of the branches of the tripod, clamps the azimuth circle, or allows it a quick or slow motion at pleasure.

To the top of the pillar, D, is fixed a horizontal brass bar, with two supporters at right angles to it, in the tops of which are centered the ends of a horizontal axis, round which the whole of the upper part of the instrument may be turned, so as to bring the plane of the circle into any position which may be required. The centre work of the upper circle, EE, is made fast to the middle of the horizontal axis, which it crosses at right angles; and at its remote end is placed a counterpoise, which balances the circle and telescopes. The circle, EE, has an index with four branches, whose verniers subdivide the circle to 10″.

The instrument has two telescopes, GG′, HH′, one in front of the circle and the other behind it; and parallel to the latter is placed the level, K. The front telescope moves freely on a spindle, within the axis of the circle. The back telescope is a little below the axis of the circle, while the level is a little above it, and both revolve on a collar which works on the outside of that axis. These can be fixed in any position by a clamp, which embraces the back edge of the circle. The circle turns freely about its axis, carrying telescopes, level, etc., without altering their position in respect to itself. There is a clamp for fixing the circle, and a tangent screw for slow motion.

(138.) By means of the two motions round a vertical and horizontal axis, the plane of the circle may be made to pass through any two points whose angular distance is required to be measured. Let P and S represent two objects whose distance from each other is to be measured, and let GNH′ represent the circle adjusted, so that its plane passes through them. Fix the front telescope, HH′, at the zero of the graduation in H, and turn the circle about its axis until the telescope HH′ is directed exactly



upon the object S. Clamp the circle in this position, and point the back telescope, GG′, upon the object P. The angle PCS will be measured by the arc GH, intercepted between the lines CP and CS. Unclamp the circle, and turn it until the back telescope, GG′, is pointed toward S. The front telescope will now come into the position CL; the zero of the graduation, which was before at H, will be removed to L. Again clamp the circle, release the front telescope, and direct it toward the object P. The arc GHL will be twice the arc required to be measured. Repeat this double observation, starting again from the point G; that is, turn the circle with its two telescopes until the front telescope is pointed upon the object S. The zero of the graduation will now be found at M. Detach the back telescope, and point it again upon the object P; the arc GM will be three times the arc GH. Unclamp the circle, and turn it until the back telescope is pointed upon S; the zero of the graduation will now be found at N. Again clamp the circle, release the front telescope, and direct it toward the object P. The arc GN, which may be read upon the limb, will be four times the arc required. By repeating the observation ten times, we shall obtain ten times the angle sought. It is not necessary to read the graduation after each observation; it is sufficient to read the resulting arc after the observations are concluded, and divide the final arc by the number of observations.

(139.) Suppose these ten observations should bring the front telescope back to the zero of the graduation from which we started, then each arc would be equal to 36°; and this result would not be affected by any error in the graduation of the circle. It is not to be expected that the telescope will, in practice, be brought round exactly to the zero; but it should be brought round as near to zero as can be done by the continued repetition of the angle PCS; then, dividing the result by the number ot repetitions, the effect of any error in the graduation of the circle will be greatly diminished, if not entirely destroyed.

In a similar manner may the zenith distance of any celestial body be measured, by employing the spirit-level attached to the back telescope to indicate a horizontal line.

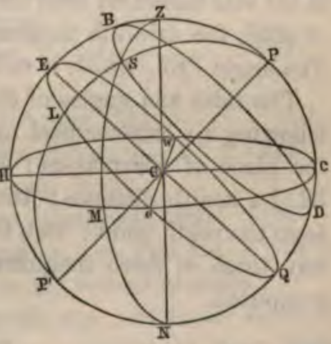
(140.) The chief advantage contemplated in the invention of the repeating circle was the annihilation of errors of graduation;

but the great improvements which have been made in recent years in graduating circles have rendered this an object of minor importance, while this instrument is liable to some serious errors of its own, so that the repeating circle is at present much less used than formerly.

CHAPTER IV.

THE DIURNAL MOTION.

(141.) If, upon a clear evening, we carefully watch the appearance of the heavenly bodies for a sufficient period, we shall find that they slowly change their places with respect to the horizon. Each star appears to describe, as far as its course lies above the horizon, a circle in the sky; but these circles are not all of the same magnitude. The apparent relative situations of the stars among each other remain unchanged; but all the stars seem to revolve with a uniform motion from east to west, as if they were attached to the internal surface of a vast hollow sphere, having the spectator in its centre, and turning around an axis inclined to the horizon, so as to pass through a fixed point called the pole. This apparent rotation of the heavens is called the *diurnal motion.*



(142.) Let C be the place of the spectator, Z his zenith, and N his nadir. Let PCP′ be the axis about which the diurnal motion is apparently performed, P the elevated pole, and P′ the depressed pole of the heavens. Then HMO, a great circle of the sphere, whose poles are Z and N, will be his celestial horizon, PO will be the altitude of the pole, OPZEH will be his meridian; and ELQ, a great circle perpendicular to PP′, will be the celestial equator. Also, if S represents the position of any star, and PSP′ be a great circle passing through it, then LS will be the declination, and PS the polar distance of the star, and BSD will be the diurnal circle it will appear to describe about the pole. O and H are the north and south points, *e* and *w* are the east and west points of the horizon. Also, if we draw the vertical circle ZSMN, OM will be the azimuth of the star, reckoned from the north point, MS its altitude, and ZS its zenith distance.

The angle ZPS, which the circle PSP′ makes with the meridian PZP′, is called the *hour angle* of the star S.

(143.) The circles thus drawn form a number of spherical triangles, the relations of whose sides and angles may be determined by spherical trigonometry. When the place of only one celestial object on the sphere is concerned, it may be determined from the triangle PZS.

In the triangle PZS, Z represents the zenith, P the elevated pole, and S the star, sun, or other celestial object. In this triangle the sides are, 1st. PZ, which, being the complement of PO, the altitude of the pole, is the complement of the latitude of the place, and is called the *co-latitude*; 2d. PS, the polar distance, or the complement of the declination of the star; and, 3d. ZS, the zenith distance, which is the complement of the altitude of the star. If the object be situated on the side of the equator opposite to that of the elevated pole, PS will be greater than 90°.

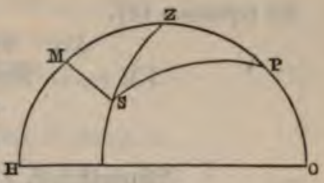
In the same triangle the angles are, 1st. ZPS, the hour angle of the star from the meridian; 2d. PZS, which is the azimuth of the star measured from the north point, and is the supplement of HZS, the azimuth measured from the south point; and, 3d. The angle PSZ, which is called the parallactic angle.

The sides and angles of this triangle, therefore, represent the following six astronomical magnitudes: 1st. The co-latitude of the place of observation; 2d. The polar distance of the star; 3d. Its zenith distance; 4th. Its hour angle; 5th. Its azimuth from the north point; and, 6th. Its parallactic angle; and when any three of these magnitudes are given, the others may be computed.

PROBLEM.

(144.) *To find the altitude, azimuth, and parallactic an of a star, its polar distance and hour angle being given well as the latitude of the place.*

Let P be the pole, Z the zenith, S the place of the star HO the horizon.



Then

PO = the latitude, which we will represent by ϕ;

PZ = the co-latitude $= 90^\circ - \phi$;

PS = the polar distance of the star;

$= 90^\circ - \delta$, where δ represents the star's declination;

ZS = zenith distance of the star, which we represent by Z;

ZPS = the star's hour angle, which we represent by P;

PZS = the azimuth of the star, counted from north, which we represent by A.

In the spherical triangle, PZS, are given two sides, PS and PZ, with the included angle, to find the other parts.

Let fall the perpendicular SM upon PZH; then, by Napier's rule,

$$\text{R. cos. P} = \text{tang. PM cot. PS.}$$

Therefore, $\text{tang. PM} = \text{cos. P tang. PS}$

$$= \text{cos. P cot. } \delta \quad \ldots\ldots\ldots \quad (1)$$

But $\text{ZM} = \text{PM} - \text{PZ}$

$$= \text{PM} + \phi - 90^\circ.$$

Then, by Trig., Art. 216,

$$\text{sin. PM} : \text{sin. ZM} :: \text{tang. SZM} : \text{tang. SPM};$$

that is, $\text{sin. PM} : \text{cos.}(\text{PM} + \phi) :: \text{tang. A} : \text{tang. P};$

$$:: \text{cot. P} : \text{cot. A} \quad \ldots\ldots \quad (2)$$

Also, Trig., Art. 216,

$$\text{cos. PM} : \text{cos. ZM} :: \text{cos. SP} : \text{cos. SZ};$$

that is, $\text{cos. PM} : \text{sin.}(\text{PM} + \phi) :: \text{sin. } \delta : \text{cos. Z} \quad \ldots\ldots \quad (3)$

Also, $\text{sin. ZS} : \text{sin. P} :: \text{sin. PZ} : \text{sin. PSZ};$

that is, $\text{sin. Z} : \text{sin. P} :: \text{cos. } \phi : \text{sin. parallactic angle} \quad . \ (4)$

When the star has south declination, cot. δ in Eq. 1 will be negative, and PM must be taken in the second quadrant.

Ex. 1. Find the altitude, azimuth, and parallactic angle of Aldebaran (Dec. 16° 13′ N.), to an observer at New York, latitude 40° 42′ N., when the star is three hours east of the meridian.

By equation (1),

	cos. 45°	= 9.849485
	cot. 16° 13′	= 0.536342
PM =	67° 38′ 31″ tang.	= 0.385827

By equation (2),

$\phi = 40° 42'$

$\mathrm{PM}+\phi = 108° 20' 31''$ cos. = 9.497879

cot. P = 0.000000

cosec. PM = 0.033941

Azimuth = S. 71° 12′ 30″ E. cot. = 9.531820

By equation (3),

sin. $(\mathrm{PM}+\varphi) = 9.977356$

sin. $\delta = 9.446025$

sec. PM = 0.419767

Zenith distance = 45° 49′ 27″ cos. = 9.843148

or Altitude = 44° 10′ 33″

By equation (4),

sin. 45° = 9.849485

cos. $\phi = 9.879746$

cosec. Z = 0.144357

Parallactic angle = 48° 22′ sin. = 9.873588

Ex. 2. Find the altitude and azimuth of Regulus (Dec. 12° 42′ N.), to an observer at Washington, latitude 38° 53′ N., when the hour angle of the star is 3h. 15m. 20s. W.

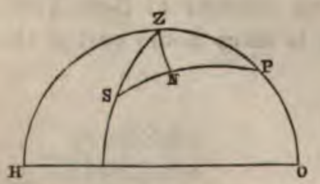
Ans. Its altitude = 39° 38′ 0″,
azimuth = S. 72° 28′ 14″ W.

Ex. 3. Find the altitude and azimuth of Fomalhaut (Dec. 30° 25′ S.), to an observer at Cambridge, latitude 42° 22′ N., when the hour angle of the star is 2h. 5m. 36s. E.

Ans. Its altitude = 11° 41′ 37″,
azimuth = S. 27° 18′ 40″ E.

Ex. 4. Find the altitude and azimuth of α Ursæ Majoris (Dec. 62° 33′ N.), to an observer at Philadelphia, latitude 39° 57′ N., when the hour angle of the star is 5h. 17m. 40s. E.

Ans. Its altitude = 39° 24′,
azimuth = N. 35° 54′ E.



(145.) When only the parallactic angle is required, it may be computed without finding the altitude or azimuth, as follows:

Draw ZN perpendicular to PS, and represent PN by x. Then, by Napier's rule,

$$\text{R. cos. P} = \text{tang. PN cot. PZ},$$

or $$\text{cos. P} = \text{tang. } x \text{ tang. } \phi\,;$$

that is, $$\text{tang. } x = \text{cos. P cot. } \phi \quad \ldots\ldots\ldots \quad (1)$$

Again, by Spher. Trig., Art. 216,

$$\text{sin. NS} : \text{sin. PN} :: \text{tang. P} : \text{tang. S},$$

or $$\text{tang. par. ang.} = \frac{\text{sin. PN tang. P}}{\text{sin. NS}} = \frac{\text{sin. } x \text{ tang. P}}{\text{cos. } (x+\delta)} \quad \ldots (2)$$

Example. Required the parallactic angle for Washington Observatory, latitude 38° 53′ 33″, the moon's hour angle being 50° and Declination 21° N.

By formula (1),

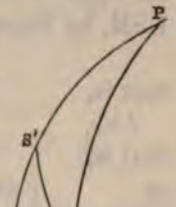
$$\begin{array}{r} \text{cot. } \phi = 0.093297 \\ \text{cos. P} = 9.808067 \\ x = 38^\circ\ 32'\ 55''\ \text{tang.} = \overline{9.901364} \end{array}$$

By formula (2),

$$\begin{array}{r} \text{sin. } x = 9.794612 \\ \text{tang. P} = 0.076186 \\ \text{sec. } 59^\circ\ 32'\ 55'' \qquad = 0.295157 \\ \text{Par. angle} = 55^\circ\ 41'\ 24''\ \text{tang.} = \overline{0.165955} \end{array}$$

As the parallactic angle is frequently required in many computations, Table XVII. has been constructed for Washington Observatory by the preceding method, except that instead of the geographical latitude, the geocentric latitude, 38° 42′ 25″, has been used. See Art. 208.

(146.) *Corollary.* By the same method we may compute the distance between two stars whose right ascensions and declinations are known.



Let P be the pole, and S and S′ two stars whose places are known. Then PS and PS′ will represent their polar distances, and SPS′ will be the difference of their right ascensions. Draw SM perpendicular to PS′ produced. Then

$$\text{R. cos. P} = \text{tang. PM cot. PS}.$$

Therefore, $$\text{tang. PM} = \text{cos. P tang. PS}.$$

Also, $$\text{S}'\text{M} = \text{PM} - \text{PS}'.$$

And $$\text{cos. PM} : \text{cos. S}'\text{M} :: \text{cos. PS} : \text{cos. S}'\text{S}.$$

Ex. 1. Required the distance from Aldebaran, R. A. 4h. 27m.

25.94s., Polar distance 73° 47′ 33″.3, to Sirius, R.A. 6h. 38m. 37.62s., Polar distance 106° 31′ 1″.8.

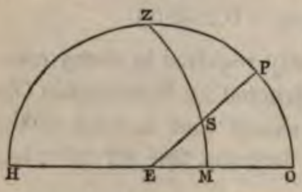
P=2h. 11m. 11.68s.= 32° 47′ 55″.2 cos.=9.9245786
PS=106° 31′ 1″.8 tang.=0.5279175
PM=109° 25′ 54″.55 tang.=0.4524961
PS′= 73° 47′ 33″.3
S′M= 35° 38′ 21″.25 cos.=9.9099313
PS=106° 31′ 1″.8 cos.=9.4537809
PM=109° 25′ 54″.55 sec.=0.4779669
SS′= 46° 0′ 42″.3 cos.=9.8416791

Ex. 2. Required the distance from Regulus, R. A. 10h. 0m. 29.11s., Polar distance 77° 18′ 41″.4, to Antares, R. A. 16h. 20m. 20.35s., Polar distance 116° 5′ 55″.5.

Ans. 99° 55′ 44″.9.

PROBLEM.

(147.) *To find the altitude and azimuth of a star when it is six hours from the meridian.*



If the star S be six hours from the meridian, then the angle ZPS=90°; the hour circle, PE, intersects the horizon in the east point, E; and the angle PEO is equal to the latitude of the place. Draw the vertical circle ZSM. Then, in the right-angled spherical triangle ESM, by Napier's rule,

R. sin. SM=sin. E sin. ES;
that is, sin. *altitude*=sin. ϕ sin. δ . . . . (1)
Also, R. cos. E=tang. EM cot. ES;
that is, tang. EM.=tang. ES cos. E,
or cotang. *azimuth*=tang. δ cos. ϕ . . . . . . (2)

Ex. 1. In Lat. 41° 18′ N., when the sun has 18° 25′ N. declination, what is his altitude and azimuth at six o'clock in the morning?

By formula (1),

sin. 41° 18′ =9.819545
sin. 18° 25′ =9.499584
Altitude= 12° 2′ 6″ sin.=9.319129

By formula (2),

tang. 18° 25′ $=9.522417$
cos. 41° 18′ $=\underline{9.875793}$
Azimuth = N. 75° 57′ 19″ E. cot. $=\overline{9.398210}$

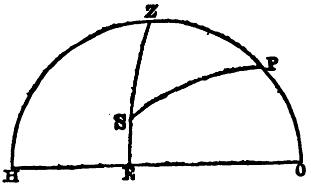
Ex. 2. Find the altitude and azimuth of Regulus (Dec. 12° 42′ N.) to an observer at Philadelphia, Lat. 39° 57′ N., when the star is six hours past the meridian.

Ans. Its altitude = 8° 6′ 56″,
azimuth = N. 80° 11′ 54″ W.

Ex. 3. Find the altitude and azimuth of Capella (Dec. 45° 50′ N.) to an observer at Cambridge, Lat. 42° 22′ N., six hours before the star comes to the meridian.

Ans. Its altitude = 28° 54′ 23″,
azimuth = N. 52° 44′ 28″ E.

PROBLEM.



(148.) *To find the altitude and hour angle of a star when it is upon the prime vertical.*

Let ZSE be the prime vertical, and S the position of the star. Draw the hour circle PS. The angle PZS will be a right angle, and we shall have, by Napier's rule,

$$\text{R. cos. P} = \text{tang. PZ cot. SP};$$

that is,
$$\text{cos. P} = \text{cot. } \phi \text{ tang } \delta \; . \; . \; . \; . \; . \; . \quad (1)$$

Also,
$$\text{R. cos. SP} = \text{cos. SZ cos. PZ},$$

or
$$\text{cos. SZ} = \frac{\text{cos. SP}}{\text{cos. PZ}};$$

that is,
$$\text{sin. } \textit{altitude} = \frac{\text{sin. } \delta}{\text{sin. } \phi} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Ex. 1. Find the altitude and hour angle of Aldebaran (Dec. 16° 13′ N.) when it is exactly east of an observer at New York, Lat. 40° 42′ N.

By formula (2),

sin. 16° 13′ $=9.446025$
sin. 40° 42′ $=\underline{9.814313}$
Altitude = 25° 21′ 27″ sin. $=\overline{9.631712}$

By formula (1),

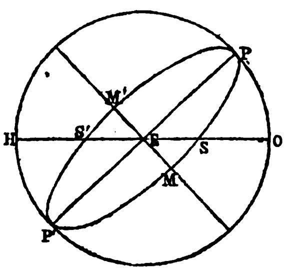
tang. $16^\circ\ 13'$ $= 9.463658$
cot. $40^\circ\ 42'$ $= 0.065433$
$70^\circ\ 14'\ 12''$ cos. $= \overline{9.529091}$
$=$ 4h. 40m. 56.8s. $=$ hour angle.

Ex. 2. Find the altitude and hour angle of Vega (Dec. $38^\circ\ 38'$ N.), when it is exactly west of an observer at Cambridge, Lat. $42^\circ\ 22'$ N.

Ans. Altitude $= 67^\circ\ 53'\ 37''$,
Hour angle $=$ 1h. 55m. 12s.

PROBLEM.

(149.) *To find the amplitude and hour angle of a star when it is in the horizon.*



Let PEP′ represent the hour circle which is six hours from the meridian, and which intersects the horizon in the east point, E. Let S or S′ be the position of a star in the horizon, and through S draw the hour circle PSP′; also, through S′ draw the hour circle PS′P′. Then, in the right-angled spherical triangle EMS or EM′S′,

EM or EM′ = the distance of the star from the six o'clock hour circle;
MS or M′S′ = the star's declination;
ES or ES′ = the star's amplitude;
= the complement of the star's azimuth;
MES = M′ES′ = the complement of the latitude.

Now, by Napier's rule,

$$\text{R. sin. MS} = \text{sin. ES sin. MES},$$

or

$$\text{sin. ES} = \text{sin. MS cosec. MES};$$

that is,

$$\text{sin. } \textit{amplitude} = \text{cos. } \textit{azimuth} = \text{sin. } \delta \text{ sec. } \phi \quad . \ . \ (1)$$

Also,

$$\text{R. sin. EM} = \text{tang. MS. cot. MES},$$

or

$$\text{sin. EM} = \text{tang. } \delta \text{ tang. } \phi \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (2)$$

$$\text{P} = 6 \text{ hours} \mp \text{EM},$$

where P represents the time from the star's rising to its passing the meridian.

Ex. 1. Find the amplitude and hour angle of Arcturus (Dec.

19° 57′ N.) when it rises to an observer at New York, Lat. 40° 42′ N.

By formula (1),

sin. 19° 57′ =9.533009
sec. 40° 42′ =0.120254
Amplitude=E. 26° 44′ 49″ N. sin. =9.653263
or Azimuth =N. 63° 15′ 11″ E.

By formula (2),

tang. 19° 57′ =9.559885
tang. 40° 42′ =9.934567
EM=18° 11′ 34″ sin. =9.494452

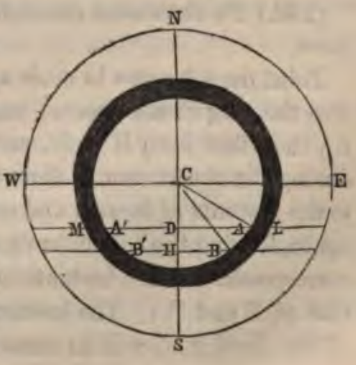
Hence the hour angle=7h. 12m. 46.3s.

Ex. 2. Find the hour angle and amplitude of Antares (Dec. 26° 6′ S.), when it sets to an observer at Philadelphia, Lat. 39° 57′ N. *Ans.* Hour angle=4h. 23m. 5.7s.
Amplitude =W. 35° 1′ 16″ S.
or Azimuth =S. 54° 58′ 44″ W.

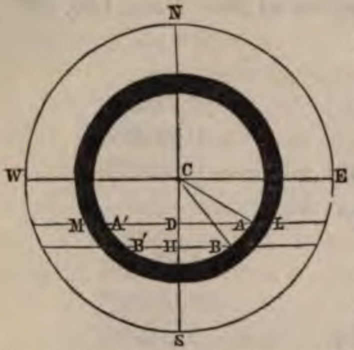
As we have frequent occasion to know the time of rising and setting of the heavenly bodies, it is convenient to have a table from which this may be ascertained without the labor of computation. Table XIX. furnishes the semi-diurnal arcs for any latitude up to 60°, and for any declination not exceeding 29°, from which, if we know the time of passing the meridian, the time of rising or setting is easily found.

To find the time of rising of the sun's upper limb, corrected for refraction, see Art. 169.

RING MICROMETER.



(150.) The ring micrometer consists of an opaque ring, inserted in the focus of a telescope, and having a diameter somewhat less than that of the field of view. When the telescope is fixed in position, by observing the instants at which two stars pass the opposite sides of either the outer or inner circle of the ring, their difference of right ascension and



declination may be computed, provided we know the diameter of the ring. The annexed figure represents the appearance of a ring suspended in the focus of a telescope, the field of view being represented by the circle NWSE. Each star is to be observed when it passes behind the ring at L, when it reappears at A; when it disappears again at A′, and when it reappears at M.

(151.) *To determine the radius of the ring.*

If there are spider lines bisecting the ring exactly in the centre, we may determine the radius by observing the time required by an equatorial star in passing centrally across the ring; or by observing the passage of any star not very near the pole, and multiplying the interval by the cosine of its declination; that is,

$$r = \text{Radius} = \frac{15}{2}(t' - t)\cos. \text{Dec.},$$

where t and t' are the times of ingress and egress of the star. See Art. 72.

The radius of the ring will retain the same value only as long as the distance of the ring from the object-glass remains unchanged. When, therefore, the radius of the ring has been once determined, the position of the tube carrying the micrometer should be accurately marked, and, in all subsequent observations, should be carefully adjusted to the same position.

(152.) *To determine the difference of right ascension of two stars.*

Point the telescope in such a manner that the stars may traverse the ring in succession; one of them, for example, from A to A′, the other from B to B′, and leave the telescope undisturbed during the observation. Note the times T and T′, corresponding to the instants of ingress and egress of the first star at A and A′; again, leaving the telescope undisturbed, note the times t and t', corresponding to the instants of ingress and egress of the second star at B and B′. The instant of passing the middle point, D, of the chord AA′, will be denoted by $\frac{1}{2}(T' + T)$; and the instant of passing the middle point, H, of the chord BB′, will be denoted by $\frac{1}{2}(t'+t)$. The difference of right ascension will therefore be $\frac{1}{2}(t'+t)-\frac{1}{2}(T'+T)$, provided the clock has no rate that sensibly affects the interval.

(153.) *To determine the difference of declination of two stars.* We must previously have an approximate knowledge of the declination of each of the stars.

Put δ = the approximate declination of the first star;
δ' = the approximate declination of the second star.

Then we shall have

$$AD=\tfrac{1}{2}(T'-T)15 \cos. \delta,$$

and

$$BH=\tfrac{1}{2}(t'-t)15 \cos. \delta'.$$

Put X = the angle ACD, and x = the angle BCH.

Then

$$\left.\begin{aligned}\sin. X&=\frac{AD}{r}=\frac{15}{2r}\cos. \delta\,(T'-T)\\ \sin. x&=\frac{BH}{r}=\frac{15}{2r}\cos. \delta'(t'-t)\end{aligned}\right\}\ldots\ldots(1)$$

Also,

$$\left.\begin{aligned}CD&=r\cos. X\\ CH&=r\cos. x\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots(2)$$

Hence DH, or the difference of declination $=r(\cos. x-\cos. X)$, when both arcs are on the same side of the centre of the ring. When they are on opposite sides, the difference of declination $=r(\cos. x+\cos. X)$.

When the observations are made with reference to the outer edge of the ring, we must proceed in the same manner; and if observations are made at both edges of the ring, a mean of the two results must be taken.

The results for right ascension will be most reliable when the stars pass near the centre of the ring; but the results for declination will be most reliable when the stars pass at a considerable distance from the centre.

(154.) The following observations of Encke's comet and a neighboring star will illustrate the use of this micrometer:

	North or South of Centre.	Outer Ring. Ingress.	Inner Ring. Ingress.	Inner Ring. Egress.	Outer Ring. Egress.	Concluded Transit over Hour-Circle.	Difference of R. A.
		h. m. s	m. s.	m. s.	m. s.	m. s.	m. s.
Star	N.	23 13 9	13 28	14 44	15 4	14 6.25	2 1.75
Comet	S.	23 15 11	15 36	16 38	17 7	16 8.0	
Star	N.	23 35 59	36 23	37 16	37 41	36 49.75	1 52.75
Comet	S.	23 37 48	38 15	39 10	39 37	38 42.5	

The observations of the first of the preceding stars with the outer ring give $\frac{1}{2}(T'+T)=14m.\ 6.5s.$; from the inner ring we obtain 14m. 6s.; the mean of the two is 14m. 6.25s., which is the time of passing the middle point of its chord. In the same manner we obtain for the comet 16m. 8.0s. Hence their difference of right ascension was 2m. 1.75s.; and, in the same manner, their difference of right ascension at the second observation was 1m. 52.75s.

The difference of declination is computed as follows, using only the observations of the inner ring:

The radius of the inner ring was 9′ 38″=578″.

The declination of the star was 32° 8′ 41″ N.

The declination of the comet at the first observation was 31° 56′ 25″ N. nearly.

The declination of the comet at the second observation was 31° 53′ 14″ N. nearly.

For the first star observation, by equation (1),

$$\begin{aligned} \tfrac{1}{2}(t'-t)=38s.&=1.57978\\ 15\ &=1.17609\\ \cos.\ \delta&=9.92773\\ \frac{1}{r}&=7.23807\\ x=56^\circ\ 36'\ 50''\ \sin.&=\overline{9.92167} \end{aligned}$$

By equation (2),

$$\begin{aligned} \cos.\ x&=9.74058\\ r&=2.76193\\ CH=318''.1&=\overline{2.50251} \end{aligned}$$

In a similar manner, for the first comet observation, we obtain

$$CD=422''.3.$$

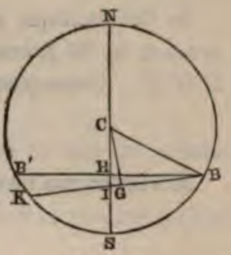
Hence, since the star and comet were on opposite sides of the centre, the difference of declination$=318''.1+422''.3=740''.4=12'\ 20''.4$.

In the same manner, we find the difference of declination at the second observation to be 15′ 29″.7.

(155.) Frequently a comet changes its right ascension and declination so rapidly that we can not assume that in one second of time it describes 15″ cos. δ in arc, and that its path is perpendicular to an hour circle. In this case, we must apply a correction to the result obtained without regarding the proper motion.

Let NS represent an hour circle, and draw BB′ perpendicular to NS.

Suppose the comet to describe the path BK instead of BB′,



Represent CG by d = the least distance of the comet from the centre of the ring; and let $\tau = \frac{1}{2}(t'-t)$ = half the interval between the ingress and egress; then

$$d^2 = r^2 - (15\tau \cos. \delta)^2.$$

Represent by Δa the increase of right ascension of the comet in a second of time; $\Delta\tau$ the change of τ caused by the change of right ascension, so that $\tau + \Delta\tau$ represents the half interval which would have been observed if there had been no change of right ascension. Then

$$\Delta\tau = -\tau\Delta a.$$

But, by differentiating the above expression for d^2, we have

$$\Delta d = -\frac{15^2\tau \cos.^2\delta}{d}\Delta\tau.$$

Hence $$\Delta d = (15\tau \cos. \delta)^2\frac{\Delta a}{d} \quad . \; . \; . \; . \; . \; . \; . \quad (A)$$

which represents the required correction of the comet's declination.

Let $\Delta\delta$ represent the change of declination of the comet in a second of time, and n the angle KBB′, which the comet's path makes with a parallel, we shall have

$$\text{tang. } n = \frac{\Delta\delta}{(15 + \Delta a)\cos. \delta};$$

or we may assume without appreciable error,

$$\text{tang. } n = \frac{\Delta\delta}{15 \cos. \delta}.$$

Let y represent GI, the portion of the comet's path between the hour circle, CI, and the perpendicular, CG, drawn from the centre upon the path, and we shall have

$$y = d \text{ tang. } n = \frac{d\Delta\delta}{15 \cos. \delta}.$$

The correction to be applied to the time of transit over the hour circle, determined without regard to proper motion, is

$$\frac{y}{15\ \text{cos.}\ \delta},$$

or $$\Delta\tau = +\frac{d\Delta\delta}{(15\ \text{cos.}\ \delta)^2}\ .\ .\ .\ .\ .\ .\ .\ (B)$$

In the example given above, the comet's motion in right ascension in 24 hours was -7m.59.25s., and in declination $-3° 5' 0''.7$. Consequently,

$$\log.\ \Delta\alpha = 7.74405n,$$

and $$\log.\ \Delta\delta = 9.10884n.$$

Moreover, we have before found,

$$\log.\ d = 2.62567$$
$$\tau = 31\text{s}.$$
$$\delta = 31° 56' 25''.$$

To compute Δd.	To compute $\Delta\tau$.
By formula A.	By formula B.
$15 = 1.17609$	$d = 2.62567$
$\tau = 31\text{s}. = 1.49136$	$\Delta d = 9.10884n$
$\text{cos.}\ \delta = 9.92870$	$\overline{1.73451n}$
$\overline{2.59615}$	$(15\ \text{cos.}\ \delta)^2 = 2.20958$
2	$\Delta\tau = -0.33\text{s}. = \overline{9.52493n}.$
$\overline{5.19230}$	
$\Delta\alpha = 7.74405n$	
$\text{comp.}\ d = 7.37433$	
$\Delta d = -2''.04 = \overline{0.31068n}.$	

At the time of these observations, the comet was moving southward from the centre of the field, so that its apparent path may be represented by BK. It passed the point G, half-way between B and K, at 16m. 8.0s. Hence it passed the point I, on the hour circle bisecting the ring, at 16m. 8.33s. Therefore the true difference of right ascension was 2m. 2.08s.

The comet's least distance from the centre of the ring was before computed to be $422''.3$, which is represented by CG. Its corrected distance is CH, which is $420''.3$. Hence the true difference of declination was

$$420''.3 + 318''.1 = 738''.4 = 12' 18''.4.$$

CHAPTER V.

TIME.

(156.) The interval between two successive returns of the vernal equinox to the same meridian is called a *sidereal* day.

The interval between two successive returns of the sun to the same meridian is called a *solar* day.

The sun completes an apparent revolution about the earth in one year, or 365 days 5 hours 48 minutes and 47.57 seconds; so that the sun's mean daily motion is found by the proportion

$$\text{one year} : \text{one day} :: 360^\circ : \text{daily motion} = 59' \, 8''.33.$$

This motion is not uniform, but is greatest when the sun is nearest the earth. Hence the solar days are unequal; and to avoid the inconvenience which would result from this fact, astronomers have recourse to a *mean* solar day, the length of which is equal to the mean or average of all the apparent solar days in a year.

(157.) The length of the mean solar day is different from that of the sidereal, because when the mean sun, in its diurnal motion, returns to the meridian, it is 59′ 8″.33 advanced eastward in right ascension.

An arc of the equator, equal to 360° 59′ 8″.33, passes the meridian in a mean solar day, while only 360° pass in a sidereal day. To find the excess of the solar day above the sidereal day, expressed in sidereal time, we have the proportion

$$360^\circ : 59' \, 8''.33 :: \text{one day} : 3\text{m}.\ 56.555\text{s}.$$

Hence 24 hours of mean solar time are equivalent to 24h. 3m. 56.555s. of sidereal time. As we have frequent occasion to convert intervals of mean solar time into intervals of sidereal time, Table IV. has been constructed, from which such intervals are found by mere inspection.

Example. Find the sidereal interval which corresponds to 15h. 20m. 20.58s. of mean solar time.

According to Table IV.,
15 hours mean solar time = 15h. 2m. 27.847s. sidereal time.
20 minutes " " " = 20 3.285 " "
20 seconds " " " = 20.055 " "
0.58 " " " = 0.582 " "
The sidereal interval = 15h. 22m. 51.769s.

To find the excess of the solar day above the sidereal day, expressed in solar time, we have the proportion

360° 59′ 8″.33 : 59′ 8″.33 :: one day : 3m. 55.909s.

Hence 24 hours of sidereal time are equivalent to 23h. 56m. 4.091s. of mean solar time. In order to facilitate the conversion of sidereal time into solar time, Table V. has been constructed, from which these intervals are found by mere inspection.

Example. Find the solar interval which corresponds to 16h. 15m. 25.66s. of sidereal time.

According to Table V.,
16 hours sidereal time = 15h. 57m. 22.727s. mean solar time.
15 minutes " " = 14 57.543 " " "
25 seconds " " = 24.932 " " "
0.66 " " = 0.658 " " "
The solar interval = 16h. 12m. 45.860s.

(158.) Throughout this work we shall suppose the student to have in his possession some astronomical ephemeris, like the Nautical Almanac. The English Nautical Almanac has been published annually since 1767, and generally appears about three years in advance of the date for which it is computed. The French *Connaissance des Temps* has been published annually since 1679, without ever having suffered a single interruption; and the Berlin Astronomisches Jahrbuch has been published annually since 1776. The first volume of the American Nautical Almanac, being for 1855, was published in February, 1853, and since that time it has been published annually. Either of these almanacs will furnish all the data which are required for the computations in this treatise. We shall, however, employ the American Nautical Almanac, whenever it can conveniently be done; and for other cases shall refer to the English Nautical Almanac.

PROBLEM.

(159.) *To convert mean solar time into sidereal time.*

When the sun is on the meridian, the sidereal time is the same as the sun's apparent right ascension.

Thus, according to the American Nautical Almanac for 1855, page 271, the sun's apparent right ascension at Washington, apparent noon, January 1, 1855, is 18h. 46m. 58.21s.; this is, therefore, the sidereal time at that instant. The sidereal time of mean noon may be found from the preceding, by applying the equation of time, reduced to its sidereal equivalent. Thus, on January 1, 1855, the equation of time is +3m. 49.72s., which is equivalent to 3m. 50.35s. sidereal time. Therefore the sidereal time of mean noon is

18h. 46m. 58.21s. − 3m. 50.35s., which equals 18h. 43m. 7.86s.; and this is the number given in the last column of page 271 of the almanac. The almanac furnishes, in like manner, the sidereal time of mean noon at Washington for every day in the year. With this assistance, we can easily convert any instant of mean solar time into its corresponding sidereal time, by the following

RULE.

Sidereal time required = sidereal time at the preceding mean noon, plus the sidereal interval corresponding to the given mean time.

Example. Convert 2h. 22m. 25.62s. mean solar time at Washington, January 2, 1855, into sidereal time.

Sidereal time at the preceding mean noon, viz., January 2 . . . . 18h. 47m. 4.42s.
Add the mean time, reduced to its sidereal equivalent by Table IV. . . . . 2h. 22m. 49.02s.
The sum is the sidereal time required 21h. 9m. 53.44s.

If the place of observation be not on the meridian of the ephemeris, the sidereal time at mean noon must be corrected by the *addition* of 9.8565s. $\left(=\frac{3\text{m. } 56.555\text{s.}}{24}\right)$ for each hour of longitude, if the place be to the west of the first meridian, but by its *substraction* if to the east.

Example. Convert 7h. 55m. 51.65s. mean time at the High

School Observatory, Philadelphia, April 19, 1855, into sidereal time.

The sidereal time at the preceding Washington mean noon is	1h. 48m. 55.82s.
Correction for 7m. 33.6s., Philadelphia east of Washington	−1.24s.
Sidereal time at the preceding Philadelphia mean noon	1h. 48m. 54.58s.
Add the mean time, reduced to its sidereal equivalent	7h. 57m. 9.82s.
The sum is the sidereal time required	9h. 46m. 4.40s.

PROBLEM.

(160.) *To convert sidereal time into mean solar time.*

If from the proposed sidereal time we subtract the sidereal time at the preceding mean noon, we shall obtain the interval from mean noon expressed in sidereal time; and if we convert this interval into its mean solar equivalent, we shall have the interval elapsed since mean noon expressed in mean time, and therefore the time which ought to be shown by a mean-time clock.

Example. Convert 21h. 9m. 53.44s. sidereal time at Washington, January 2, 1855, into mean solar time.

Sidereal time given	21h. 9m. 53.44s.
Sidereal time at preceding mean noon	18h. 47m. 4.42s.
Interval in sidereal time from mean noon	2h. 22m. 49.02s.
Equivalent in mean solar time by Table V.	2h. 22m. 25.62s.

which is therefore the mean solar time required.

(161.) If we subtract the sidereal time at mean noon from twenty-four hours, and convert this interval into its solar equivalent, we shall have the mean time of transit of the first point of Aries, which may be called the *mean time at sidereal noon.* It is the time which ought to be shown by a mean-time clock, at the moment that a clock adjusted to sidereal time indicates exactly 0h. 0m. 0s. The mean time of transit of the first point of Aries is given in the English Nautical Almanac for every day of the year, on page xx. of each month. It is omitted in the American Almanac for 1855, but is inserted in the Almanac for 1856, on page III. of each month, under the title mean time of sidereal 0h. This quantity is found as follows:

The sidereal time at Greenwich, mean noon, January 1, 1855, is 18h. 42m. 17.25s. Subtracting this from 24 hours, we have 5h. 17m. 42.75s., which, reduced to its equivalent solar interval, is 5h. 16m. 50.70s., which is, therefore, the mean time of transit of the first point of Aries for January 1, 1855, at Greenwich, and is so given on page xx. of the English Almanac. With this assistance, we can easily convert any instant of sidereal time into its corresponding mean solar time, by the following

RULE.

The mean solar time required = mean time at the preceding sidereal noon, plus the mean interval corresponding to the given sidereal time.

Example. Convert 21h. 8m. 55.39s. sidereal time at Greenwich, January 2, 1855, into mean time.

Mean time at the preceding sidereal noon, January 1	5h. 16m. 50.70s.
Add the given sidereal time reduced to its equivalent mean time	21h. 5m. 27.51s.
The sum is the mean time required, January 2	2h. 22m. 18.21s.

(162.) If the place of observation be not on the meridian of the ephemeris, the mean time of the transit of the first point of Aries must be corrected by the *subtraction* of 9.8296s. $\left(=\frac{3\text{m. } 55.909\text{s.}}{24}\right)$ for each hour of longitude, if the place be to the west of the first meridian, but by its *addition* if to the east.

Example. Convert 22h. 11m. 37.68s. sidereal time at High School Observatory, Philadelphia, October 17, 1855, into mean time.

The mean time at the preceding Greenwich sidereal noon is	10h. 20m. 32.74s.
Correction for 5h. 0m. 37.6s., Philadelphia west of Greenwich	−49.25s.
Mean time at the preceding Philadelphia sidereal noon	10h. 19m. 43.49s.
Add the sidereal time, reduced to its mean equivalent	22h. 7m. 59.52s.
The sum is the mean time required . .	8h. 27m. 43.01s.

PROBLEM.

(163.) *To find the time by observation.*

First Method.—By equal altitudes of a star on opposite sides of the meridian.

Observe the times when the star has equal altitudes before and after passing the meridian; the arithmetical mean between these times is the time of the star's passing the meridian. By comparing this time with the known place of the star, we may obtain the error of the clock.

Example. The numbers in column first of the following table show the times when Arcturus had the altitudes contained in column second, on the east of the meridian. Column third shows the times when it had the same altitudes on the west of the meridian. Column fourth shows the sums of these times, the average of which is 28h. 7m. 42.5s.; consequently the star passed the meridian at 14h. 3m. 51.25s. by the clock.

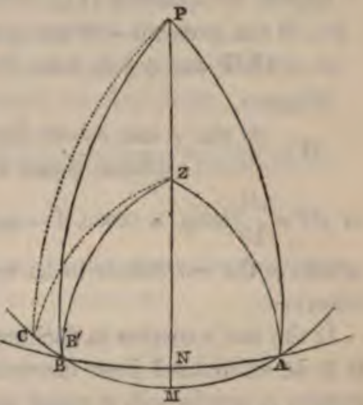
East.			Altitude.		West.			Sum.		
h.	*m.*	*s.*	°	′	*h.*	*m.*	*s.*	*h.*	*m.*	*s.*
10	55	49.2	43	10	17	11	53.0	28	7	42.2
	57	59.5	43	30		9	43.0			42.5
11	0	9.7	43	50		7	32.5			42.2
	2	20.7	44	10		5	22.2			42.9
	6	43.7	44	50		0	59.0			42.7
					Mean			28	7	42.5
					Meridian passage=			14	3	51.25

If we suppose the clock regulated to sidereal time, and the right ascension of the star to be 14h. 9m. 0.16s., then the clock was slow 5m. 8.91s.

(164.) *Second Method.—By equal altitudes of the sun.*

Since the declination of the sun changes from morning to evening, the time of the sun's arriving at a given altitude is affected by this motion, and we must compute the correction to be applied to the mean of the times observed. This may be done by the following method:

Let PZM be the meridian of the place of observation, P the pole, Z the zenith, AMB a small circle parallel to the horizon, ANB the parallel described by a star in its diurnal motion, and cutting the former circle in A and B. If ZA is found by observation equal to ZB, then, since PZ is constant, if the polar distance, PA, does not change, the two triangles, PZA, PZB, will be mutually equilateral, and, consequently, the angle ZPA = ZPB; that is, the hour angle of a star from the meridian is the same for equal altitudes on the east and west sides of the meridian; and this is the case with all the fixed stars, but not with the sun. Suppose the polar distance of the sun has diminished during the interval, then, when the western hour angle, ZPB, is equal to the eastern, ZPA, the sun will be at B′, nearer to the zenith; and when the sun reaches the circle AMB at C, the hour angle ZPC will be greater than the hour angle ZPA or ZPB.



It is necessary, then, to compute the angle BPC.

Put ϕ = the latitude of the place; δ = the declination of the sun when on the meridian; $d\delta$ = the increase of declination from the meridian to the afternoon observation; P = the hour angle from the meridian, supposing no change in the declination; dP = the increase of the hour angle in time caused by the change of declination; and Z = the observed zenith distance.

Now, in the triangle APZ, Trig., Art. 225,

$$\cos. AZ = \cos. PZ \cos. AP + \sin. PZ \sin. AP \cos. APZ,$$

or

$$\cos. Z = \sin. \phi \sin. \delta + \cos. \phi \cos. \delta \cos. P \quad . \; . \; . \; . \; . \; (1)$$

Also, in the triangle CPZ,

$$\cos. CZ = \cos. PZ \cos. CP + \sin. PZ \sin. CP \cos. CPZ,$$

or

$$\begin{aligned}\cos. Z &= \sin. \phi \sin. (\delta + d\delta) + \cos. \phi \cos. (\delta + d\delta) \cos. (P + dP) \\ &= \sin. \phi \sin. \delta \cos. d\delta + \sin. \phi \cos. \delta \sin. d\delta \\ &\quad + \cos. \phi (\cos. \delta \cos. d\delta - \sin. \delta \sin. d\delta)(\cos. P \cos. dP \\ &\qquad - \sin. P \sin. dP).\end{aligned}$$

But since the variations of δ and P, in the present case, are necessarily small, we may put

$$\cos. d\delta = 1; \; \cos. dP = 1; \; \sin. d\delta = d\delta \sin. 1''; \; \sin. dP = 15 dP \sin. 1''.$$

Therefore,

$$\cos. Z=\sin. \phi \sin. \delta+d\delta \sin. 1'' \sin. \phi \cos. \delta+\cos. \phi \cos. \delta \cos. P$$
$$-d\delta \sin. 1'' \cos. \phi \sin. \delta \cos. P$$
$$-15dP \sin. 1'' \cos. \phi \cos. \delta \sin. P.$$

Hence, by equation (1),

$$0=d\delta \sin. \phi \cos. \delta-d\delta \cos. \phi \sin. \delta \cos. P$$
$$-15dP \cos. \phi \cos. \delta \sin. P.$$

Whence

$$dP=\frac{d\delta \sin. \phi \cos. \delta-d\delta \cos. \phi \sin. \delta \cos. P}{15 \cos. \phi \cos. \delta \sin. P},$$

$$\text{or } dP=\frac{d\delta}{15}(\text{tang. } \phi \text{ cosec. } P-\text{tang. } \delta \cot. P) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

which is the correction to be applied to the mean of the times observed.

If the sun's motion in declination is northward, this correction is to be subtracted from the mean of the times observed; if the motion is southward, it must be added.

Ex. 1. At a place in Lat. 54° 20′ N., the sun was found to have equal altitudes at 8h. 59m. 4s. A.M. and at 3h. 0m. 40s. P.M. It is required to find the time of noon, the declination of the sun being 19° 48′ 29″ N., and the decrease of declination between the two observations being 192″.

By equation (2),

$\text{tang. } \phi=0.14406$	$\text{tang. } \delta=9.55652$
$\text{cosec. } P=0.14900$	$\cot. P=9.99697$
$\frac{d\delta}{15}=6.4=0.80618$	$\frac{d\delta}{15}=0.80618$
$12.57\text{s.}=\overline{1.09924}$	$2.29\text{s.}=\overline{0.35967}$

Hence $12.57-2.29=10.28\text{s.}=dP.$

This correction is to be added to the mean of the times observed, because the sun's motion was southward.

The mean of the observed times is 11h. 59m. 52s.; therefore the time of apparent noon was 0h. 0m. 2.28s., or the clock was 2.28s. too fast by apparent time.

(165.) In order to facilitate the preceding computations, various tables have been devised, but the one which has been chiefly used was first proposed by Gauss. Table XI. is from the American Nautical Almanac for 1856, and was furnished by Professor Chauvenet. It differs from the table of Gauss only in using the hourly change of the sun's declination instead of twice the daily change. This table was constructed as follows:

Put T = the interval of time between the morning and afternoon observations, expressed in hours.

μ = the hourly change of the sun's declination.

Then, since $d\delta$ represents the increase of declination from the meridian to the afternoon observation, we shall have

$$d\delta=\tfrac{1}{2}\mu T.$$

And since P represents the hour angle from the meridian expressed in arc, we shall have

$$P=7\tfrac{1}{2}T.$$

Hence the correction to be added to the mean of the times observed to obtain the time of apparent noon is

$$x=-\frac{\mu T \text{ tang. } \phi}{30 \text{ sin. } 7\frac{1}{2}T}+\frac{\mu T \text{ tang. } \delta}{30 \text{ tang. } 7\frac{1}{2}T},$$

or
$$x=-\mu \text{ tang. } \phi\frac{T}{30 \text{ sin. } 7\frac{1}{2}T}+\mu \text{ tang. } \delta\frac{T}{30 \text{ tang. } 7\frac{1}{2}T}.$$

Let us make

$$A=\frac{T}{30 \text{ sin. } 7\frac{1}{2}T}, \text{ and } B=\frac{T}{30 \text{ tang. } 7\frac{1}{2}T},$$

and we shall have

$$x=-A\mu \text{ tang. } \phi+B\mu \text{ tang. } \delta.$$

Table XI. furnishes the values of A and B for all values of T from 2 hours to 24 hours. The following is the method of computing A and B:

Let T = 2 hours.

$$\begin{array}{rr}
7\tfrac{1}{2}T=15^\circ \text{ sin.}=9.4130 & 7\tfrac{1}{2}T=15^\circ \text{ tang.}=9.4280 \\
30=\underline{1.4771} & 30=\underline{1.4771} \\
\underline{0.8901} & \underline{0.9051} \\
2=0.3010 & 2=0.3010 \\
\text{log. } A=\overline{9.4109} & \text{log. } B=\overline{9.3959}
\end{array}$$

which are the values of log. A and log. B, given in the table for an interval of 2 hours; and in the same manner were the other numbers computed. If we employ the numbers of this table, the computation of Ex. 1 will proceed as follows:

The interval of time between the morning and afternoon observations being 6h. 1m. 36s., we have, by Table XI., log. A = 9.4520, and log. B = 9.2999; and by the Nautical Almanac, $\mu=-31''.85$. The operation, therefore, will stand thus:

$$\begin{array}{cc} \text{log. A}=9.4520 & \text{log. B}=9.2999 \\ \mu=-31.85=1.5031n & \mu=-31.85=1.5031n \\ \text{tang. }\phi=\underline{0.1441} & \text{tang. }\delta=\underline{9.5565} \\ -12.57\text{s.}=1.0992n & -2.29\text{s.}=0.3595n \end{array}$$

Hence $x=12.57-2.29=10.28$, the same correction as found on page 128.

The following rule for the signs of the two terms of the correction for equal altitudes may be found convenient:

The sign of the first term is positive from the summer to the winter solstice, and negative from the winter to the summer solstice.

The sign of the second term is positive from the equinoxes to the solstices, and negative from the solstices to the equinoxes.

(166.) The following is the most convenient mode of taking these observations. Having brought the lower limb of the sun, as seen reflected from the sextant mirror, into approximate contact with the upper limb, as seen reflected from the mercury, move the vernier forward, and set the zero to coincide with some convenient division upon the limb. Wait for the instant of contact, and note the time by the chronometer. Move the vernier forward 10′ or 20′, and note the instant of contact as before, making the successive observations at equal intervals of 10′ or 20′. It is by no means necessary that the sextant should indicate the true altitude of the body, for it is the peculiar excellence of this method that it merely requires the observations to be made at the *same* altitude on both sides of the meridian.

Ex. 2. At Pembina, in Lat. 48° 58′ 34″, the following double altitudes of the sun's upper limb were observed August 22d, 1849:

A.M.	Double Altitudes.	P.M.
h. m. s.	° ′ ″	h. m. s.
9 0 50	78 36 45	2 33 55
9 1 55	78 53 45	2 32 47
9 3 14	79 14 15	2 31 35
9 4 25	79 32 30	2 30 28
9 5 9	79 43 45	2 29 23
9 6 8	80 2 15	2 28 40
9 6 48	80 10 45	2 28 2
9 8 12	80 34 45	2 26 40

It is required to find the error of the chronometer, the declination of the sun being 11° 39′ 44″ N., and the hourly decrease of declination being 50″.77.

When we have a number of observations made at short intervals of time, as in the present instance, it is most convenient to take the average of all the morning observations, which in the present case is 9h. 4m. 35.1s.; and also the average of the evening observations, which in the present case is 2h. 30m. 11.2s., and regard them as constituting one complete observation. The mean of these times is 11h. 47m. 23.15s.; the correction of the hour angle is found to be 13.98s. Therefore the time of apparent noon was 11h. 47m. 37.13s., or the chronometer was slow by apparent time 12m. 22.87s.

Ex. 3. It is required to find the error of the chronometer from the following observations of the sun's lower limb, made October 8th, 1852, in Lat. 30° 4′ N.; the sun's declination at noon, October 8th, being 6° 7′ S., and decreasing 57″.17 per hour.

A.M.			Double Altitudes.		P.M.		
h.	*m.*	*s.*	°	′	*h.*	*m.*	*s.*
21	7	27	73	0	2	33	59
	8	24		20		33	3
	9	23		40		32	5
	10	18	74	0		31	9
	11	16		20		30	12
	12	11		40		29	14
	13	11	75	0		28	13
	14	9		20		27	15
	15	10		40		26	15
	16	6	76	0		25	20

Ans. The mean of the observed times is 23h. 50m. 43.0s.; the correction of the hour angle is +10.45s. Hence the time of apparent noon was 23h. 50m. 53.45s.; and since the equation of time was −12m. 34.77s., the chronometer was 3m. 28.22s. too fast by mean time.

(167.) It frequently happens that clouds prevent our taking the afternoon observations corresponding to the morning observations; but if the clouds subsequently disperse, we may still take a series of western altitudes, and wait about 18 hours to observe the corresponding eastern altitudes. If the observations are made upon a star, the mean of the observed times will give the time of passage over the lower meridian. If the observations are made upon the sun, the correction to the mean of the observed times will still be given by formula (2), page 128. If the sun is moving northward, it will be further from the upper meridian at the time of the eastern observation than at the time of the western, that is, it will be nearer to the lower meridian. Hence the correction given by formula (2) must be added to the mean of the observed times; and if the interval between the observations exceeds 12 hours, B will be negative, because cot. P will be negative. Hence the correction to be added algebraically to the mean of the observed times, to obtain the time of apparent midnight, is

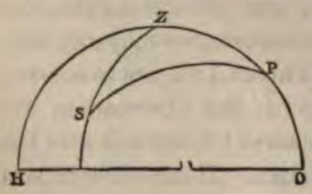
$$x' = \mathrm{A}\mu \text{ tang. } \phi + \mathrm{B}\mu \text{ tang. } \delta.$$

Ex. 4. It is required to find the error of the chronometer from the following observations of the sun's lower limb, made October 8th and 9th, 1852, in latitude 30° 4′ N., the sun's declination at midnight being −6° 19′, and decreasing 57″.06 per hour.

October 8th, P.M	Double Altitudes.	October 9th, A.M.
h. m. s.	° ′	h. m. s.
2 33 59	73 0	21 8 43
33 3	20	9 41
32 5	40	10 39
31 9	74 0	11 36
30 12	20	12 34
29 14	40	13 31
28 13	75 0	14 30
27 15	20	15 28
26 15	40	16 28
25 20	76 0	17 26

Ans. The mean of the observed times is 11h. 51m. 22.05s.; the correction of the hour angle is −37.12s. Hence the time of apparent midnight was 11h. 50m. 44.93s.; and since the equation of time was −12m. 42.82s., the chronometer was 3m. 27.75s. too fast by mean time.

(168.) *Third Method.—By a single altitude of the sun or a star.*



Let PZH be the meridian of the place of observation, P the pole, Z the zenith, and S the place of the sun or star. If the zenith distance, SZ, has been measured and corrected for refraction, then in the spherical triangle, ZPS, the three sides are known, viz.,

PZ = the co-latitude = ψ;
ZS = the true zenith distance = z;
PS = the north polar distance of the star = d.

In this triangle we can compute the angle ZPS, which is the distance of the star from the meridian.

By Trig., Art. 226,

$$\sin.\ \tfrac{1}{2}A = \sqrt{\frac{\sin.\ (S-b)\sin.\ (S-c)}{\sin.\ b\ \sin.\ c}}.$$

Put $$2S = z + d + \psi;$$

then $$\sin.\ \tfrac{1}{2}P = \sqrt{\frac{\sin.\ (S-\psi)\sin.\ (S-d)}{\sin.\ \psi\ \sin.\ d}}.$$

Ex. 1. At a place in Lat. 25° 40′ N., the sun's correct central altitude was found to be 10° 6′ 27″, when his declination was 8° 5′ 56″ S. What was his distance from the meridian?

$$\begin{array}{rl@{\qquad}r}
d = & 98^\circ\ \ 5'\ 56'' & \sin.\ (S-\psi) = 9.922746 \\
z = & 79^\circ\ 53'\ 33'' & \sin.\ (S-d) = 9.593007 \\
\psi = & \underline{64^\circ\ 20'\ \ 0''} & \text{cosec. } \psi = 0.045117 \\
 & 242^\circ\ 19'\ 29'' & \text{cosec. } d = \underline{0.004353} \\
S = & 121^\circ\ \ 9'\ 44'' & 2)\,\underline{9.565223} \\
S-\psi = & 56^\circ\ 49'\ 44'' & \tfrac{1}{2}P = 37^\circ\ 18'\ 53''.5\ \sin. = 9.782612 \\
S-d = & 23^\circ\ \ 3'\ 48'' & P = 74^\circ\ 37'\ 47'', \\
 & & \text{or 4h. 58m. 31.1s.} = \text{apparent time.}
\end{array}$$

It may be found convenient to employ in our computation ϕ, the latitude of the place, and δ, the declination of the star, rather than the co-latitude and polar distance. For this purpose, we have only to substitute in the preceding formula δ for $90^\circ - d$, and ϕ for $90^\circ - \psi$, and we shall obtain

$$\sin.\ \tfrac{1}{2}P = \sqrt{\frac{\sin.\left[\dfrac{z+(\phi-\delta)}{2}\right]\times\sin.\left[\dfrac{z-(\phi-\delta)}{2}\right]}{\cos.\ \phi\ \cos.\ \delta}}.$$

Ex. 2. At a place in Lat. 52° 13′ 26″ N., at 3h. 21m. 13.4s. P.M. by the clock, the corrected zenith distance of the sun's centre was found to be 75° 16′ 15″, when his declination was 9° 33′ 30″ S. Required the correction of the clock.

Ans. The true hour angle was 3h. 21m. 22.7s.; hence the clock was 9.3s. slow.

Ex. 3. On the 4th of March, 1850, at 13h. 16m. 45.12s. by the sidereal clock, the zenith distance of α Lyræ was observed at Greenwich to be 54° 16′ 14″.58; it is required to determine the error of the clock, supposing the star's R. A. to be 18h. 31m. 50.84s., and its declination 38° 38′ 39″.4 N.

Ans. The clock was slow 19.64s.

Ex. 4. The following double altitudes of the sun's upper limb were observed August 29, 1849, in Lat. 48° 17′ N.:

Times.			Double Altitudes.		
h.	m.	s.	°	′	″
8	58	58	78	7	45
9	0	15	78	27	10
9	1	14	78	45	45
9	2	10	78	57	30
9	3	5	79	12	30
9	4	39	79	35	45
9	5	54	80	54	45
9	6	41	80	8	45
9	7	37	80	21	15
9	8	25	80	37	15

It is required to determine the error of the chronometer, the sun's declination being 9° 16′ 20″ N., and his semi-diameter 15′ 52″. *Ans.* The chronometer was slow 23m. 4.44s.

The most favorable opportunity for determining the time from altitudes of the sun or a star is when it rises or falls most rapidly. This happens when the sun or star is passing the prime vertical; that is, when it is nearly east or west. The sun's altitude should not be less than 10 degrees, on account of the irregular refraction near the horizon. In general, two or three hours from the meridian will be sufficient.

(169.) *Corollary.* By the same method we may compute the time at which the sun's upper limb rises or sets, when allowance is made for refraction. The effect of refraction is to cause the sun to appear above the sensible horizon sooner in the morning and later in the afternoon than he actually is; and when the sun's upper limb coincides with the horizon, the centre is about 16′ below. At the instant, therefore, of sunrise or sunset, his centre is 90° 50′ from the zenith; the semi-diameter being about 16′, and the horizontal refraction 34′.

Ex. 1. Required the time of sunset at New York, Lat. 40° 42′, at the summer solstice.

$$
\begin{array}{lrll}
\text{Here} & \psi = & 49^\circ\ 18' & \text{sin.}\ (S-\psi)=9.908141 \\
 & d = & 66^\circ\ 32' & \text{sin.}\ (S-d)=9.777444 \\
 & z = & \underline{90^\circ\ 50'} & \text{cosec.}\ \psi=0.120254 \\
 & S = & 103^\circ\ 20' & \text{cosec.}\ d=\underline{0.037492} \\
 & S-\psi = & 54^\circ\ \ 2' & 2)\ 9.843331 \\
 & S-d = & 36^\circ\ 48' & \tfrac{1}{2}P=\ 56^\circ\ 36'\ \text{sin.}=\overline{9.921666} \\
 & & & P=113^\circ\ 12'=7\text{h.}\ 33\text{m.}
\end{array}
$$

Hence the sun sets at 7h. 33m. apparent time; or, adding 1m. for equation of time, we have 7h. 34m. mean time.

Ex. 2. Required the mean time of sunset at New Orleans, Lat 29° 58′ at the winter solstice; mean time being one minute slow of apparent time. *Ans.* 5h. 5m.

(170.) The preceding methods are adapted to the use of travelers and navigators, as the observations may all be made with a sextant. In fixed observatories the time is habitually found by a transit instrument, which is the most accurate method known, as well as the most convenient.

Fourth Method.—To determine time by the transit instrument.

The instant of the sun's passing the meridian is the time of apparent noon; and hence, if we compare the sun's passage over the meridian with a chronometer, we shall obtain the deviation of the chronometer from apparent solar time. If to this we apply the equation of time with its proper sign, we shall obtain the error of the chronometer in mean time.

Ex. 1. The sun was observed to pass the meridian at 11h. 59m. 18.7s. by chronometer, the equation of time being +13m. 22.5s. Required the error of the chronometer.

Ans. 0m. 41.3s. slow for apparent time;
14m. 3.8s. slow for mean time.

Ex. 2. The sun was observed to pass the meridian at 11h. 56m. 12.21s. by chronometer; the equation of time being −3m. 56.26s. Required the error of the chronometer.

Ans. 3m. 47.79s. slow for apparent time;
0m. 8.47s. fast for mean time.

In a fixed observatory it is most convenient for ordinary purposes to employ sidereal time. The error of a sidereal clock or chronometer is found in the manner already explained, except that we must know the right ascension of the object observed. The right ascension of the sun and 100 fixed stars is given for every day of the year, in both the English and American Nautical Almanacs; and the right ascension of 1500 stars is given in the catalogue at the close of this volume.

Ex. 3. The star Rigel was observed to pass the meridian of Greenwich, February 6, 1851, at 5h. 6m. 35.41s. by a sidereal clock, the star's right ascension being 5h. 7m. 22.97s. Required the error of the clock. *Ans.* 47.56s. slow.

Ex. 4. The sun's centre passed the meridian of Greenwich, May 15, 1851, at 3h. 25m. 35.17s. by the sidereal clock, the sun's right ascension being 3h. 26m. 33.78s. Required the error of the clock. *Ans.* 58.61s. slow.

(171.) The error of the clock may be deduced from the transit of any star whose right ascension is known; but the places of all stars contained in the catalogues are not equally well determined; and it is obviously proper that the stars whose places are best determined should be preferred for this purpose. The places of the 100 stars in the Nautical Almanac are considered to be better known than any others. At the Greenwich Observatory, the error of the clock is determined exclusively by the Nautical Almanac stars; and only those are used whose declination is less than 40 degrees.

At the Oxford Observatory, the stars used for finding the clock error are chiefly the Nautical Almanac stars, but occasionally other stars are employed.

At the Edinburgh Observatory, only Nautical Almanac stars are used for determining the correction of the clock, and of this list only those are employed whose places are considered to be best determined.

At the Washington Observatory, the error of the clock is determined from observations of stars of the American Ephemeris, situated within 40 degrees of the equator.

CHAPTER VI.

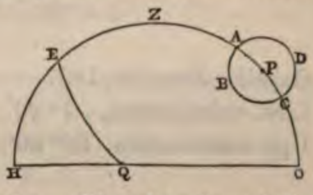
LATITUDE.

(172.) THE latitude of a place is equal to the elevation of the pole above the horizon, and this altitude could be easily determined if the pole were a visible point. But as there is no star exactly at the pole, its position must be determined by observations of stars at a distance from it.

FIRST METHOD.

By transits of a circumpolar star both above and below the pole.

The best method of determining the latitude of a place, so as to be independent of the declination of the star observed, and also as free as possible from the errors of refraction, is by observations of a circumpolar star at the time of its upper and lower culminations. These observations may be made by means of a mural circle, or any graduated circle.



Let HZPO represent a meridian, HO the horizon of the place of observation, P the place of the pole, and ABCD the circle described by a circumpolar star in its diurnal motion. The elevation of the pole PO is equal to half the sum of AO and CO, corrected for refraction.

Let A and A′ represent the altitudes of a circumpolar star at its upper and lower culminations; also, let r and r' be the refractions corresponding to these altitudes; then

$$\phi=\tfrac{1}{2}(\mathrm{A}+\mathrm{A}'-r-r'),$$

both altitudes being measured from the north horizon.

If zenith distances instead of altitudes are observed, the colatitude will be,

$$\psi=90^\circ-\phi=\tfrac{1}{2}(\mathrm{Z}+\mathrm{Z}'+r+r').$$

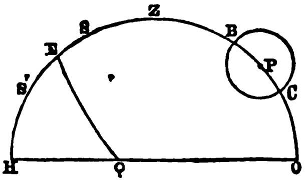
The refraction is derived from Table VIII.

Examples. The following observations were made at Greenwich Observatory:

	Observed Altitude.	Refraction.	True Altitude.
Polaris, May 9, 1851.			
Lower culmination,	50° 0′ 8″.49 −	48″.08 =	49° 59′ 20″.41
Upper culmination,	52° 58′ 38″.31 −	42″.16 =	52° 57′ 56″.15
		Sum =	102° 57′ 16″.56
		Latitude =	51° 28′ 38″.28
δ Ursæ Minoris, January 22, 1851.			
Lower culmination,	48° 5′ 18″.60 −	53″.80 =	48° 4′ 24″.80
Upper culmination,	54° 53′ 33″.22 −	42″.49 =	54° 52′ 50″.73
		Sum =	102° 57′ 15″.53
		Latitude =	51° 28′ 37″.77
β Cephei, March 17, 1847.			
Low. culmination,	31° 23′ 32″.85 −	1′ 35″.22 =	31° 21′ 57″.63
Upp. culmination,	71° 35′ 36″.53 −	0′ 19″.22 =	71° 35′ 17″.31
		Sum =	102° 57′ 14″.94
		Latitude =	51° 28′ 37″.47
α Cephei, March 17, 1847.			
Low. culmination,	23° 27′ 6″.03 −	2′ 13″.53 =	23° 24′ 52″.50
Upp. culmination,	79° 32′ 33″.71 −	0′ 10″.66 =	79° 32′ 23″.05
		Sum =	102° 57′ 15″.55
		Latitude =	51° 28′ 37″.77
Capella, June 14, 1837.			
Low. culmination,	7° 25′ 0″.72 −	6′ 52″.78 =	7° 18′ 7″.94
Upp. culmination,	95° 39′ 2″.42 +	5″.49 =	95° 39′ 7″.91
		Sum =	102° 57′ 15″.85
		Latitude =	51° 28′ 37″.92

As the refraction for the last star is large, the result of that observation is less reliable than the others. Those stars are accordingly to be preferred whose polar distance is the least.

SECOND METHOD.



(173.) *By simple meridian altitudes.*

Let PZH represent the meridian of the place of observation, HO the horizon, Z the zenith, P the place of the pole, EQ the equator, S or S′ a star on the meridian, SE or S′E its declination (δ), SP or S′P its distance from the pole (d), which is the complement of δ; the arc EH is the complement of the latitude (ϕ), or $90^\circ-\phi$.

We measure the altitude (A) of the object S or S′, or its zenith distance (Z), and correct it for refraction and parallax, if the parallax is appreciable. Then it is evident that

$$EH=SH-SE=S'H+S'E.$$

These two equations are included in the same expression by regarding the declination negative when it is south of the equator. Thus,

$$90^\circ-\phi=A-\delta,$$

or

$$\phi=90^\circ+\delta-A.$$

But

$$Z=90^\circ-A.$$

Hence

$$\phi=\delta+Z,$$

for stars which culminate south of the zenith, where δ must have the negative sign when the declination is south.

If the star passes the meridian between the north pole and the zenith, as, for example, at B, then we shall have

$$PO=BO-BP;$$

that is,

$$\phi=A-d.$$

But

$$A=90^\circ-Z, \text{ and } d=90^\circ-\delta.$$

Hence

$$\phi=\delta-Z.$$

If the star passes the meridian below the north pole, then we shall have

$$PO=CO+PC;$$

that is,

$$\phi=A+d=180^\circ-\delta-Z.$$

Hence we shall have

$\phi=\delta+Z$ if the observations be made to the south;

$\phi=\delta-Z$ if to the north, *above* the pole;

$\phi=180^\circ-(\delta+Z)$ if to the north, *below* the pole.

The following observations were made at Greenwich in 1851:

Stars South of the Zenith.

February 10, 1851.

	Observed Zenith Distance.	Refraction.	True Zenith Distance.
Pollux.	23° 5′ 24″.05+	25″.80=	23° 5′ 49″.85
		Star's declination=	28° 22′ 47″.70
		Latitude =	51° 28′ 37″.55

July 10, 1851.

Observed Zenith Distance. Refraction. True Zenith Distance.

Antares . . . . 77° 30′ 11″.54 + 4′ 15″.48 = 77° 34′ 27″.02

Star's declination = −26° 5′ 48″.50

Latitude = 51° 28′ 38″.52

June 30, 1851.

Sun. Observed zenith distance of upper limb, 27° 59′ 39″.53

semi-diameter, +15′ 46″.05

refraction, +29″.49

parallax, −3″.93

true zenith distance of centre = 28° 15′ 51″.14

Sun's declination = +23° 12′ 47″.30

Latitude = 51° 28′ 38″.44

The method of computing the sun's parallax will be explained in Article 206.

Star North of the Zenith, and above the Pole.

June 29, 1851.

Observed Zenith Distance. Refraction. True Zenith Distance.

α Ursæ Majoris, 11° 4′ 39″.01 + 10″96 = 11° 4′ 49.″97

Star's declination = +62° 33′ 26″.69

Latitude = 51° 28′ 36″.72

Star below the Pole.

January 27, 1851.

Observed Zenith Distance. Refraction. True Zenith Distance.

β Ursæ Minoris, 53° 44′ 24″.60 + 1′ 20″.69 = 53° 45′ 45″.29

Star's declination = 74° 45′ 37″.79

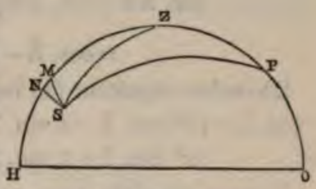
Sum = 128° 31′ 23″.08

Latitude = 51° 28′ 36″.92

THIRD METHOD.

(174.) *By circum-meridian altitudes.*

The preceding method gives but one value of the latitude, because the star can only be observed at the instant when it crosses the meridian. But where the observer is furnished with an altitude and azimuth instrument, a repeating circle or sextant, we may render any number of observations made on each side of the meridian, and at a short distance from it, equal in accuracy to those which are made at the moment of culmination. For this purpose, we must know the distance (in time) of the star from the meridian at the instant of each observation, and we can compute the correction which ought to be applied to the zenith distance observed.



Let P be the pole, Z the zenith of the place of observation, PZM a meridian, S a star near to the meridian, M the point where this star crosses the meridian, and PS an hour circle passing through the star. Suppose the zenith distance, ZS, of the star has been measured, and corrected for refraction, and also for parallax, when the sun or a planet has been observed; it is required to compute the zenith distance, ZM, of the star when on the meridian. Now from the figure we perceive that

$$PS=90^\circ-\delta,$$
$$PZ=90^\circ-\phi,$$
$$ZM=PM-PZ=\phi-\delta=(Z),$$

the zenith distance of the star on the meridian.

With Z as a centre, describe the arc SN, and the point N will be at the same altitude as S. It is required to compute $MN=x$, the quantity which the star must rise from S, before it reaches the meridian.

By Trig., Art. 225,

$$\cos. a=\cos. b \cos. c+\sin. b \sin. c \cos. A.$$

But $\cos. A=1-2 \sin.^2\tfrac{1}{2} A.$ Trig., Art. 74.

Hence

$$\cos. a=\cos. b \cos. c+\sin. b \sin. c-2 \sin. b \sin. c \sin.^2\tfrac{1}{2}A.$$

Also, $\cos. b \cos. c+\sin. b \sin. c=\cos. (b-c).$ Trig., Art. 72.

Hence $\cos. a=\cos. (b-c)-2 \sin. b \sin. c \sin.^2\tfrac{1}{2}A.$

Applying this formula to the triangle PZS, and representing the angle ZPS by P, we have

$$\cos. ZS=\cos. (PS-PZ)-2 \sin. PZ \sin. PS \sin.^2\tfrac{1}{2}P$$
$$=\cos. Z-2 \cos. \phi \cos. \delta \sin.^2\tfrac{1}{2}P \quad . \; . \; . \; . \; . \; . \quad (1)$$

But $ZS=ZM+x.$

Hence

$$\cos. ZS=\cos. ZM \cos. x-\sin. ZM \sin. x.$$ Trig., Art. 72.

But, since x is supposed to be a small arc, we may put

$$x=\sin. x,$$

and $$\cos. x = 1 - \frac{x^2}{2} +, \text{ etc.} \quad \text{Calculus, p. 174.}$$

Hence we obtain

$$\cos. ZS = \cos. ZM(1 - \frac{x^2}{2} +, \text{etc.}) - x \sin. ZM$$
$$= \cos. Z - \tfrac{1}{2}x^2 \cos. Z - x \sin. Z.$$

Therefore equation (1) becomes

$$\cos. Z - \tfrac{1}{2}x^2 \cos. Z - x \sin. Z = \cos. Z - 2 \cos. \phi \cos. \delta \sin.^2 \tfrac{1}{2}P,$$

or $$\tfrac{1}{2}x^2 \cos. Z + x \sin. Z = 2 \cos. \phi \cos. \delta \sin.^2 \tfrac{1}{2}P \quad . \; . \quad (2)$$

If we neglect the term containing x^2, and suppose x to be expressed in seconds, we shall have

$$x = \frac{2 \sin.^2 \tfrac{1}{2}P \cos. \phi \cos. \delta}{\sin. 1'' \sin. Z},$$

which formula is sufficiently accurate, when the hour angle does not exceed ten minutes. If a further approximation is required, it may be obtained as follows:

Divide equation (2) by sin. Z, and we obtain

$$x + \tfrac{1}{2}x^2 \cot. Z = \frac{2 \sin.^2 \tfrac{1}{2}P \cos. \phi \cos. \delta}{\sin. Z}.$$

Represent the second member of this equation by B, and $\tfrac{1}{2}$ cot. Z by A, then $$x + Ax^2 = B,$$
or $$x = B - Ax^2.$$

But B is the approximate value of x before found; hence, for a second approximation, we shall have

$$x = B - AB^2;$$

or, supposing x to be expressed in seconds,

$$x = \frac{2 \sin.^2 \tfrac{1}{2}P \cos. \phi \cos. \delta}{\sin. 1'' \sin. Z} - \left(\frac{\sin.^2 \tfrac{1}{2}P \cos. \phi \cos. \delta}{\sin. Z}\right)^2 \frac{2 \cot. Z}{\sin. 1''} \quad . \; (3)$$

which is the correction to be subtracted from the zenith distances observed near the meridian, for an upper culmination, in order to obtain the true meridional zenith distance.

Ex. 1. On the 23d of February, 1850, the zenith distance of α Orionis was observed at Greenwich, 20m. 26.25s. before coming to the meridian, to be 44° 18′ 30″.31, the declination of the star being 7° 22′ 14″.74 N. Required the reduction to the meridian and the resulting latitude.

Here P = 20m. 26.25s.; therefore $\tfrac{1}{2}$P = 10m. 13.12s., which, reduced to arc, is 2° 33′ 16″.9.

$$\begin{array}{rl}
\sin.\quad 2^\circ\, 33'\, 16''.9 & = 8.649071 \\
 & \, 8.649071 \\
\cos.\ \phi,\ 51^\circ\, 28'\, 38'' & = 9.794366 \\
\cos.\ \delta,\ \ 7^\circ\, 22'\, 15'' & = 9.996396 \\
\text{cosec.}\, z,\ 44^\circ\ \ 6'\, 23'' & = 0.157394 \\ \hline
m & = 7.246298 \\
2\ \text{cosec.}\ 1'' & = 5.615455 \\ \hline
727''.37 & = 2.861753
\end{array}
\qquad
\begin{array}{rl}
m^2 & = 4.4926 \\
\cot.\ z & = 0.0135 \\
2\ \text{cosec.}\ 1'' & = 5.6155 \\ \hline
1''.32 & = 0.1216
\end{array}$$

Therefore

$$x = 727''.37 - 1''.32 = 726''.05.$$

Hence we have

Observed zenith distance	$=44^\circ\ 18'\ 30''.31$
Reduction to the meridian	$=\ \ -12'\ \ 6''.05$
Corrected zenith distance	$=44^\circ\ \ 6'\ 24''.26$
Star's declination	$=\ 7^\circ\ 22'\ 14''.74$
Latitude	$=51^\circ\ 28'\ 39''.00$

(175.) To diminish the labor of these reductions, Table X. has been computed, in which Part I. gives the value of $\frac{2 \sin.^2 \frac{1}{2} P}{\sin. 1''}$; and the argument of the table is the distance (in time) of the star from the meridian. This value (or the sum of those values divided by the number of observations, if more than one observation has been made) must be multiplied by $\frac{\cos. \phi \cos. \delta}{\sin. Z}$. Part II. of the table contains the value of $\frac{2 \sin.^4 \frac{1}{2} P}{\sin. 1''}$, which must be multiplied by $\left(\frac{\cos. \phi \cos. \delta}{\sin. Z}\right)^2 \cot. Z$. This second correction may generally be omitted when the distance from the meridian does not exceed ten minutes.

Ex. 2. The following observations of Polaris at its upper culmination were made at Washington Observatory, November 25, 1845, the altitude of the star having been observed at each vertical wire of the mural circle. In the following table, column first shows the wire at which the observation was made, column second shows the hour angle of the star from the meridian, and column third shows the observed zenith distances corrected for error of runs:

Wire.	Hour angles.	Zenith Distances.	Table X.
	m. s.	° ′ ″	″
1	29 25.5	49 35 27.92	1697.65
2	19 34.5	35 2.86	751.95
3	9 49.5	34 48.12	189.50
4	0 13.5	34 43.74	0.10
5	9 33.5	34 48.27	179.35
6	19 13.5	35 2.56	725.25
7	29 0.5	35 28.07	1649.95
Means		49 35 3.07	741.96

Column fourth contains the numbers from Table X. corresponding to the hour angles in column second. The mean of these numbers is 741″.96. The reduction to the meridian is then computed as follows:

$$
\begin{aligned}
741''.96 &= 2.87038\\
\cos.\ \phi,\ 38^\circ\ 53'\ 39'' &= 9.89115\\
\cos.\ \delta,\ 88^\circ\ 29'\ 34'' &= 8.42000\\
\text{cosec. } z,\ 49^\circ\ 35'\ 55'' &= 0.11832\\
\text{Reduction} = 19''.95 &= \overline{1.29985}
\end{aligned}
$$

The latitude will then be obtained as follows:

Mean of observed zenith distances $= 49^\circ\ 35'\ \ 3''.07$
Reduction to the meridian $= \quad -19''.95$
Refraction $= \quad +\ 1'\ 11''.08$
Corrected zenith distance $= 49^\circ\ 35'\ 54''.20$
Star's declination $= 88^\circ\ 29'\ 34''.15$
Latitude $= 38^\circ\ 53'\ 39''.95$

(176.) In the preceding reductions it is necessary to know the distance (in time) of the sun or star from the meridian, or the hour angles, at the moment of each observation. These hour angles are determined by the chronometer; and it is desirable that its motion should correspond to that of the object observed; that is, if the sun be the object, the chronometer should be adjusted to mean solar time; and if a star be the object, the chronometer should be adjusted to sidereal time. This, however, is not necessary, since a correction may be readily applied so as to reduce the rate either from mean solar to sidereal, or from sidereal to mean solar time. A further correction is also necessary in all cases where the chronometer has a gaining or a losing rate on either mean solar or sidereal time. This correction is obtained in the following manner:

If the clock in 24 hours loses r seconds, then, instead of 86400 beats in a day, it will make only $86400-r$. The true value of an hour angle, P, noted by such a clock, is

$$\mathrm{P}.\frac{86400}{86400-r}, \text{ or } \mathrm{P}\left(1+\frac{r}{86400-r}\right);$$

that is, the observed hour angle should be multiplied by the factor

$$1+\frac{r}{86400-r}.$$

The principal term of formula (3) for the reduction on page 142 contains the factor $\sin.^2\tfrac{1}{2}\mathrm{P}$, which must therefore be multiplied by the square of the above factor, which is nearly equal to

$$1+\frac{2r}{86400-r}.$$

If the clock indicates mean solar time, and we are observing a star, the clock loses 235.909s. in 24 hours, when compared with the progress of the star, and we must take $r=235.909$s., and the preceding factor becomes

$$1.005476,$$

and its logarithm is

$$0.0023715.$$

In a similar manner we obtain the correction for a loss or gain of 1, 2, 3, etc., seconds per day of the chronometer. This correction has been computed, and is appended to Table X., which gives the logarithm of the factor to be employed for a daily rate of the clock or chronometer, amounting to ±30 seconds. These values are in all cases additive.

Ex. 3. On the 18th of October, 1841, near the River St. John's, in latitude about 46° 53′ 30″, the following observations were made on the star α Ceti, declination 3° 28′ 8″.2 N. Column first shows the number of the observation, column second shows the hour angle of the star from the meridian, and column third shows the observed altitude corrected for the error of the sextant. The chronometer was regulated to mean solar time, and had a daily losing rate of 2.7s.

Obs.	Hour Angles.	Altitudes observed.	Table X.	
			Part I.	Part II.
	m. s.	° ′ ″	″	″
1	14 30.0	46 28 45	412.70	0.41
2	10 50.2	46 31 45	230.54	0.13
3	5 0.0	46 34 35	49.10	0.01
4	1 59.7	46 35 25	7.77	0.00
5	2 3.0	46 35 30	8.20	0.00
6	5 6.5	46 34 45	51.25	0.01
7	7 21.7	46 33 50	106.45	0.03
8	13 7.3	46 30 0	337.97	0.28
9	16 40.1	46 26 35	545.39	0.72
Means		46 32 21.1	194.37	0.18

Column fourth contains the numbers from Part I., Table X., corresponding to the hour angles in column second. Column fifth contains the numbers from Part II., Table X. The reduction to the meridian is then computed as follows:

cos. ϕ, 46° 53′ 30″ $=9.834662$
cos. δ, 3° 28′ 8″ $=9.999204$ $m^2=9.993$
cosec. z, 43° 25′ 22″ $=0.162805$ cot. $z=0.023$
$m=\overline{9.996671}$ $0.18=9.255$
$194''.37=2.288629$ $0.19=\overline{9.271}$
On account of mean solar time $=0.002371$
On account of rate of clock $=0.000027$
$193''.95=\overline{2.287698}$

Therefore $x=193''.95-0''.19=193''.76.$

Hence we have the following results:

Observed zenith distance $=43°\ 27'\ 38''.9$
Reduction to the meridian $=\quad -3'\ 13''.8$
Refraction $=\quad +\ 55''.8$
Corrected zenith distance $=\overline{43°\ 25'\ 20''.9}$
Star's declination $=\ 3°\ 28'\ 8''.2$
Latitude $=\overline{46°\ 53'\ 29''.1}$

(177.) When the sun is the object observed, we must take into account the change of declination during the interval of the observations; for the observed altitude, corrected in the manner before explained, will not be equal to the meridian altitude, but will differ from it by the change in the sun's declination. Let the change of the sun's declination in one minute of time be denoted by $d\delta$, which is positive when the sun is approaching the elevated pole; and if P is the sun's hour angle at the time of observation, which is negative before the sun arrives at the meridian and afterward positive, the whole change of declination is $Pd\delta$, which is the correction to be applied to the altitude found by Art. 176 to obtain the true meridian altitude. When several observations have been made, the mean of the values found by Art. 176 is to be diminished by the mean of the values of $Pd\delta$. But the hour angles have contrary signs on opposite sides of the meridian; hence, if we make $\mathrm{E}=$ the sum of the hour angles observed on the east side of the meridian, and $\mathrm{W}=$ the sum of the hour angles observed on the west side, $(\mathrm{E}-\mathrm{W})d\delta$ will be the correction for the sum of the observed distances. If we make $n=$ the number of the observations, the mean correction to be applied to the mean of all the observed zenith distances will be

$$\frac{d\delta}{n}(\mathrm{E}-\mathrm{W}),$$

where E and W are expressed in minutes of time.

Ex. 4. At a station in Lat. 51° 32′ N. nearly, the correct central altitudes of the sun on the 11th of March were determined by observation, as follows:

Altitudes. ° ′ ″	Hour Angles. m. s.	By Table X. ″
34 54 46	9 41 E.	184.1
55 26	8 19 E.	135.8
56 8	6 39 E.	86.8
56 31	5 16 E.	54.5
56 53	3 49 E.	28.6
57 6	2 47 E.	15.2
57 18	0 19 W.	0.2
57 11	2 5 W.	8.5
57 3	3 9 W.	19.5
56 48	4 36 W.	41.5
56 26	6 8 W.	73.9

The sun's meridian declination was 3° 30′ 38″ S., and it was decreasing at the rate of 0″.98 in a minute. What was the true latitude?

Entering Table X. with the hour angles given above, we obtain the values set down in the last column, the sum of which, being divided by 11, will give 58″.96; whence, by formula (3), page 142, we obtain the reduction to the meridian, 44″.7.

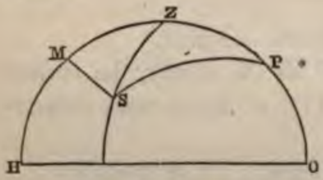
The sum of the eastern hour angles, diminished by the sum of the western, and divided by 11, gives 1m. 50.3s., which, multiplied by 0″.98, gives 1″.8 for the correction for change of declination.

Hence we have the following results:

Mean of the observed altitudes = 34° 56′ 30″.5
Reduction to the meridian = +44″.7
Correction for change of declination = +1″.8
Concluded meridian altitude = 34° 57′ 17″.0
Zenith distance = 55° 2′ 43″
Sun's declination = − 3° 30′ 38″
Latitude = 51° 32′ 5″ N.

FOURTH METHOD.

(178.) *By a single altitude, the time of observation being known.*



Let Z be the zenith of the observer, P the pole, S a star whose altitude is measured at a known instant of time. Then, in the spherical triangle ZPS, we have given $PS=90°-\delta$, $ZS=Z$, and the hour angle ZPS, to find PZ.

From S let fall the perpendicular SM upon PZ produced. Then, by Napier's rule, we shall have

$$\text{R. cos. P} = \text{tang. PM cot. PS} = \text{tang. PM tang. } \delta.$$

Hence $$\text{tang. PM} = \text{cos. P cot. } \delta \quad \ldots\ldots\ldots \quad (1)$$

$$MZ = PM - PZ = PM + \phi - 90°.$$

Also, Trig., Art. 216,

$$\text{cos. PM} : \text{cos. ZM} :: \text{cos. PS} : \text{cos. ZS},$$

or $$\text{cos. PM} : \text{sin. } (PM+\phi) :: \text{sin. } \delta : \text{cos. Z}.$$

Hence $$\text{sin. } (PM+\phi) = \frac{\text{cos. Z cos. PM}}{\text{sin. } \delta} \quad \ldots\ldots \quad (2)$$

Equation (1) furnishes the value of PM, and equation (2) furnishes the value of $PM+\phi$. The difference between these quantities is ϕ, the latitude required.

Ex. 1. At 1h. 14m. 11.6s. apparent time, the true altitude of the sun was 33° 40′ 35″.5, and his declination 5° 15′ 28″.0 S. Required the latitude of the place.

By equation (1),

$$\begin{array}{rl}
\text{cos. } 18^\circ\ 32'\ 54'' & = 9.976834 \\
\text{cot. } 5^\circ\ 15'\ 28'' & = \underline{1.036099n} \\
\text{PM} = 95^\circ\ 32'\ 39''\ \text{tang.} & = 1.012933n
\end{array}$$

By equation (2),

$$\begin{array}{rl}
\text{cos. } 56^\circ\ 19'\ 24''.5 & = 9.743904 \\
\text{cos. PM} & = 8.985035n \\
\text{cosec. } \delta & = \underline{1.037930n} \\
\text{PM} + \phi = 144^\circ\ 13'\ 28''\ \text{sin.} & = 9.766869 \\
\text{PM} = & \underline{\ 95^\circ\ 32'\ 39''} \\
\phi = & \ 48^\circ\ 40'\ 49''
\end{array}$$

Ex. 2. At a place in Lat. 42° 34′ N. nearly, the altitude of Aldebaran (Dec. 16° 12′ 26″ N.) was found by observation to be 39° 2′ 10″, when its hour angle was 3h. 25m. 40s. What was the latitude of the place? *Ans.* 42° 34′ 56″.

Ex. 3. At a place in Lat. 41° 25′ nearly, the altitude of Regulus (Dec. 12° 41′ 18″ N.) was found by observation to be 41° 5′ 20″, when its hour angle was 3h. 2m. 21s. What was the latitude of the place? *Ans.* 41° 25′ 47″.

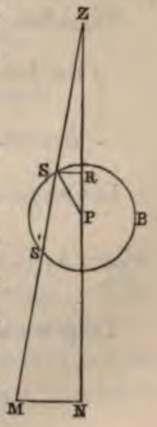
Ex. 4. On the 27th of February, 1850, the zenith distance of Procyon (Dec. 5° 36′ 6″.7 N.) was observed at Greenwich to be 48° 48′ 34″.06, when its hour angle from the meridian was 1h. 20m. 18.13s. It is required to deduce the latitude from this observation. *Ans.*

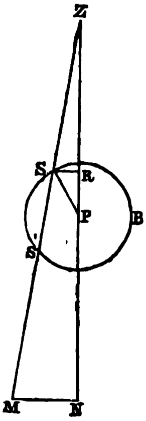
This method is deficient in accuracy when the observations are made far from the meridian, because a small error in the hour angle produces a large error in the computed value of the latitude. The observations should, therefore, always be made as near as possible to the meridian.

FIFTH METHOD.

(179.) *By observations of the pole star at any time of the day.*

Let P be the pole, Z the zenith, ZPN the meridian of the place of observation, and S the pole star in any point of its diurnal circle, SBS′. Then we shall have ZP = 90° − φ, ZS = 90° − H, H being the observed height of the star corrected for refraction. Represent the polar distance, PS, by *d*. Since the arc *d* is at present less than 90′, the sides ZP,





ZS, differ only by a small arc, x, which we propose to calculate.

Let $ZP-ZS=x$; that is, $H-\phi=x$,

or $\phi=H-x$.

Represent the hour angle ZPS by P. The spherical triangle ZPS furnishes by Trig., Art. 225,

$$\cos. ZS=\cos. PS \cos. PZ+\sin. PS \sin. PZ \cos. P,$$

or

$$\sin. H=\cos. d \sin. (H-x)+\sin. d \cos. (H-x) \cos. P.$$

By substituting the values of sin. $(H-x)$, and cos. $(H-x)$, Trig., Art. 72, and dividing the whole equation by sin. H, we obtain

$$1=\cos. d\ (\cos. x-\sin. x \cot. H)$$
$$+\sin. d\ (\cos. x \cot. H+\sin. x) \cos. P,$$

or

$$1=\cos. x\ (\cos. d+\sin. d \cot. H \cos. P)$$
$$-\sin. x\ (\cos. d \cot. H-\sin. d \cos. P).$$

Let us put

$$a=\cos. d+\sin. d \cot. H \cos. P,$$
$$b=\cos. d \cot. H-\sin. d \cos. P,$$

and we have

$$1=a \cos. x-b \sin. x \ldots\ldots\ldots\ldots (1)$$

But by Calculus, Art. 228,

$$\sin. y=y-\frac{y^3}{6}+\frac{y^5}{120}-, \text{ etc.}$$

$$\cos. y=1-\frac{y^2}{2}+\frac{y^4}{24}-, \text{ etc.}$$

Therefore

$$a=1+d \cos. P \cot. H-\frac{d^2}{2}-\frac{d^3}{6} \cos. P \cot. H+, \text{ etc.}$$

$$b=\cot. H-d \cos. P-\frac{d^2}{2} \cot. H+\frac{d^3}{6} \cos. P+, \text{ etc.}$$

Let us now assume

$$x=Ad+Bd^2+Cd^3+, \text{ etc.} \ldots\ldots\ldots (2)$$

where A, B, and C represent unknown coefficients independent of d.

Then we shall have

$$\cos. x=1-\frac{A^2d^2}{2}-ABd^3+, \text{ etc.}$$

$$\sin.\ x = \mathrm{A}d + \mathrm{B}d^2 + \left(\mathrm{C} - \frac{\mathrm{A}^3}{6}\right)d^3 +, \text{ etc.}$$

Substituting in equation (1) the values of a, b, sin. x, and cos. x, arranging the terms in the order of the powers of d, and retaining all the terms which contain the first three powers of d, we obtain

$$\begin{aligned} 1 = 1 + \cos.\ \mathrm{P} \cot.\ \mathrm{H}.d - \frac{d^2}{2} - \frac{d^3}{6} \cos.\ \mathrm{P} \cot.\ \mathrm{H} \\ - \frac{\mathrm{A}^2 d^2}{2} - \frac{\mathrm{A}^2 d^3}{2} \cos.\ \mathrm{P} \cot.\ \mathrm{H} - \mathrm{AB}d^3 \\ - \mathrm{A} \cot.\ \mathrm{H}.d + \mathrm{A} \cos.\ \mathrm{P}.d^2 + \frac{\mathrm{A}d^3}{2} \cot.\ \mathrm{H} + \mathrm{B}d^3 \cos.\ \mathrm{P} \\ - \mathrm{B} \cot.\ \mathrm{H}.d^2 - \left(\mathrm{C} - \frac{\mathrm{A}^3}{6}\right) \cot.\ \mathrm{H}.d^3. \end{aligned}$$

Since this equation must be verified by any value of d, the terms involving the same powers of d must cancel each other. Algebra, Art. 300.

Hence,

First. $\cos.\ \mathrm{P} \cot.\ \mathrm{H} - \mathrm{A} \cot.\ \mathrm{H} = 0$; whence $\mathrm{A} = \cos.\ \mathrm{P}$.

Second. $-\frac{1}{2} - \frac{\mathrm{A}^2}{2} + \mathrm{A} \cos.\ \mathrm{P} - \mathrm{B} \cot.\ \mathrm{H} = 0.$

Therefore,

$$\mathrm{B} \cot.\ \mathrm{H} = \cos.^2 \mathrm{P} - \frac{\cos.^2 \mathrm{P}}{2} - \frac{1}{2} = \frac{\cos.^2 \mathrm{P} - 1}{2} = -\frac{\sin.^2 \mathrm{P}}{2}.$$

Hence $\mathrm{B} = -\frac{\sin.^2 \mathrm{P}}{2} \tang.\ \mathrm{H}.$

Third. $-\frac{\cos.\ \mathrm{P}}{6} - \frac{\mathrm{A}^2 \cos.\ \mathrm{P}}{2} + \frac{\mathrm{A}}{2} - \left(\mathrm{C} - \frac{\mathrm{A}^3}{6}\right) = 0.$

Whence, substituting the value of A already found,

$$\begin{aligned} 3\mathrm{C} &= \cos.\ \mathrm{P} - \cos.^3 \mathrm{P} \\ &= \cos.\ \mathrm{P}(1 - \cos.^2 \mathrm{P}) \\ &= \cos.\ \mathrm{P} \sin.^2 \mathrm{P}. \end{aligned}$$

Therefore,

$$\mathrm{C} = \tfrac{1}{3} \cos.\ \mathrm{P} \sin.^2 \mathrm{P}.$$

Substituting these values in equation (2), we obtain

$$x = d \cos.\ \mathrm{P} - \tfrac{1}{2} \sin.^2 \mathrm{P} \tang.\ \mathrm{H}.d^2 + \tfrac{1}{3} \cos.\ \mathrm{P} \sin.^2 \mathrm{P}d^3.$$

In order that x and d may be expressed in seconds of arc, we must change these letters into x sin. 1″ and d sin. 1″; whence we have

$$\phi = \mathrm{H} - d \cos. \mathrm{P} + \tfrac{1}{2} \sin. 1'' (d \sin. \mathrm{P})^2 \tan g. \mathrm{H}$$
$$- \tfrac{1}{3} \sin.^2 1'' (d \cos. \mathrm{P}) (d \sin. \mathrm{P})^2 \ldots\ldots \quad (3)$$

The last term of this equation never amounts to half a second, and may therefore generally be omitted.

Ex. 1. The altitude of the pole star being found 46° 17′ 28″, the hour angle 5h. 42m. 4.4s. from the upper culmination, and the polar distance 1° 28′ 7″.68; required the latitude of the place.

Computation by formula (3),

$d = 5287''.68 = 3.72327$	$d = 3.7233$	$d \cos. \mathrm{P} = 2.616$
$\cos. \mathrm{P}, 85° 31' 6'' = 8.89287$	$\sin. \mathrm{P} = 9.9987$	$(d \sin. \mathrm{P})^2 = 7.444$
$413''.2 = 2.61614$	3.7220	$\tfrac{1}{3} \sin.^2 1'' = 8.894$
	3.7220	$0''.1 = 8.954$
	tang. H, 0.0196	
	$\tfrac{1}{2} \sin. 1'' = 4.3845$	
	$70''.5 = 1.8481$	

Result.

Observed altitude,	H	= 46° 17′ 28″.0
first correction,	=	− 6′ 53″.2
second correction,	=	+ 1′ 10″.5
third correction,	=	− 0″.1
Latitude	=	46° 11′ 45″.2

The computation may also be performed by the formulas of Art. 178.

cos. P = 8.892874
cot. δ = 8.408935
PM = 6′ 53″.3 tang. = 7.301809

sin. H = 9.8590542
cos. PM = 9.9999991
cosec. δ = 0.0001427
PM + ϕ = 46° 18′ 38″.5 = 9.8591960
PM = 6′ 53″.3
Latitude = 46° 11′ 45″.2

The method of Art. 178 is about as convenient as the one here explained, except that, when great accuracy is demanded, the former method requires logarithms to seven places.

Ex. 2. The altitude of the pole star being 43° 2′ 38″ when

the hour angle was 76° 0′ 2″ from the upper culmination, and its Dec. 88° 31′ 52″.32; required the latitude of the observer.
Ans. 42° 42′ 18″.2.

Ex. 3. The altitude of the pole star being found 39° 1′ 39″, the hour angle 5h. 36m. 41s. from the upper culmination, and the polar distance 1° 28′ 7″.68; required the latitude of the place. *Ans.* 38° 53′ 36″.2.

SIXTH METHOD.

(180.) *By observing the difference of the meridional zenith distances of two stars on opposite sides of the zenith.*

If we select two stars whose places are well known, one of which culminates to the north, and the other to the south of the observer, at nearly the same distances from the zenith, and within a short interval of time, and measure accurately the *difference* of their zenith distances, the latitude of the place of observation may thence be easily deduced. If we represent the zenith distance of the northern star by Z_n, and that of the southern star by Z_s; also the declination of the northern star by δ_n, and that of the southern star by δ_s, then, by Art. 173, we shall have

$$\phi=\delta_s+Z_s;$$

$$\phi=\delta_n-Z_n.$$

Hence $$2\phi=\delta_s+\delta_n+Z_s-Z_n;$$

that is, the sum of the declinations of the two stars (which are given by the catalogue), added to the difference of their zenith distances, gives twice the latitude of the place.

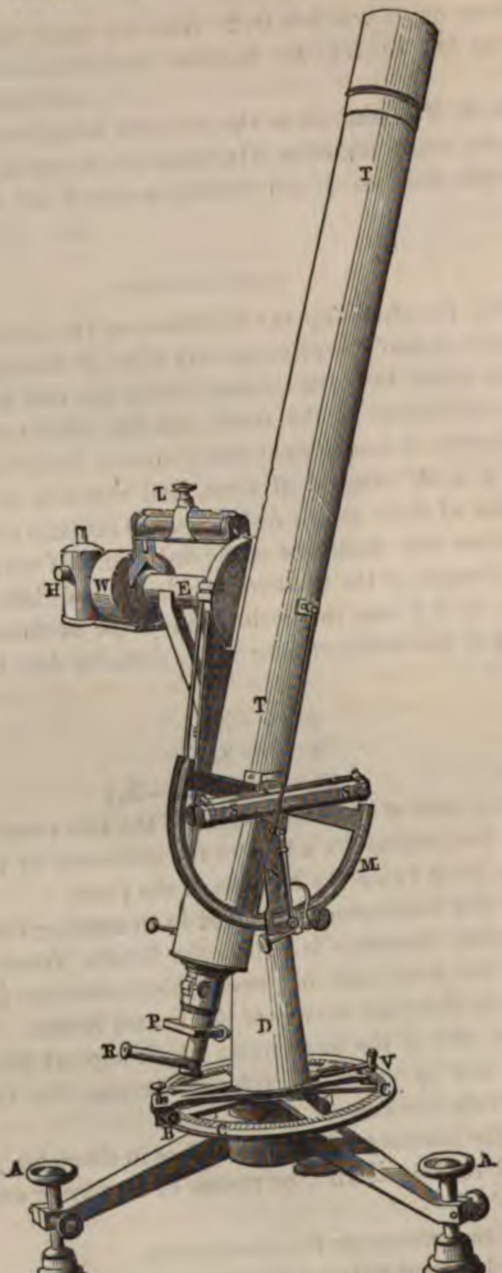
(181.) The instrument employed in measuring the difference of the zenith distances is called the Zenith Telescope. The figure on the next page represents this instrument in the form now used in the coast survey of the United States.

A, A are two of the feet screws which support the entire instrument, and by which the column carrying the telescope is rendered truly vertical.

C, C is the horizontal circle, 12 inches in diameter, graduated to 10′, and reading to 10″, by means of its vernier and microscope V.

B is the tangent screw for slow motion.

This circle serves to mark the position of the meridian, when


T
L
H
W
E
T
M
P
D
R
V
C
K
O
B
A
A

it has once been determined, and likewise enables the observer to turn the telescope promptly through 180° in azimuth.

D is the vertical column which supports the telescope, and about which the telescope turns freely in azimuth.

E is a horizontal axis, to one end of which is attached the telescope, TT, which is counterpoised by the weight W, at the other end.

This axis is hollow, and through it passes the light of the lamp, H, to illumine the wires of the telescope. The telescope has a focal length of about 40 inches, and an aperture of 3 inches.

L is a level resting upon the horizontal axis, by means of which the column D is rendered truly vertical.

M is a graduated semicircle attached to the telescope, and having a vernier, N, with a microscope. This semicircle serves as a finder for setting the telescope to the altitude of the stars to be observed.

S, S is a very delicate level attached to the semicircle.

P is the parallel wire micrometer for measuring small differences of altitude, having three fixed vertical, and two movable horizontal wires.

R is the diagonal eye-piece, which is made of unusual length, so that the micrometer may not interfere with the observations. The eye-pieces employed have a field of view of from 10′ to 15′.

(182.) *Method of Observation.*—Select a pair of stars, the difference of whose zenith distances does not exceed a convenient range of the micrometer, say ten minutes, one of which culminates to the north, and the other south of the zenith. Having leveled the instrument, set the telescope to an altitude midway between the two stars, and bring the bubble of the level S to the middle of its scale. Bring the telescope into the plane of the meridian by setting the vernier of the horizontal circle to the point previously determined. As the first star enters the field of view, follow its image with one of the horizontal wires, and bisect it at the instant it crosses the middle vertical wire. Record the position of the level S, noting the divisions corresponding to each extremity of the bubble. Turn the telescope 180° in azimuth, being careful to preserve the same inclination to the horizon, and make a similar observation upon the second star, bisecting it with the other horizontal wire.

A comparison of the readings of the two micrometer screws will give the difference of zenith distance of the two stars, which must be corrected by the readings of the level, if the readings at each extremity are not the same in both cases; and also for the difference of the refractions of the two stars.

The stars should be so selected that their zenith distances may be as small as practicable, and should in no case exceed 25 degrees.

The following observations, made at Mount Independence, Maine, one of the coast survey stations, September 25, 1849, will illustrate this method. The pair of stars employed consisted of Nos. 6983 and 6996 of the British Association catalogue, whose apparent places were

No.	Right Ascension.	Declination.
	h. m. s.	° ′ ″
6983	20 10 50.05	47 15 40.70
6996	20 12 48.19	40 16 19.21

The formula for latitude is

$$2\phi=\delta_s+\delta_n+Z_s-Z_n.$$

Here $\delta_s+\delta_n=87° \; 31' \; 59''.91$; Z_s-Z_n was found, by observation, equal to $-50''.29$. Therefore $2\phi=87° \; 31' \; 9''.62$. The observations indicated no correction for level; and the correction for difference of refraction was $-0''.02$. Hence the final latitude is $43° \; 45' \; 34''.80$.

September 27th, the same stars were again observed, when $\delta_s+\delta_n$ equaled $87° \; 32' \; 0''.40$; Z_s-Z_n was found equal to $-49''.43$. The correction for level was $+0''.90$, and for refraction $-0''.02$, from which we deduce the latitude, $43° \; 45' \; 35''.92$.

(183.) This method of determining latitude possesses the following advantages: 1. It eliminates almost entirely the effect of atmospheric refraction, since we only require the difference of refraction of the two stars. With a zenith distance of 25 degrees, and a difference of altitude of 24′ between the two stars, this difference of refraction does not exceed half a second of arc. The observations are generally made much nearer to the zenith than 25°, and the difference of altitude is commonly but a few minutes.

2. The angular measurements required are made by means of a micrometer, so that there is no occasion for a large gradu-

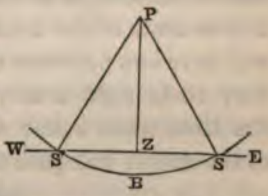
ated circle, the semicircle attached to the telescope being used merely as a finder.

The chief objection to this method is, that the resulting latitude is affected by any error which may exist in the assumed declinations of the stars employed, and we are generally obliged to make our selections from stars whose places have not been determined with the greatest accuracy. When accurate determinations of the stars employed can be obtained with the large instruments of a fixed observatory, this objection is mostly obviated.

SEVENTH METHOD.

(184.) *By observations with a transit instrument in the prime vertical.*

This method supposes the transit instrument to be placed with its supports north and south, so that the telescope, when directed toward the horizon, points due east and west. We must then observe the passage of some known star over the same wires when the telescope is pointing west. From these observations we may determine the latitude of the place, or the declination of the star, when either of these quantities is known.



Let P represent the pole of the earth, Z the zenith of the observer, EZW the prime vertical, which is also the line described in the heavens by the transit; and let the arc SBS′ be the path of a star which culminates a little south of the zenith. Let the times at which a star crosses the field of the transit at S and S′ be noted; then will the angle SPS′, which is the difference of those times, be known. Then, in the right-angled spherical triangle PZS, by Napier's rule,

$$\text{R.}\cos.\ \text{ZPS} = \text{tang.}\ \text{PZ}\ \cot.\ \text{PS}.$$

Put ZPS = P = half the sidereal interval between the times of east and west transit;

$\delta = 90° - \text{PS} =$ the declination of the star;

$\phi = 90° - \text{PZ} =$ the latitude of the place.

Then

$$\cos.\ \text{P} = \cot.\ \phi\ \text{tang.}\ \delta;$$

or
$$\text{tang. }\phi=\frac{\text{tang. }\delta}{\text{cos. P}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

which is the same as given in Art. 148.

Ex. 1. On the 16th of December, 1844, the transit of α Lyræ over the prime vertical of Cambridge was observed at 16h. 34m. 47.3s.; and again at 20h. 25m. 14.0s.; the declination of the star being 38° 38′ 42″.05. Required the latitude of the observatory.

Here P = 1h. 55m. 13.35s. in time, or 28° 48′ 20″.25 in arc.

$$\begin{aligned}\text{tang. }\delta,\ 38^\circ\ 38'\ 42''.05 &= 9.9028601\\ \text{cos. P},\ 28^\circ\ 48'\ 20''.25 &= 9.9426327\\ \text{Latitude} = 42^\circ\ 22'\ 48''.3\ \text{tang.} &= \overline{9.9602274}\end{aligned}$$

Ex. 2. On the 4th of January, 1846, the transit of α Lyræ over the prime vertical of Washington was observed at 18h. 27m. 0.35s.; and again at 19h. 28m. 1.0s.; the declination of the star being 38° 38′ 42″.37. Required the latitude of the observatory.

Here P = 0h. 30m. 30.325s. in time, or 7° 37′ 34″.87 in arc.

$$\begin{aligned}\text{tang. }\delta,\ 38^\circ\ 38'\ 42''.37 &= 9.9028615\\ \text{cos. P},\ \ 7^\circ\ 37'\ 34''.87 &= 9.9961414\\ \text{Latitude} = 38^\circ\ 53'\ 37''.1\ \text{tang.} &= \overline{9.9067201}\end{aligned}$$

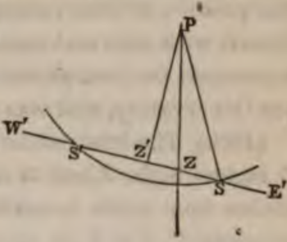
(185.) When these observations are made for the determination of latitude, it is best to select a star which culminates but a little south of the zenith, as the same error in the observations will have less influence upon the result. The transit instrument may be brought nearly into the prime vertical, by computing the time when a star which culminates several degrees south of the zenith will pass the prime vertical. The formula

$$\text{cos. P} = \text{cot. }\phi\ \text{tang. }\delta$$

gives the hour angle between the meridian and the time of transit over the prime vertical. The right ascension of the star, minus the hour angle, gives the time of the east transit; and the right ascension, plus the hour angle, gives the time of west transit.

(186.) When the instrument is brought nearly into the prime vertical, the error in azimuth may be determined as follows: Half the sum of the times of transit over the east and west verticals, gives the time of transit over the meridian of the instrument. This result should be equal to the right ascension of the star, corrected for the error of the clock. If the two results are not equal, their difference shows the angle which the meridian of the instrument makes with the true meridian.

If the plane of the telescope deviates much from the prime vertical, the co-latitude deduced will be sensibly too small. Suppose the axis deviates to the east of north, and that the telescope describes a vertical circle, passing through E′ZW′; then will PZ′, which bisects SS′, be the co-latitude which results from the above formula.



The correction for this deviation may be computed as follows: Take the half sum of the times of transit over the east and west verticals, correct it for the error of the clock, and subtract the result from the star's right ascension. The difference will be the angle ZPZ′. Now, from the right-angled triangle PZZ′, we have

$$\text{tang. PZ cos. ZPZ}' = \text{tang. PZ}' = \text{tang. PS} \times \text{cos. SPZ}',$$

or

$$\text{tang. } \phi = \frac{\text{tang. } \delta \times \text{cos. ZPZ}'}{\text{cos. SPZ}'}.$$

The angle ZPZ′ is the same for all stars, and it is better to deduce its value from a star which culminates several degrees south of the zenith, since the same error in the observations will have less influence upon the azimuth deduced.

(187.) If we reverse the telescope upon its supports, any error of collimation or inequality of pivots will produce exactly a contrary effect on the latitude. Observations, therefore, of two stars on the same day, in reversed positions of the telescope, or of the same star on following days, in reversed positions of the telescope, will correct each other, and the mean will give the true latitude, if the declination of the star is accurately known. This is one of the best methods of determining the latitude with a portable instrument.

In the equation

$$\text{cos. P} = \text{cot. } \phi \text{ tang. } \delta,$$

either ϕ or δ may be computed when the other quantity is known. Hence, in a fixed observatory, when the latitude is well determined, the declinations of stars may be determined with great precision by a transit instrument, adjusted to the prime vertical.

But to accomplish this object in the best manner requires an instrument of a peculiar construction. The instrument should admit of having the level applied to it while the telescope is in the position of observation, and it should also admit of being reversed with ease and rapidity. The figure on the opposite page represents the instrument used for this purpose at the Washington Observatory, and was made by Pistor and Martins, of Berlin.

(188.) The instrument rests on a block of granite, MM, 6 feet 5 inches high, 3 feet 3 inches from east to west, and 3 feet 7 inches from north to south. This block is cut so as to form two columns 4½ feet high, separated by a cavity which contains the reversing apparatus.

S is the axis of the instrument, terminating in two pivots, B, B, 3.6 inches in diameter; to one of which is attached the telescope, T, to the other the cylinder, U, which counterpoises the telescope. The telescope is 6½ feet focal length, and 4.8 inches clear aperture.

V, V are the Y's which support the axis, and C, C are friction rollers, with grooves for relieving the Y's. They are regulated by the counterpoises W, W, all of which are carried by the reversing apparatus.

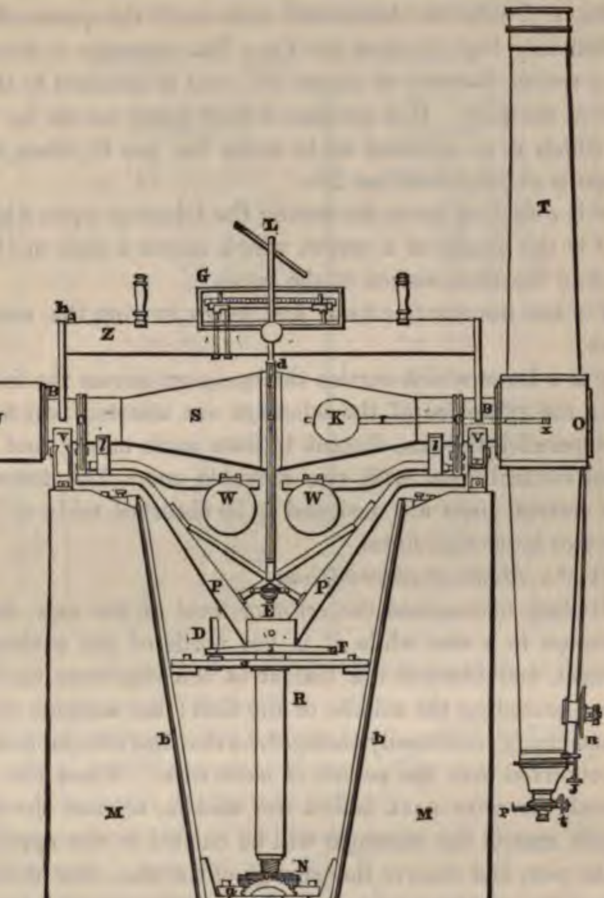
The axis, S, is hollow, and contains a lever, *r*, one end of which expands into a fork, and is firmly secured at *x* to each side of the telescope tube. To the other end of the lever is attached the counterpoise, K, which transfers the weight of the telescope to that part of the pivot which rests immediately upon the Y's. A similar counterpoise is placed on the other side, to produce the same effect with reference to the cylinder U.

Z is the striding level, which rests permanently upon the pivots B, B during the observations; and L is a mirror for illuminating the level divisions by means of a lamp. The level tube is protected by a glass case, G, and there is a cross level at *h*.

About the middle of the axis, at *d*, is a clamp for slow motion of the telescope, and a screw, with a Hook's joint, at E.

The reversing apparatus, P, P, turns on an inverted cone, working in the hollow cylinder, R, and is strengthened by the cross iron bars, *a*, *a*, *a*, which are supported by the flat iron bars, *b*, *b*.

H is a crank which turns a cog-wheel at N, which, by means



L

of a screw, lifts the hollow cylinder, R, and, by means of the forks, *l*, *l*, lifts the horizontal axis until the pivots, B, B, are sufficiently high to clear the Y's. The telescope is then turned to a zenith distance of about 45°, and is revolved to the other side of the pier. It is prevented from going too far by the arm F, which is so adjusted as to strike the pin D, when the telescope is exactly over the Y's.

n is a finding circle for setting the telescope upon a star.

J is the handle of a screw, which moves a slide at O for regulating the illumination of the wires.

t is the micrometer head and screw moving the micrometer wire.

p is a lever which carries the eye-piece across the field.

In the eye-piece of the telescope are inserted two horizontal and parallel threads, distant 1′ from each other; and also 15 fixed vertical lines, with one movable one. The transits over the vertical lines are designed to be observed midway between the two horizontal lines.

(189.) *Mode of observation.*

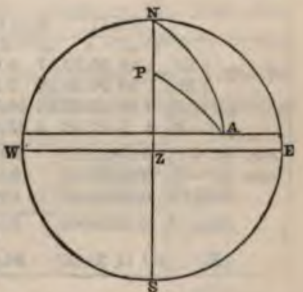
Having determined the error of level of the axis, direct the telescope to a star while it is yet north of the eastern prime vertical, and observe the transit of the star over each of the wires preceding the middle of the field; the altitude of the telescope being continually changed, so that the oblique transit may be observed over the centre of each wire. When the star has passed the wire next before the middle, reverse the axis, by which means the telescope will be carried to the opposite side of the pier, and observe the passage of the star, now on the south side of the eastern prime vertical, over the same wires as before, but in the opposite order. Determine again the error of level of the axis. When the star is approaching the western prime vertical from the south, the instrument being still in its second position, ascertain again the error of level of the axis. Again observe the transit of the star over the first seven wires preceding the middle of the field; reverse the instrument to its first position, and observe the transit of the star, now on the north side of the western prime vertical, over the same wires. Finally, ascertain the error of level of the axis in the last position.

The following observations were made by Struve, with the prime vertical transit of the Pulkova Observatory. The numbers in the last column are read from below, upward.

January 15, 1842. *o* Draconis.

Wires.	EAST VERTICAL.			WEST VERTICAL.		
	Telescope S.			Telescope S.		
	h.	*m.*	*s.*	*h.*	*m.*	*s.*
I.	17	54	30.7	19	42	51.4
II.		55	8.65		42	13.65
III.		55	44.4		41	38.0
IV.		56	22.25		40	59.85
V.		57	0.6		40	21.7
VI.		57	40.9		39	41.4
VII.	17	58	19.5	19	39	2.7
	Telescope N.			Telescope N.		
VII.	18	1	4.0	19	36	17.85
VI.		1	45.5		35	37.0
V.		2	29.8		34	52.35
IV.		3	12.7		34	9.3
III.		3	57.6		33	24.7
II.		4	39.8		32	42.1
I.	18	5	26.35	19	31	55.6
	Level $= +0''.687$			Level $= +0''.923$		

(190.) The reduction of the observations is made as follows, each wire being treated separately.



Let NESW represent the horizon, NS the meridian, EW the prime vertical, P the pole, and A the place of the star at its transit over one of the wires of the telescope. Join PA and NA by arcs of great circles. The projection of each wire on the sky is a small circle, whose pole is the north point, N, of the horizon. If c represent the angular distance of one of the wires from the line of collimation, $90° - c$ will be the radius NA of the small circle, when the star is seen on it, north of the prime vertical, and $90° + c$ when the star is south of the prime vertical.

In the triangle PNA, by Trig., Art. 225, we have

$$\cos. \text{NA} = \cos. \text{NP} \cos. \text{PA} + \sin. \text{NP} \sin. \text{PA} \cos. \text{NPA}.$$

Let $\phi = \text{NP}$ the latitude of the place;

$\delta = 90^\circ - \text{PA} =$ the star's declination;

$t, t' =$ the hour angles SPA from the meridian, at the two observations over the same wire, in the direct and reversed positions of the axis.

Then, when the star is north of the prime vertical,

$$\cos.\,(90^\circ - c) = \sin.\,c = \cos.\,\phi\,\sin.\,\delta - \sin.\,\phi\,\cos.\,\delta\,\cos.\,t\,;$$

and, when the star is south of the prime vertical,

$$\cos.\,(90^\circ + c) = -\sin.\,c = \cos.\,\phi\,\sin.\,\delta - \sin.\,\phi\,\cos.\,\delta\,\cos.\,t'.$$

Adding these two equations, we obtain

$$0 = 2\,\sin.\,\delta\,\cos.\,\phi - \cos.\,\delta\,\sin.\,\phi\,(\cos.\,t + \cos.\,t'),$$

or, Trig., Art. 75,

$$\text{tang.}\,\delta\,\cot.\,\phi = \frac{\cos.\,t + \cos.\,t'}{2} = \cos.\,\frac{t'+t}{2}\,\cos.\,\frac{t'-t}{2}. \quad (2.)$$

This formula will furnish the declination when the latitude is known, or the latitude when the declination is known. The latitude of the Pulkova instrument is 59° 46′ 18″. t' represents half the interval between the first transit east and the second transit west; and t is half the interval between the second transit east and the first transit west.

(191.) The following is Struve's reduction of the preceding observations, a correction of +0.09s. being applied to the interval W.−E. for rate of clock.

	Wire I.	Wire II.	Wire III.	Wire IV.	Wire V.	Wire VI.	Wire VII.
	h m s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.
W.−E. $2t'$	1 48 20.79	47 5.09	45 53.69	44 37.69	43 21.19	42 0.59	40 43.29
W.−E. $2t$	1 26 29.34	28 2.39	29 27.19	30 56.69	32 22.64	33 51.59	35 13.94
$\frac{1}{2}(t'+t)$	0 48 42.53	48 46.87	48 50.22	48 53.60	48 55.96	48 58.05	48 59.31
$\frac{1}{2}(t'-t)$	0 5 27.86	4 45.67	4 6.62	3 25.25	2 44.64	2 2.25	1 22.33
cos. $\frac{1}{2}(t'+t)$	9.9901167	0871	0642	0411	0250	0107	0020
cos. $\frac{1}{2}(t'-t)$	9.9998765	9063	9301	9516	9688	9828	9922
tang. ϕ	0.2345728	5728	5728	5728	5728	5728	5728
tang. δ	0.2245660	5662	5671	5655	5666	5663	5670
δ	59° 11′ 39″.00	39″.04	39″.23	38″.90	39″.12	39″.06	39″.21

The mean error of level of the instrument may be applied to ϕ, or we may apply a correction to the declination obtained with a constant value of ϕ. If the inclination of the axis be denoted by I, which is the mean of the two inclinations, telescope N and telescope S, then $\phi + \text{I}$ should be used in place of ϕ, in formula (2), Art. 190. Now, by formula (1), Art. 184, we have

$$\text{tang.}\,\delta = \text{tang.}\,\phi\,\cos.\,\text{P}.$$

By differentiating, supposing P constant, we obtain

$$d\delta \text{ sec. }^2\delta = d\phi \text{ sec. }^2\phi \text{ cos. P.}$$

Hence

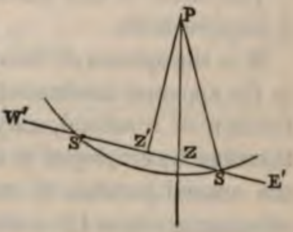
$$d\delta = d\phi \frac{\text{sec. }^2\phi \text{ tang. }\delta}{\text{sec. }^2\delta \text{ tang. }\phi} = d\phi \frac{\text{cos. }\delta \text{ sin. }\delta}{\text{cos. }\phi \text{ sin. }\phi} = d\phi \frac{\text{sin. }2\delta}{\text{sin. }2\phi},$$

or

$$d\delta = \frac{\text{sin. }2\delta}{\text{sin. }2\phi}\text{I.}$$

In the preceding observations the mean inclination of the axis was +0″.805.

The mean value of δ	=59° 11′ 39″.071
Correction for inclination of axis	+0″.814
Observed declination	=59° 11′ 39″.885



(192.) The declination thus found is not correct, unless the telescope is truly adjusted to the prime vertical. Suppose there is an error in the azimuth of the instrument equal to a or 90° − PZZ′; then, in the triangle PZZ′,

$$\text{tang. P} = \frac{\text{cot. PZZ}'}{\text{cos. PZ}} = \frac{\text{tang. }a}{\text{sin. }\phi}.$$

If the error in azimuth be small, we may assume

$$\text{P} = \frac{a}{\text{sin. }\phi},$$

which represents the angle at the pole, between the true meridian and the meridian of the instrument. The instant of the star's passage over the meridian of the instrument is equal to the half sum of the east and west transits. Thus, in the preceding observation, we have

Wires.	Telescope S.	Telescope N.
I.	18h. 48m. 41.10s.	18h. 48m. 40.93s.
II.	41.15s.	41.25s.
III.	41.20s.	41.07s.
IV.	41.05s.	41.00s.
V.	41.15s.	41.15s.
VI.	41.15s.	40.95s.
VII.	41.10s.	40.97s.
Mean,	18h. 48m. 41.13s.	41.05s.

Mean, 18h. 48m. 41.09s.

The instant of meridian passage requires a small correction for the difference of inclinations of the axis in the two verticals.

This correction in the present case amounts to -0.08s.; and hence the true time of meridian passage by the instrument is

18h. 48m. 41.01s.

The star's right ascension, corrected for error of the clock, was

18h. 48m. 41.86s.

Hence $P=-0.85$s. in time, is the angle of the two meridians.

For the azimuth of the axis of rotation, reckoned from the south round by the west,

$$a=15P \sin. \phi, \text{ in arc.}$$

In the present case,

$$a=-11''.0, \text{ in arc.}$$

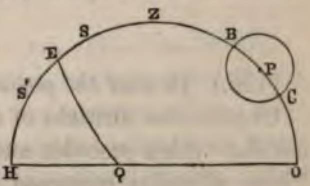
The effect of this small azimuthal error upon the declination is inappreciable.

It is the opinion of Struve that, with this instrument, changes in the apparent declination of zenith stars, amounting to a small fraction of a second, may be detected. This instrument may therefore be employed to determine the aberration of light, and the annual parallax of zenith stars. The bright star α Lyræ culminates about 15′ south of the zenith of Washington Observatory, and this star has been observed by Professor Hubbard with great care, for the purpose of determining its annual parallax, which, according to the Pulkova observations, amounts to about 0″.2

CHAPTER VII.

ECLIPTIC.

(193.) When an observer has obtained the latitude of his station, he is prepared with an astronomical circle to determine the apparent declinations of the heavenly bodies. For the elevation of the equator, EH, is the complement of PO, the elevation of the pole; and if from SH, the altitude of a star, we subtract EH, the elevation of the equator, we shall obtain the star's declination. This rule will hold for all the heavenly bodies at their upper culmination, if we measure their altitude from the south horizon. Or, if we represent the latitude by ϕ, and the zenith distance by Z, when a body culminates south of the zenith, we have



$$\delta=\phi-Z.$$

If it culminate north of the zenith, and *above* the pole,

$$\delta=\phi+Z.$$

If it culminate north of the zenith, and *below* the pole,

$$\delta=180^\circ-(\phi+Z).$$

(194.) If the declination of the sun be observed during a whole year, whenever it passes the meridian, upon comparing the results it will be found that, on the 22d of December, the declination has its greatest value on the southern side of the equator; that it diminishes till the 21st of March, when the declination is exactly or nearly zero; and that it afterward increases on the northern side of the equator till June 21. From this time the declination diminishes till the 23d of September, when it is again zero, and increases again on the southern side of the equator till the 22d of December.

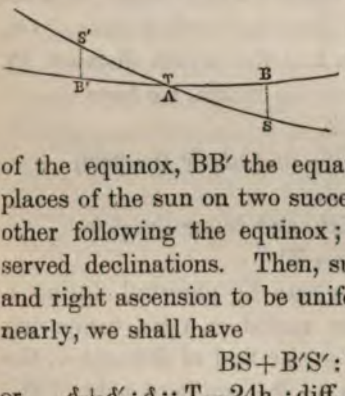
The greatest observed northern and southern declinations of the sun constitute approximate values of the angles at which the plane of the ecliptic and the plane of the equator intersect each other; and the times at which the declinations are nearly zero, are the approximate times when the sun, in ascending and descending, crosses the plane of the earth's equator; but as the observations are only made at the instants of apparent noon at the station, it is not probable that the greatest or least declination will take place precisely at the instant of observation; and therefore a computation must be made to obtain these elements with sufficient accuracy.

Right ascension is reckoned from the vernal equinox; and a clock regulated to exact sidereal time should indicate 0h. 0m. 0s. when the vernal equinox is passing the meridian.

PROBLEM.

(195.) *To find the position of the equinoctial points.*

Observe the altitude of the sun when on the meridian upon the day which precedes and the day which follows the equinox. These altitudes, corrected for refraction and parallax, will furnish the declinations δ and δ', one south and the other north. Let T represent the interval between the observations, expressed in sidereal time. Let A be the place of the equinox, BB′ the equator, SS′ the ecliptic, S and S′ the places of the sun on two successive days, one preceding and the other following the equinox; also, let BS and B′S′ be the observed declinations. Then, suppose the motion in declination and right ascension to be uniform at this time, as they are very nearly, we shall have



$$BS+B'S' : BS :: BB' : BA,$$

or $\delta+\delta' : \delta :: T-24\text{h.} : \text{diff. right asc. between B and A.}$

Ex. 1. On the 20th of March, 1851, the sun's declination at noon was observed at Greenwich to be 16′ 33″.29 S., and March 21st it was 7′ 7″.54 N.; also the sidereal interval between the observations was 24h. 3m. 38.23s. What was the sun's right ascension at noon, March 21st?

In this case we shall have the proportion

$$16' \, 33''.29+7' \, 7''.54 : 7' \, 7''.54 :: 3\text{m. } 38.23\text{s.} : 65.67\text{s.}$$

Therefore the sun's right ascension at noon, March 21st, was

0h. 1m. 5.67s.

Ex. 2. On the 22d of September, 1851, at noon, the sun's declination observed at Greenwich was 3′ 51″.53 N., and on the 23d it was 19′ 33″.76 S.; also the sidereal interval of the transits was 24h. 3m. 36.13s. What was the sun's right ascension at noon, September 23d?

Ans. 12h. 3m. 0.52s.

Ex. 3. On the 22d of September, 1846, at noon, the sun's declination, observed at Washington, was 17′ 2″.80 N., and on the 23d it was 6′ 21″.56 S.; also, the sidereal interval of the transits was 24h. 3m. 35.50s. What was the sun's right ascension at the second observation?

Ans. 12h. 0m. 58.55s.

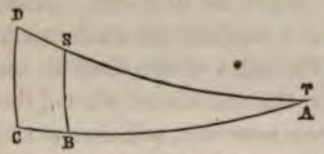
(196.) These computations should be made both for March and September, when the sun crosses the equator. If the sidereal clock were correct, it would be found to indicate 12 hours when the sun is on the meridian at the autumnal equinox; from which we infer that the two equinoctial points are distant from one another 180 degrees. If now we observe some star which passes the meridian about the same time with the vernal equinox (as, for example, α Andromedæ), its right ascension will be known; and having settled the right ascension of one star, the right ascension of other stars may thence be deduced. Thus, taking the apparent right ascension of α Andromedæ on January 31, 1853, to be 0h. 0m. 46.10s., let the index of the clock be set to that time when α Andromedæ is on the meridional wire of the transit telescope. The clock, if it goes correctly, will denote the right ascension of other stars when they are bisected by the meridional wire. Thus, on the above day,

Aldebaran passing the meridional wire at 4h. 27m. 29.19s.
Capella " " " " " 5h. 5m. 50.26s.
Rigel " " " " " 5h. 7m. 28.58s.
Sirius " " " " " 6h. 38m. 40.82s.

these times would be the apparent right ascensions of those stars.

The star selected by any astronomer to regulate the right ascensions of other stars is called his fundamental star. Dr. Maskelyne, at the Greenwich Observatory, employed α Aquilæ for this purpose; while, at the Washington Observatory, α Andromedæ is employed.

(197.) When a number of stars have had their right ascensions determined by referring them to some fundamental star, they will all be charged with the error which may happen to belong to this star; and it is an object of the utmost importance to ascertain the existence and quantity of such error. The difficulty lies in determining accurately the position of the first point of Aries, from which the right ascensions of all the stars are counted. The course pursued, therefore, by astronomers, is first to find the sun's right ascension, by comparing the transit of his centre with the transit of the fundamental star, or with the transits of several principal stars, related to it by known differences; and, secondly, to compute from his observed declination the right ascension belonging to the moment of the meridian passage. These operations should be performed on several days, near both the vernal and autumnal equinox. The right ascensions derived from a comparison with the stars should agree with those derived from the observed declinations of the sun. If there be a constant difference, this will be the correction to be applied to the assumed right ascension of the fundamental star. The sun's right ascension is deduced from his declination in the following manner:



Let AC represent a part of the equator, AD a part of the ecliptic, and A be the first point of Aries. Suppose the sun to be at S, and draw SB perpendicular to AC; then AB will be the right ascension of the sun, and SB his declination.

But, by Napier's rule,

$$\text{rad.} \times \text{sin. AB} = \text{cotang. SAB} \times \text{tang. SB};$$

that is,

$$\text{sin. R. A.} = \text{cotang. obliquity} \times \text{tang. dec.}$$

Or, representing the sun's declination by δ, and the obliquity of the ecliptic by ω, we have

$$\text{sin. R. A.} = \frac{\text{tang. } \delta}{\text{tang. } \omega}.$$

Ex. 1. The following observations of the sun's centre were made at Greenwich in 1851:

Date.	Sun's R. A. observed.	Sun's Dec. observed.
	h. m. s.	° ′ ″
Sept. 15	11 34 16.15	2 47 0.18 N.
16	37 51.22	2 23 50.70 N.
21	55 48.55	0 27 15.35 N.
22	59 24.19	0 3 51.53 N.
23	12 3 0.32	0 19 33.76 S.

It is required to find the mean correction of the right ascensions, the obliquity of the ecliptic being 23° 27′ 28″.15.

The computation for September 15 is as follows:

tang. δ, 2° 47′ 0″.18 = 8.6867922
tang. ω, 23° 27′ 28″.15 = 9.6374270
sin. R. A., 11h. 34m. 16.13s. = 9.0493652

The observed right ascension was 11h. 34m. 16.15s.
Error of the observed R. A. +0.02s.
In the same way the 2d observation gives +0.03s.
" " 3d " " −0.19s.
" " 4th " " −0.24s.
" " 5th " " −0.01s.
The mean = −0.08s.

That is, the observed right ascensions appear to be too small by 0.08s.

Similar observations should be made at each equinox every year, until it appears that no further correction is required.

Ex. 2. The following observations of the sun's centre were made at Washington in 1846:

Date.	Sun's R. A.	Sun's Dec.
	h. m. s.	° ′ ″
Sept. 16	11 35 49.40	+2 36 56.46
21	11 53 46.89	+0 40 29.52
22	11 57 22.79	+0 17 2.80
23	12 0 58.29	−0 6 18.71
25	12 8 10.58	−0 53 11.50
28	12 18 59.37	−2 3 28.11

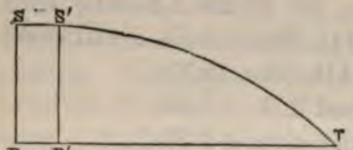
It is required to find the mean correction of the right ascensions, the obliquity of the ecliptic being 23° 27′ 25″.88.

Ans. +0.06s.

PROBLEM.

(198.) *To find the obliquity of the ecliptic.*

Observe the right ascension and declination of the sun near one of the solstices. If the sun were exactly at the solstice at one of the observations, the observed declination would be the obliquity required. But as such a coincidence can seldom happen, it is customary to take observations on several days both before and after the solstice, and compute the reduction to the solstice. This may be done in the following manner:



Let AB represent the equator, AS the ecliptic, A the vernal equinox, S the solstice, and S′ the place of the sun near the solstice. Let fall the perpendicular S′B′ upon the equator. Then, by Napier's rule,

$$\text{R. sin. AB}'=\text{tang. S}'\text{B}' \text{ cot. BAS.}$$

Put δ=the observed declination; h=BB′=6h.—the sun's right ascension; ω=the obliquity of the ecliptic; and x=the required correction to obtain the declination at the solstice. Then

$$\text{sin. AB}'=\text{tang. }\delta\text{ cot. }\omega,$$

or

$$\cos h=\frac{\text{tang. }\delta}{\text{tang. }\omega}.$$

By Trig., Art. 76,

$$\frac{\text{sin. }(\omega-\delta)}{\text{sin. }(\omega+\delta)}=\frac{\text{tang. }\omega-\text{tang. }\delta}{\text{tang. }\omega+\text{tang. }\delta}=\frac{1-\frac{\text{tang. }\delta}{\text{tang. }\omega}}{1+\frac{\text{tang. }\delta}{\text{tang. }\omega}}=\frac{1-\text{cos. }h}{1+\text{cos. }h}$$

$$=\frac{2\text{ sin. }^2\tfrac{1}{2}h}{2\text{ cos. }^2\tfrac{1}{2}h}\quad\text{Trig., Art. 74, }=\text{tang. }^2\tfrac{1}{2}h;$$

that is,

$$\text{sin. }(\omega-\delta)=\text{tang. }^2\tfrac{1}{2}h\text{ sin. }(\omega+\delta).$$

When the required correction is small, we may put $\omega-\delta$ for sin. $(\omega-\delta)$, and dividing by sin. $1''$, to have x expressed in seconds, we obtain

$$x=\omega-\delta=\frac{\text{tang. }^2\tfrac{1}{2}h\text{ sin. }(\omega+\delta)}{\text{sin. }1''}\quad.\ .\ .\ .\ .\ (1)$$

which is the correction in seconds to be added to the observed declination, to obtain the obliquity.

Ex. 1. In June, 1851, the following observations were made at Greenwich:

June 17.	Sun's R. A.	5h.	40m.	59.17s.	Dec.	23°	23′	7″.57
“ 19.	“	5	49	18.63	“		26	4 .87
“ 21.	“	5	57	37.76	“		27	22 .56
“ 26.	“	6	18	25.27	“		23	21 .25
“ 27.	“	6	22	34.12	“		21	18 .59
“ 28.	“	6	26	43.59	“		18	53 .14
“ 30.	“	6	35	1.20	“		12	47 .16

It is required to determine the obliquity of the ecliptic.

Assume for the obliquity the greatest observed declination, or 23° 27′ 22″.56. Then, for June 17th, the reduction will be as follows:

h=19m. 0.83s.=4° 45′ 12″.45; $\frac{1}{2}h$=2° 22′ 36″.2.

By formula (1),

tang. $\frac{1}{2}h$=8.618105
8.618105
sin. 46° 50′ 30″.13=9.863005
cosec. 1″=5.314425
Correction, 259″.20=2.413640

This correction being added to the observed declination, 23° 23′ 7″.57, gives

The obliquity of the ecliptic=23° 27′	26″.77
In like manner the 2d observation gives	26 .80
“ “ 3d “ “	26 .59
“ “ 4th “ “	24 .55
“ “ 5th “ “	23 .78
“ “ 6th “ “	25 .26
“ “ 7th “ “	26 .39
The mean is 23° 27′	25″.73

(199.) It may be thought that this method involves a vicious principle, inasmuch as it requires a knowledge of ω to enable us to find the value of ω. But it will be noticed that only an approximate knowledge of ω is required to furnish a very accurate value of the correction x. In reducing the preceding observation of June 17, an error of one minute in the assumed value of ω will occasion an error of less than one tenth of a second in the computed reduction.

This is but one example of a very common case in astronomy, in which we employ an approximate value of an unknown quantity to obtain a more accurate determination.

Ex. 2. The following observations were made at Greenwich in 1850:

June 17.	Sun's R. A.	5h.	41m.	59.36s.	Dec.	23°	23′	29″.18
" 18.	"	5	46	8.94	"		25	7 .16
" 20.	"	5	54	27.74	"		27	0 .73
" 21.	"	5	58	37.40	"		27	23 .24
" 22.	"	6	2	46.82	"		27	17 .83
" 24.	"	6	11	5.83	"		25	54 .95
" 25.	"	6	15	14.88	"		24	38 .84
" 26.	"	6	19	24.40	"		22	55 .04

Required the obliquity of the ecliptic.

Ans. 23° 27′ 23″.88.

Ex. 3. The following observations were made at Washington in 1846:

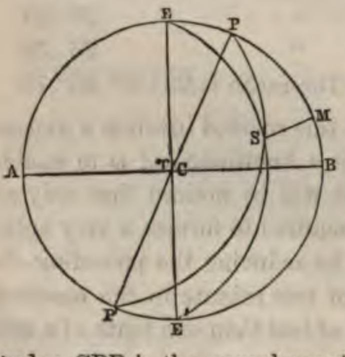
Dec. 18.	Sun's R. A.	17h.	44m.	37.21s.	Dec.	23°	24′	32″.69
" 21.	"	17	57	56.81	"		27	20 .43
" 22.	"	18	2	23.51	"		27	19 .64
" 23.	"	18	6	50.17	"		26	49 .82

Required the obliquity of the ecliptic.

Ans. 23° 27′ 23″.20.

PROBLEM.

(200.) *To find the longitude and latitude of a star, when its right ascension and declination are known.*



Let P represent the pole of the equator, E the pole of the ecliptic, C the first point of Aries, PSP′ an hour circle passing through the star S, and ESE′ a circle of latitude passing through the same star. Then AEBE′ represents the solstitial colure, EP represents the obliquity of the ecliptic, PS the polar distance of the star, ES its co-latitude; SPB is the complement of its right ascension, and SEB is the complement of its longitude. Draw SM perpendicular to PB. Represent PM by a; also represent the longitude of the star S by L, its latitude by l, and the obliquity of the ecliptic by ω.

Now, by Napier's rule, we have

$$\text{R. cos. SPM} = \text{tang. PM cot. PS};$$

that is,
$$\text{sin. R. A.} = \text{tang. } a \text{ tang. Dec.},$$

or
$$\text{tang. } a = \text{sin. R. A. cot. Dec.} \quad \ldots\ldots \quad \text{(A)}$$

Also,
$$\text{EM} = \text{EP} + \text{PM} = a + \omega.$$

Again, Trig., Art. 216, Cor. 3,

$$\text{sin. EM} : \text{sin. PM} :: \text{tang. SPM} : \text{tang. SEM};$$

that is,

$$\text{sin. } (a+\omega) : \text{sin. } a :: \text{cot. R. A.} : \text{cot. L} :: \text{tang. L} : \text{tang. R. A.},$$

or
$$\text{tang. L} = \frac{\text{tang. R.A. sin. } (a+\omega)}{\text{sin. } a} \quad \ldots\ldots \quad (1)$$

Also,
$$\text{tang. EM cot. ES} = \text{R. cos. SEM};$$

that is,
$$\text{tang. } l = \text{cot. } (a+\omega) \text{ sin. L} \quad \ldots\ldots \quad (2)$$

Also, Trig., Art. 216,

$$\text{cos. PM} : \text{cos. EM} :: \text{cos. PS} : \text{cos. ES};$$

that is,
$$\text{sin. } l = \frac{\text{cos. } (a+\omega) \text{ sin. Dec.}}{\text{cos. } a} \quad \ldots\ldots\ldots \quad (3)$$

And
$$\text{R. cos. SEP} = \text{tang. EM cot. ES};$$

that is,
$$\text{sin. L} = \text{tang. } (a+\omega) \text{ tang. } l \quad \ldots\ldots\ldots \quad (4)$$

Ex. 1. On the 1st of January, 1851, the R. A. of Capella was 5h. 5m. 42.03s., and its Dec. 45° 50′ 22″.4 N.; required its latitude and longitude, the obliquity of the ecliptic being 23° 27′ 25″.47.

By equation (A),

R. A. 76° 25′ 30″.45	sin. = 9.9876948
Dec. 45° 50′ 22″.4	cot. = 9.9872707
a = 43° 20′ 58″.31	tang. = 9.9749655
ω = 23° 27′ 25″.47	
$a+\omega$ = 66° 48′ 23″.78	

By equation (1),

	tang. R. A. = 0.6171524
	sin. $(a+\omega)$ = 9.9634009
	cosec. a = 0.1633928
L = 79° 46′ 40″.93	tang. = 0.7439461

By equation (2), cot. $(a+\omega)=9.6319144$
sin. $\mathrm{L}=9.9930515$
$l=22°\ 51'\ 48''.14$ tang. $=9.6249659$

By equation (3), cos. $(a+\omega)=9.5953154$
sin. Dec. $=9.8557564$
sec. $a=0.1383583$
$l=22°\ 51'\ 48''.14$ sin. $=9.5894301$

By equation (4), tang. $(a+\omega)=0.3680856$
tang. $l.=9.6249659$
$\mathrm{L}=79°\ 46'\ 41''.00$ sin. $=9.9930515$

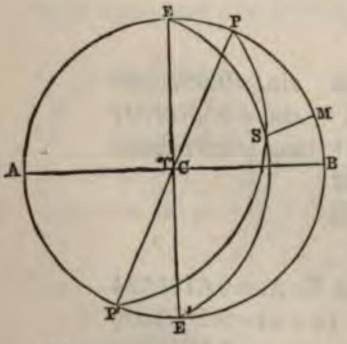
Formulas (3) and (4) give nearly the same result as formulas (1) and (2). Formulas (1) and (2) are, however, to be preferred, because the tangents vary more rapidly than the sines, especially near 90°.

Formulas which furnish the value of an unknown quantity by means of its tangent or cotangent, are generally more accurate than those which furnish it by means of its sine or cosine.

Ex. 2. On the 1st of January, 1851, the R. A. of Regulus was 10h. 0m. 25.87s., and its Dec. 12° 41′ 32″.7 N. Required its latitude and longitude, the obliquity of the ecliptic being 23° 27′ 25″.47. *Ans.* Latitude, 0° 27′ 35″.3 N.
Longitude, 147° 45′ 30″.3.

PROBLEM.

(201.) *To find the right ascension and declination of a star when its latitude and longitude are known.*



Using the same figure as in the last problem, and employing the same notation, except that we represent EM by a, we obtain

tang. EM cot. ES $=$ R. cos. SEM; that is,

$$\text{tang. EM} = \text{tang. } a = \text{sin. L cot. } l \quad . \ . \ . \ . \quad (\mathrm{A})$$

Also,

$$\mathrm{PM}=\mathrm{EM}-\mathrm{EP}=a-\omega.$$

Again, sin. PM : sin. EM :: tang. SEM : tang. SPM;
that is,

sin. $(a-\omega)$: sin. a :: cot. L : cot. R. A. :: tang. R. A. : tang. L.

Therefore, $$\text{tang. R. A.}=\frac{\text{tang. L sin. }(a-\omega)}{\text{sin. }a} \quad \ldots\ldots (1)$$

Also, tang. PM cot. PS = R. cos. SPM;

that is, $$\text{tang. Dec.}=\text{cot. }(a-\omega)\text{ sin. R. A.} \quad \ldots\ldots (2)$$

Also, cos. EM : cos. PM :: cos. ES : cos. PS;

that is, $$\text{sin. Dec.}=\frac{\text{cos. }(a-\omega)\text{ sin. }l}{\text{cos. }a} \quad \ldots\ldots (3)$$

And R. cos. SPM = tang. PM cot. PS;

that is, $$\text{sin. R. A.}=\text{tang. }(a-\omega)\text{ tang. Dec.} \quad \ldots (4)$$

Ex. 1. On the 1st of January, 1851, the longitude of Capella was 79° 46′ 40″.93, and its latitude 22° 51′ 48″.14 N.; required its right ascension and declination, the obliquity of the ecliptic being 23° 27′ 25″.47.

By equation (A),

sin. L = 9.9930515
cot. l = 0.3750341
a = 66° 48′ 23″.78 tang. = 0.3680856
ω = 23° 27′ 25″.47
$a-\omega$ = 43° 20′ 58″.31

By equation (1),

tang. L = 0.7439461
sin. $(a-\omega)$ = 9.8366072
cosec. a = 0.0365991
R. A. 76° 25′ 30″.45 tang. = 0.6171524

By equation (2),

cot. $(a-\omega)$ = 0.0250345
sin. R. A. = 9.9876948
Dec. 45° 50′ 22″.4 tang. = 0.0127293

By equation (3),

sin. l = 9.5894301
cos. $(a-\omega)$ = 9.8616417
sec. a = 0.4046846
Dec. = 45° 50′ 22″.4 sin. = 9.8557564

By equation (4),

tang. $(a-\omega)$ = 9.9749655
tang. Dec. = 0.0127293
R. A. = 76° 25′ 30″.4 sin. = 9.9876948

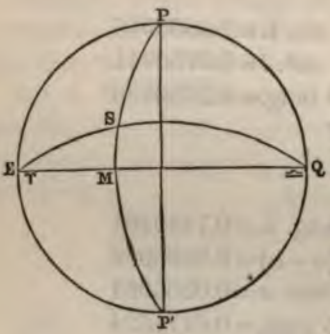
In this example we have reproduced the results of *Ex.* 1 in the preceding problem. Equations (1) and (2) are to be preferred to equations (3) and (4), for the reason given in the preceding Article.

Ex. 2. On the 1st of January, 1851, the longitude of Regulus was 147° 45′ 30″.3, and its latitude 0° 27′ 35″.3 N.; required its right ascension and declination, the obliquity of the ecliptic being 23° 27′ 25″.47.

Ans. Right ascension, 10h. 0m. 25.87s.
Declination, 12° 41′ 32″.7 N.

PROBLEM.

(202.) ***To compute the longitude, right ascension, and declination of the sun, any one of these quantities, together with the obliquity of the ecliptic, being given.***



Let EPQP′ represent the equinoctial colure, EMQ the equator, ESQ the ecliptic, E the first point of Aries, S the place of the sun, PSP′ an hour circle passing through the sun; then EM is the sun's right ascension, SM his declination, ES his longitude, and MES the obliquity of the ecliptic. Then, in the triangle ESM, we have

$$\text{tang. ME cot. SE} = \text{R. cos. E};$$

that is, representing the obliquity by ω,

$$\text{tang. R. A.} = \text{tang. Long. cos. } \omega \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and $$\text{tang. Long.} = \frac{\text{tang. R. A.}}{\cos.\ \omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also, $$\text{R. sin. ME} = \text{tang. MS cot. E};$$

that is, $$\text{sin. R. A.} = \text{tang. Dec. cot. } \omega \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and $$\text{tang. Dec.} = \text{sin. R. A. tang. } \omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Also, $$\text{R. sin. MS} = \text{sin. E sin. ES};$$

that is, $$\text{sin. Dec.} = \text{sin. } \omega \text{ sin. Long.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and $$\text{sin. Long.} = \frac{\text{sin. Dec.}}{\text{sin. } \omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Also, $\text{R. cos. ES} = \text{cos. ME cos. MS};$

that is, $\text{cos. Long.} = \text{cos. R. A. cos. Dec.} \;.\;.\;.\;.\;.\; (7)$

and $\text{cos. R. A.} = \dfrac{\text{cos. Long.}}{\text{cos. Dec.}} \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (8)$

Ex. 1. On the 1st of June, 1852, at Greenwich mean noon, the sun's right ascension was 4h. 38m. 0.88s., and his declination 22° 7′ 13″.7 N.; required his longitude.

By formula (7),

$$\begin{array}{r} \text{cos. R. A.} = 69^\circ\ 30'\ 13''.2 = 9.5442510 \\ \text{cos. Dec.} = \underline{9.9667958} \\ \text{Longitude} = 71^\circ\ 4'\ 20''.3\ \text{cos.} = 9.5110468 \end{array}$$

Ex. 2. On the 1st of January, 1852, the sun's right ascension was 18h. 44m. 49.47s., and his declination 23° 3′ 28″.0 S.; required his longitude.

Ans. 280° 18′ 2″.4.

Ex. 3. On the 20th of May, 1852, the sun's longitude was 59° 33′ 42″.5, and the obliquity of the ecliptic 23° 27′ 29″.06; required his right ascension and declination.

By formula (1),

$$\begin{array}{r} \text{tang. Long.} = 0.2309234 \\ \cos.\ \omega = \underline{9.9625359} \\ 57^\circ\ 21'\ 32''.94\ \text{tang.} = 0.1934593 \end{array}$$

R. A. = 3h. 39m. 26.20s.

By formula (5),

$$\begin{array}{r} \text{sin. Long.} = 9.9355960 \\ \sin.\ \omega = \underline{9.5999681} \\ \text{Dec. } 20^\circ\ 4'\ 21''.96\ \text{sin.} = 9.5355641 \end{array}$$

Ex. 4. On the 27th of October, 1852, the sun's longitude was 214° 14′ 45″.2, and the obliquity of the ecliptic 23° 27′ 30″.69; required his right ascension and declination.

Ans. Right ascension, 14h. 7m. 56.39s,
Declination, 12° 56′ 43″.1 S.

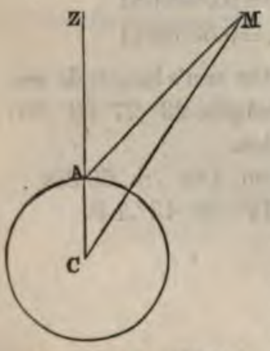
CHAPTER VIII.

PARALLAX.

(203.) THE fixed stars are so distant from the earth, that their relative positions are sensibly the same, from whatever point of the earth's surface we may view them. It is otherwise with the sun, moon, and planets, which are near enough (especially the moon) to be displaced by change of station on our globe. Two spectators, situated on different points of the earth's surface, and viewing the moon at the same instant, do not see it in the same direction. In order that astronomers residing at different points of the earth's surface may be able to compare their observations, it is necessary to take account of this effect of the difference of their stations, and it is convenient to adopt some centre of reference common to all the world, to which each astronomer may reduce his observations. The common point of reference universally agreed upon is the centre of the earth; and the difference between the apparent positions of a heavenly body, as seen from the surface or the centre of the earth, is called its parallax.

PROBLEM.

(204.) *To find the parallax of the moon, etc., in altitude.*



Let C represent the centre of the earth, A the place of the observer on its surface, M the moon, and CAZ the direction of a perpendicular to the surface at A. Then will the moon be seen from A in the direction AM, and its apparent zenith distance will be ZAM; whereas, if seen from the centre of the earth, it would appear in the direction CM, with an angular distance from the zenith of A equal to ZCM; so that ZAM−ZCM, or AMC, is the parallax.

Let us put $r=\text{AC}$, the radius of the earth;
$\text{R}=\text{CM}$, the moon's distance from the earth's centre;
$z=\text{ZCM}$, the moon's true zenith distance;
$z'=\text{ZAM}$, the moon's apparent zenith distance;
$q=\text{AMC}$, the moon's parallax in altitude.

In the triangle ACM, we have

$$\text{CM} : \text{CA} :: \sin.\ \text{CAM} : \sin.\ \text{AMC},$$

or

$$\sin.\ q=\frac{r}{\text{R}} \sin.\ z';$$

that is, the sine of the parallax in altitude $=\dfrac{\text{Radius of earth}}{\text{Distance of body}}$ × sine of the apparent zenith distance.

The parallax, therefore, for a given place, and a given distance of the body observed, is proportional to the sine of its apparent zenith distance, and is therefore the greatest when the body is observed in the act of rising or setting, in which case its parallax is called its *horizontal* parallax.

If we designate by p the horizontal parallax, we shall have, when $z'=90°$,

$$\sin.\ p=\frac{r}{\text{R}}.$$

Hence

$$\sin.\ q=\sin.\ p \sin.\ z' \quad \ldots\ldots\ldots \quad (1)$$

that is, *the sine of the parallax in altitude is equal to the sine of the horizontal parallax, into the sine of the apparent zenith distance.*

(205.) This formula furnishes the parallax when the apparent zenith distance is known, but when the true zenith distance is given we require a different formula, which is obtained as follows:

The angle $\text{ZAM}=\text{ACM}+\text{AMC}$;

that is, $z'=z+q$.

Hence

$$\sin.\ q=\sin.\ p \sin.\ (z+q)$$
$$=\sin.\ p \sin.\ z \cos.\ q+\sin.\ p \cos.\ z \sin.\ q,$$ by Trig., Art. 72.

Dividing each member by $\cos.\ q$, we obtain

$$\text{tang.}\ q=\sin.\ p \sin.\ z+\sin.\ p \cos.\ z\ \text{tang.}\ q.$$

Whence

$$\text{tang.}\ q=\frac{\sin.\ p \sin.\ z}{1-\sin.\ p \cos.\ z} \quad \ldots\ldots\ldots \quad (2)$$

which formula furnishes the parallax in altitude when the true zenith distance is known; but the expression is not convenient for computation by logarithms. If we divide the numerator of this expression by the denominator, we shall have

$$\text{tang. } q = \text{sin. } p \text{ sin. } z + \text{sin. }^2 p \text{ sin. } z \text{ cos. } z + \text{sin. }^3 p \text{ sin. } z \text{ cos. }^2 z +, \text{ etc.}$$

But by the Calculus, Art. 324, Ex. 2,

$$q = \text{tang. } q - \frac{\text{tang. }^3 q}{3} +, \text{ etc.}$$

Hence

$$q = \text{sin. } p \text{ sin. } z + \text{sin. }^2 p \text{ sin. } z \text{ cos. } z + \text{sin. }^3 p \text{ sin. } z \text{ cos. }^2 z - \frac{\text{sin. }^3 p \text{ sin. }^3 z}{3} +, \text{ etc. } \ldots\ldots \text{(A)}$$

But by Trig., Art. 73,

$$\text{sin. } z \text{ cos. } z = \frac{\text{sin. } 2z}{2}.$$

Also, Trig., Art. 79,

$$\text{sin. } 3z = 4 \text{ sin. } z \text{ cos. }^2 z - \text{sin. } z,$$

and $$\text{cos. }^2 z - 1 = -\text{sin. }^2 z.$$

Therefore $$\text{sin. } z \text{ cos. }^2 z - \text{sin. } z = -\text{sin. }^3 z;$$

that is, $$\text{sin. } 3z = 3 \text{ sin. } z \text{ cos. }^2 z - \text{sin. }^3 z,$$

or $$\frac{\text{sin. } 3z}{3} = \text{sin. } z \text{ cos. }^2 z - \frac{\text{sin. }^3 z}{3}.$$

Therefore, by substitution in equation (A), we obtain

$$q = \text{sin. } p \text{ sin. } z + \frac{\text{sin. }^2 p \text{ sin. } 2z}{2} + \frac{\text{sin. }^3 p \text{ sin. } 3z}{3} +, \text{ etc.}$$

If we wish to have q expressed in seconds, we must divide by sin. 1″, and we shall have

$$q = \frac{\text{sin. } p \text{ sin. } z}{\text{sin. } 1''} + \frac{\text{sin. }^2 p \text{ sin. } 2z}{\text{sin. } 2''} + \frac{\text{sin. }^3 p \text{ sin. } 3z}{\text{sin. } 3''} +, \text{ etc. . (3)}$$

which furnishes the parallax in terms of the true zenith distance by a series which converges rapidly.

(206.) The parallax of the sun and planets is so small, that we may employ the more convenient formula,

$$q = p \text{ sin. } z \ldots\ldots \text{(4)}$$

without sensible error. But for the moon, when the apparent zenith distance (as affected by parallax) is known, we must make use of formula (1); but when we know only the true zenith distance, we must adopt formula (2) or (3).

Ex. 1. If the horizontal parallax of Venus is 30″, what is its parallax for an altitude of 30° ?

Solution.—By formula (4),
log. 30″=1.4771
sin. 60°=9.9375
Ans. 26″.0=$\overline{1.4146}$

Ex. 2. If the horizontal parallax of Mars is 10″, what is its parallax for an altitude of 24° ?

Solution.
log. 10″=1.0000
sin. 66°=9.9607
Ans. 9″.1=$\overline{0.9607}$

In this manner was computed Table XV., showing the parallax of the sun and planets at different altitudes.

Ex. 3. If the horizontal parallax of the sun is 8″.6, what is its parallax for an altitude of 16° ? *Ans.* 8″.27.

Ex. 4. If the moon's horizontal parallax is 60′ 41″.5, what is her parallax when her apparent zenith distance is 80° 19′ 19″ ?

Solution.—By formula (1), sin. 60′ 41″.5=8.246833
sin. 80° 19′ 19″ =9.993775
Ans. 59′ 49″.67 sin.=$\overline{8.240608}$

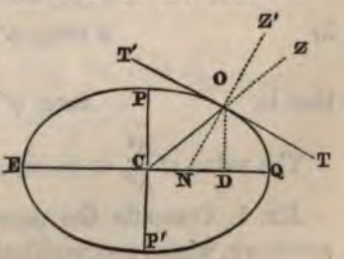
Ex. 5. If the moon's horizontal parallax is 60′ 41″.5, what is her parallax when her true zenith distance is 79° 19′ 29″.33 ?

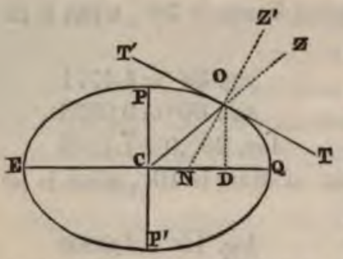
Solution.—By formula (3),

sin. 60′ 41″.5=8.246833	sin. ${}^2p$=6.4937
sin. 79° 19′ 29″.33=9.992418	sin. 158° 39′=9.5612
cosec. 1″=5.314425	cosec. 2″=5.0134
59′ 38″.29=3578″.29=$\overline{3.553676}$	+11″.70=$\overline{1.0683}$
	sin. ${}^3p$=4.740
	sin. 237° 58′=9.928*n*
	cosec. 3″=4.837
	−0″.32=$\overline{9.505}$

Hence 59′ 38″.29+11″.70−0″.32=59′ 49″.67 *Ans.*

(207.) If the earth were a sphere, a plumb line suspended at O would take the direction OC, passing through C, the centre of the sphere; and if produced upward, it would meet the heavens in Z. This line, ZOC, would also be perpendicular to





the tangent line TOT'. But since the earth is a spheroid, the meridian, PEP'Q, is an ellipse, and a plumb line at O being perpendicular to a tangent line, TT', takes the direction of the normal line, ON; and NO being produced, meets the celestial sphere in Z'. The latitude obtained by observation will be expressed by the angle Z'NQ. This is called the apparent or *geographical* latitude; while Z being the geocentric zenith, the angle ZCQ is called the *geocentric* latitude. The angle CON is the angle which a vertical line makes with the radius of the earth, and is called *the angle of the vertical.*

PROBLEM.

(208.) *To find the angle of the vertical.*

Let PEP'Q be a section of the earth by a plane passing through the poles. This section is an ellipse, whose semi-major axis, CQ, is the radius of the equator, and whose semi-minor axis, CP, is half the polar diameter. If O be the position of an observer whose latitude is ϕ, TT', a tangent to the ellipse at the point O, will represent a horizontal line, and Z'O, which is perpendicular to TT', will be the direction of a plumb line. Represent the angle OCQ, or the geocentric latitude, by ϕ', and draw the ordinate OD. Let A and B represent the semi-axes of the ellipse.

By An. Geom., Art. 80, the subnormal $\text{ND}=\frac{\text{B}^2}{\text{A}^2}x$, where x represents the abscissa CD.

But $$\text{OD}=\text{CD tang. OCD}=\text{ND tang. OND},$$

or $$x \text{ tang. } \phi'=\frac{\text{B}^2}{\text{A}^2}x \text{ tang. } \phi;$$

that is, $$\text{tang. } \phi'=\frac{\text{B}^2}{\text{A}^2} \text{ tang. } \phi.$$

The value of $\frac{\text{B}^2}{\text{A}^2}$, as determined by Bessel, is 0.9933254.

Ex. 1. Compute the geocentric latitude of Cambridge Observatory, whose geographical latitude is 42° 22′ 48″.6.

Solution.

$$\log.\frac{B^2}{A^2}=9.99709164$$

$$\text{tang. } 42^\circ\ 22'\ 48''.6=\underline{9.96022854}$$

$$\textit{Ans. } 42^\circ\ 11'\ 21''.05 \text{ tang.}=9.95732018$$

Hence the angle of the vertical is 11′ 27″.55.

Ex. 2. Compute the angle of the vertical for latitude 40°?

Solution.

$$\log.\frac{B^2}{A^2}=9.99709164$$

$$\text{tang. } 40^\circ=\underline{9.92381353}$$

$$\textit{Ans. } 39^\circ\ 48'\ 40''.24 \text{ tang.}=9.92090517$$

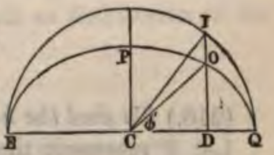
Hence the angle of the vertical is 11′ 19″.76.

In the same manner was computed column second of Table XII., showing the angle of the vertical for every degree of latitude.

(209.) The horizontal parallax of the moon is the angle which the earth's radius would subtend to an observer at the moon. It is, therefore, not the same for all places on the earth, but varies with the earth's radius. It is necessary, therefore, to compute the earth's radius for the place of the observer.

PROBLEM.

To compute the radius of the earth.



Let EPQ be half of the ellipse formed by a section of the earth through the poles. On EQ describe a semicircle, and produce OD to meet the circumference in I. Join CO and CI. Represent the angle OCD by ϕ', and the radius OC by r. Then, in the triangle OCD, we have

$$\text{CD}=\text{OC} \cos.\ \phi'=r \cos.\ \phi'\,;$$

$$\text{OD}=\text{OC} \sin.\ \phi'=r \sin.\ \phi'.$$

Also, by Conic Sections, Ellipse, Prop. 12, Cor. 3,

$$\text{B}:\text{A}::\text{DO}:\text{DI}=\frac{\text{A}\,.\,r\,.\sin.\ \phi'}{\text{B}}.$$

But $$\text{CD}^2+\text{DI}^2=\text{CI}^2=\text{A}^2.$$

Therefore $$r^2 \cos.\ ^2\phi'+\frac{\text{A}^2}{\text{B}^2}\,.\,r^2 \sin.\ ^2\phi'=\text{A}^2.$$

But, by Art. 208, $\frac{A^2}{B^2}=\frac{\text{tang. }\phi}{\text{tang. }\phi'}$.

Hence $r^2 \cos.{}^2\phi' + r^2 \frac{\sin.{}^2\phi'}{\text{tang. }\phi'} \text{tang. }\phi = A^2$;

or, multiplying by cos. ϕ,

$$r^2 \cos. \phi' (\cos. \phi' \cos. \phi + \sin. \phi' \sin. \phi) = A^2 \cos. \phi.$$

Hence, by Trig., Art. 72,

$$r^2 \cos. \phi' \cos. (\phi' - \phi) = A^2 \cos. \phi.$$

Therefore $$r^2 = \frac{A^2 \cos. \phi}{\cos. \phi' \cos. (\phi' - \phi)} \quad \ldots\ldots\ldots \quad (1)$$

Ex. 1. Compute the earth's radius for Cambridge Observatory, the equatorial radius being taken as unity.

The angle of the vertical was found in the preceding article.

By formula (1),

$$\begin{array}{r}
\cos. \phi = 9.8684615 \\
\sec. \phi' = 0.1302220 \\
\sec. (\phi' - \phi) = \underline{0.0000024} \\
2)\,\overline{9.9986859} \\
\log. r = \overline{9.9993429}
\end{array}$$

Ex. 2. Compute the earth's radius for latitude 40°.

$$\begin{array}{r}
\cos. 40° = 9.8842540 \\
\sec. 39° \; 48' \; 40''.24 = 0.1145491 \\
\sec. 11' \; 19''.76 = \underline{0.0000024} \\
2)\,\overline{9.9988055} \\
\log. r = \overline{9.9994027}
\end{array}$$

PROBLEM.

(210.) *To find the horizontal parallax for any place.*

Let P represent the horizontal parallax for a place on the equator, p the same for a place in any other latitude; let r and r' be the radii of the earth for the two stations; then, by Art. 204, $$\text{R} \sin. \text{P} = r;$$

and, for the same reason,

$$\text{R}. \sin. p = r'.$$

Therefore, $$\sin. p = \frac{r'}{r} \sin. \text{P};$$

or, calling the equatorial radius unity,

$$\sin. p = r' \sin. \text{P}.$$

As r' is nearly equal to unity, we may, without appreciable error, adopt the more convenient formula,

$$p=r'.\mathrm{P};$$

that is, the moon's horizontal parallax for any given latitude is equal to the horizontal parallax at the equator, multiplied by the radius of the earth at the given latitude, the radius at the equator being considered as unity.

Ex. 1. When the equatorial horizontal parallax is 53′, what is the horizontal parallax for Cambridge Observatory?

Solution. $53'=3180''=3.5024271$

$r'=9.9993429$

Ans. $3175''.19=\overline{3.5017700}$

Ex. 2. When the equatorial horizontal parallax is 59′, what is the horizontal parallax for latitude 40°?

Solution. $59'=3540''=3.5490033$

$r'=9.9994027$

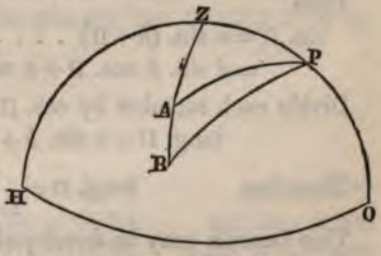
Ans. $3535''.13=\overline{3.5484060}$

It is this corrected value of the equatorial parallax which should be employed in all computations which involve the parallax of a particular place.

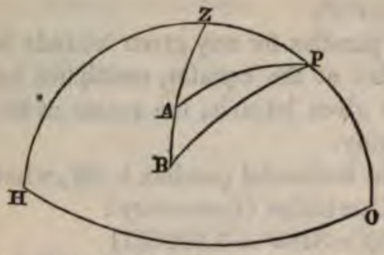
Since the effect of parallax is confined to a vertical plane, when the moon is on the meridian there is no parallax in right ascension, but its effect is wholly on the declination. In every other position of the moon (the vertical circle passing through the moon being inclined to the circle of right ascension), parallax affects the right ascension as well as declination of the moon.

PROBLEM.

(211.) *To compute the parallax in right ascension.*



Let HZO be a meridian, Z the geocentric zenith of the place of observation, and P the pole of the equator. Let A be the true place of the moon seen from the centre of the earth, and B the apparent place seen from the surface; then will the arc AB be the parallax in altitude; and the true hour angle, ZPA, is changed by parallax into ZPB. Therefore APB is the parallax in right ascension.



Let us represent the horizontal parallax of the place by p; the parallax in right ascension by Π; the hour angle, ZPA (which is equal to the sidereal time, *minus* the moon's true right ascension), by h; the moon's declination (which equals $90° - \text{AP}$) by δ; and the *geocentric* latitude of the place of observation by ϕ'. The angle ZPB will then be equal to $h+\Pi$.

In the spherical triangle ABP, we have

$$\text{sin. AP} : \text{sin. B} :: \text{sin. AB} : \text{sin. APB} = \text{sin. } \Pi = \frac{\text{sin. AB sin. B}}{\text{sin. AP}}.$$

Also, in the spherical triangle BPZ, we have

$$\text{sin. BZ} : \text{sin. BPZ} :: \text{sin. PZ} : \text{sin. B} = \frac{\text{sin. PZ sin. } (\text{APZ}+\Pi)}{\text{sin. BZ}}.$$

Therefore $\text{sin. } \Pi = \dfrac{\text{sin. AB}}{\text{sin. AP}} \cdot \dfrac{\text{sin. PZ sin. } (h+\Pi)}{\text{sin. BZ}}.$

But, by Art. 204,

$$\text{sin. AB} = \text{sin. } p \text{ sin. BZ}.$$

Hence $\text{sin. } \Pi = \dfrac{\text{sin. } p \text{ sin. BZ sin. PZ sin. } (h+\Pi)}{\text{sin. AP sin. BZ}}$

$$= \frac{\text{sin. } p \text{ cos. } \phi' \text{ sin. } (h+\Pi)}{\text{cos. } \delta}.$$

Let us put $a = \dfrac{\text{sin. } p \text{ cos. } \phi'}{\text{cos. } \delta}.$

Then

$$\text{sin. } \Pi = a \text{ sin. } (h+\Pi) \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$= a \text{ sin. } h \text{ cos. } \Pi + a \text{ cos. } h \text{ sin. } \Pi. \quad \text{Trig., Art. 72.}$$

Divide each member by cos. Π, and we have

$$\text{tang. } \Pi = a \text{ sin. } h + a \text{ cos. } h \text{ tang. } \Pi.$$

Therefore $\text{tang. } \Pi = \dfrac{a \text{ sin. } h}{1 - a \text{ cos. } h} \quad \ldots\ldots \quad (2)$

This formula may be developed in a series, as in Art. 205, and we shall obtain

$$\Pi = \frac{a \text{ sin. } h}{\text{sin. } 1''} + \frac{a^2 \text{ sin. } 2h}{\text{sin. } 2''} + \frac{a^3 \text{ sin. } 3h}{\text{sin. } 3''} +, \text{ etc.} \quad \ldots \quad (3)$$

Equation (1) will furnish the parallax in right ascension, when we know the *apparent* hour angle (as affected by parallax); but when we know only the true hour angle, h, we must employ equation (2) or equation (3).

Ex. 1. Find the moon's parallax in right ascension for the High School Observatory, Philadelphia, Lat. 39° 57′ 7″ N., when the horizontal parallax of the place is 59′ 36″.8, the moon's Dec. 24° 5′ 11″.6 N., and the moon's hour angle 61° 10′ 47″.4.

The *geocentric* latitude of the place is 39° 45′ 47″.5.

By formula (3),

$\sin. p=8.239048$		
$\cos. \phi'=9.885754$		
$\sec. \delta=0.039563$	$a^2=6.3287$	$a^3=4.493$
$a=\overline{8.164365}$	$\sin. 2h=9.9267$	$\sin. 3h=8.791n$
$\sin. h=9.942572$	$\text{cosec. } 2''=5.0134$	$\text{cosec. } 3''=4.837$
$\text{cosec. } 1''=5.314425$	$+18''.57=\overline{1.2688}$	$-0''.01=\overline{8.121n}$
$2638''.53=\overline{3.421362}$		

Hence

$$\Pi=2638''.53+18''.57-0''.01=44'\ 17''.09.$$

Therefore the moon's *apparent* hour angle is

$$61^\circ\ 10'\ 47''.4+44'\ 17''.1$$
$$=61^\circ\ 55'\ \ 4''.5.$$

With this hour angle the parallax may be computed by formula (1), thus:

$$a=8.164365$$
$$\sin. (h+\Pi)=9.945604$$
$$\textit{Ans. } \sin. 44'\ 17''.09=\overline{8.109969}$$

Ex. 2. Find the moon's parallax in right ascension for the High School Observatory, Philadelphia, when the horizontal parallax of the place is 57′ 7″.5, the moon's Dec. 26° 23′ 3″.6 N., and hour angle 32° 39′ 49″.5.

Solution.

$\sin. p.=8.220532$		
$\cos. \phi'=9.885754$		
$\sec. \delta=0.047773$	$a^2=6.3081$	$a^3=4.462$
$a=\overline{8.154059}$	$\sin. 2h=9.9584$	$\sin. 3h=9.996$
$\sin. h=9.732158$	$\text{cosec. } 2''=5.0134$	$\text{cosec. } 3''=4.837$
$\text{cosec. } 1''=5.314425$	$19''.05=\overline{1.2799}$	$0''.20=\overline{9.295}$
$1587''.24=\overline{3.200642}$		

Hence $\Pi=1587''.24+19''.05+0''.20=26'\ 46''.49$.

Therefore the moon's apparent hour angle is 33° 6′ 36″.0.

By formula (1), $a=8.154059$

$$\text{sin. }(h+\Pi)=9.737390$$

$$\textit{Ans.}\ \text{sin. }26'\ 46''.49=\overline{7.891449}$$

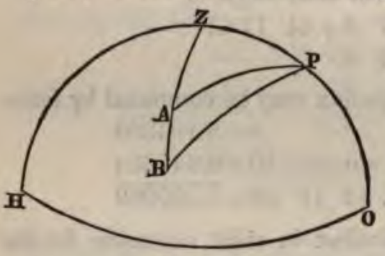
Ex. 3. Find the moon's parallax in right ascension for Western Reserve College, Ohio, Lat. 41° 14′ 42″, when the horizontal parallax of the place is 59′ 36″.5; the moon's Dec. 24° 4′ 41″.7 N., and hour angle 68° 9′ 51″.9. *Ans.* 45′ 56″.5.

Ex. 4. Find the moon's parallax in right ascension for Western Reserve College, Ohio, when the horizontal parallax of the place is 57′ 7″.7; the moon's Dec. 26° 24′ 31″.5 N., and hour angle 23° 13′ 12″.0. *Ans.* 19′ 12″.6.

In this manner was computed Table XVI., showing the moon's parallax in right ascension for Cambridge Observatory for all hour angles from the meridian to the horizon.

PROBLEM.

(212.) *To compute the moon's parallax in declination.*



Let Z be the geocentric zenith of the place of observation, P the pole of the equator, A the true place of the moon, and B its apparent place; then AB is the parallax in altitude, and BP−AP is the parallax in declination.

Represent the *true* declination, zenith distance, and hour angle of the moon by δ, Z, and h; and the *apparent* values of the same quantities by the same letters accented, δ', Z′, and h'. Also, let π represent the parallax in declination.

By Trig., Art. 225, we have

$$\text{cos. A}=\frac{\text{cos. }a-\text{cos. }b\text{ cos. }c}{\text{sin. }b\text{ sin. }c}.$$

This formula, applied successively to the triangles APZ and BPZ, gives

$$\text{cos. AZP}=\frac{\text{cos. AP}-\text{cos. PZ cos. AZ}}{\text{sin. PZ sin. AZ}}=\frac{\text{cos. BP}-\text{cos. PZ cos. BZ}}{\text{sin. PZ sin. BZ}}.$$

Therefore

$$\frac{\sin.\ \delta - \sin.\ \phi' \cos.\ Z}{\sin.\ Z} = \frac{\sin.\ \delta' - \sin.\ \phi' \cos.\ Z'}{\sin.\ Z'};$$

that is,

$$\sin.\ \delta \sin.\ Z' - \sin.\ \phi' \sin.\ Z' \cos.\ Z = \sin.\ \delta' \sin.\ Z - \sin.\ \phi' \sin.\ Z \cos.\ Z',$$

or

$$\sin.\ \delta \sin.\ Z' - \sin.\ \phi' (\sin.\ Z' \cos.\ Z - \sin.\ Z \cos.\ Z') = \sin.\ \delta' \sin.\ Z.$$

But by Trig., Art. 72, the factor included within the parenthesis is equal to sin. (Z′−Z), which, by Art. 204, is equal to sin. q, or sin. p sin. Z′.

Therefore

$$\sin.\ \delta \sin.\ Z' - \sin.\ p \sin.\ \phi' \sin.\ Z' = \sin.\ \delta' \sin.\ Z,$$

or

$$\sin.\ \delta' \sin.\ Z = \sin.\ Z' (\sin.\ \delta - \sin.\ p \sin.\ \phi') \quad . \ . \ \text{(A)}$$

Now, in order to eliminate Z′ and Z, we have, in the spherical triangles AZP and BZP,

$$\sin.\ AZP = \frac{\sin.\ AP \sin.\ APZ}{\sin.\ AZ} = \frac{\sin.\ BP \sin.\ BPZ}{\sin.\ BZ}.$$

Therefore

$$\sin.\ BP \sin.\ AZ = \frac{\sin.\ BZ \sin.\ AP \sin.\ APZ}{\sin.\ BPZ},$$

or

$$\cos.\ \delta' \sin.\ Z = \frac{\sin.\ Z' \cos.\ \delta \sin.\ h}{\sin.\ h'} \quad . \ . \ . \ . \ \text{(B)}$$

Dividing equation (A) by equation (B), we obtain

$$\text{tang.}\ \delta' = \frac{\sin.\ \delta - \sin.\ p \sin.\ \phi'}{\sin.\ h \cos.\ \delta} . \sin.\ h'.$$

Hence

$$\text{tang.}\ \delta' = \left(\frac{\sin.\ \delta}{\cos.\ \delta} - \frac{\sin.\ p \sin.\ \phi'}{\cos.\ \delta}\right)\frac{\sin.\ h'}{\sin.\ h}$$

$$= \left(1 - \frac{\sin.\ p \sin.\ \phi'}{\sin.\ \delta}\right)\frac{\sin.\ h'}{\sin.\ h} \text{tang.}\ \delta \quad . \ . \ . \ (1)$$

(213.) Formula (1) furnishes the apparent declination in terms of the true declination, the true hour angle and the apparent hour angle, which is obtained by the preceding problem. It is the simplest formula known for the parallax in declination; but in order to obtain all the accuracy which is required in many computations, it is necessary to have a table of sines and tangents to seven decimal places for every second of the quadrant. It is, therefore, sometimes more convenient to have a formula which shall furnish the parallax directly.

From the last equation but one we have

$$\frac{\text{tang. }\delta'\text{ sin. }h}{\text{sin. }h'}=\text{tang. }\delta-\frac{\text{sin. }p\text{ sin. }\phi'}{\text{cos. }\delta},$$

whence

$$\text{tang. }\delta-\frac{\text{tang. }\delta'\text{ sin. }h}{\text{sin. }h'}=\frac{\text{sin. }p\text{ sin. }\phi'}{\text{cos. }\delta},$$

or

$$\text{tang. }\delta-\text{tang. }\delta'+\text{tang. }\delta'-\frac{\text{tang. }\delta'\text{ sin. }h}{\text{sin. }h'}=\frac{\text{sin. }p\text{ sin. }\phi'}{\text{cos. }\delta}.$$

But $\text{tang. }\delta-\text{tang. }\delta'=\dfrac{\text{sin. }(\delta-\delta')}{\text{cos. }\delta\text{ cos. }\delta'}$. Trig., Art. 76.

Therefore

$$\frac{\text{sin. }(\delta-\delta')}{\text{cos. }\delta\text{ cos. }\delta'}=\frac{\text{sin. }p\text{ sin. }\phi'}{\text{cos. }\delta}-\frac{\text{tang. }\delta'}{\text{sin. }h'}(\text{sin. }h'-\text{sin. }h).$$

But by Trig., Art. 75,

$$\begin{aligned}\text{sin. }h'-\text{sin. }h&=2\text{ sin. }\tfrac{1}{2}(h'-h)\text{ cos. }\tfrac{1}{2}(h'+h)\\&=2\text{ sin. }\tfrac{1}{2}\Pi\text{ cos.}(h+\tfrac{1}{2}\Pi).\end{aligned}$$

Therefore

$$\frac{\text{sin. }\pi}{\text{cos. }\delta\text{ cos. }\delta'}=\frac{\text{sin. }p\text{ sin. }\phi'}{\text{cos. }\delta}-\frac{2\text{ sin. }\frac{1}{2}\Pi\text{ cos. }(h+\frac{1}{2}\Pi)\text{ tang. }\delta'}{\text{sin. }h'}.$$

But by Trig., Art. 74,

$$2\text{ sin. }\tfrac{1}{2}\Pi=\frac{\text{sin. }\Pi}{\text{cos. }\frac{1}{2}\Pi},\text{ which, by Art. 211}=\frac{\text{sin. }p\text{ cos. }\phi'\text{ sin. }h'}{\text{cos. }\delta\text{ cos. }\frac{1}{2}\Pi}.$$

Therefore

$$\begin{aligned}\text{sin. }\pi=&\text{sin. }p\text{ sin. }\phi'\text{ cos. }\delta'\\&-\text{sin. }p\text{ cos. }\phi'\text{ cos. }(h+\tfrac{1}{2}\Pi)\text{ sec. }\tfrac{1}{2}\Pi\text{ sin. }\delta'\quad.\quad(\text{C})\end{aligned}$$

Let us put

$$\text{cot. }b=\text{cos. }(h+\tfrac{1}{2}\Pi)\text{ cot. }\phi'\text{ sec. }\tfrac{1}{2}\Pi\quad.\ .\ .\ .\ .\ .\ .\ .\quad(\text{D})$$

Then

$$\begin{aligned}\text{sin. }\pi&=\text{sin. }p\text{ sin. }\phi'\text{ cos. }\delta'-\text{sin. }p\text{ sin. }\phi'\text{ sin. }\delta'\text{ cot. }b\\&=\text{sin. }p\text{ sin. }\phi'\left(\frac{\text{cos. }\delta'\text{ sin. }b-\text{sin. }\delta'\text{ cos. }b}{\text{sin. }b}\right)\\&=\frac{\text{sin. }p\text{ sin. }\phi'}{\text{sin. }b}\text{ sin. }(b-\delta').\end{aligned}$$

By Trig., Art. 72.

Let us also put $c=\dfrac{\text{sin. }p\text{ sin. }\phi'}{\text{sin. }b}$.

Then $\text{sin. }\pi=c\text{ sin. }(b-\delta+\pi)\quad.\ .\ .\ .\ .\ .\ .\quad(2)$

But by Trig., Art. 72,

$$\text{sin. }(b-\delta+\pi)=\text{sin. }(b-\delta)\text{ cos. }\pi+\text{cos. }(b-\delta)\text{ sin. }\pi.$$

Therefore

$$\sin.\ \pi = c\ \sin.\ (b-\delta)\ \cos.\ \pi + c\ \cos.\ (b-\delta)\ \sin.\ \pi.$$

Dividing by cos. π, we have

$$\text{tang.}\ \pi = c\ \sin.\ (b-\delta) + c\ \cos.\ (b-\delta)\ \text{tang.}\ \pi.$$

Whence

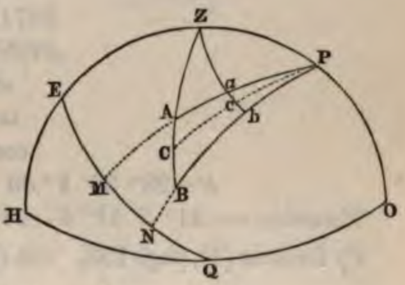
$$\text{tang.}\ \pi = \frac{c\ \sin.\ (b-\delta)}{1-c\ \cos.\ (b-\delta)}\ \ldots\ldots\ldots\ldots\ (3)$$

Developing this formula in a series, as in Art. 205, we obtain

$$\pi = \frac{c\ \sin.\ (b-\delta)}{\sin.\ 1''} + \frac{c^2\ \sin.\ 2(b-\delta)}{\sin.\ 2''} + \frac{c^3\ \sin.\ 3(b-\delta)}{\sin.\ 3''} +, \text{etc.}\ (4)$$

Equation (2) will furnish the parallax in declination when we know the apparent declination (as affected by parallax). But when we know only the true declination, we must employ equation (1), or (3), or (4).

(214.) The auxiliary angle b, introduced in equation (D), has a geometrical signification.



If we draw the arc PC, bisecting the angle APB, we shall have APC $=\frac{1}{2}\Pi$, and ZPC $=h+\frac{1}{2}\Pi$, whence equation (D) becomes

$$\cot.\ b = \cos.\ \text{ZPC}\,.\,\text{tang. PZ sec. APC},$$

or

$$\cot.\ b\ \cos.\ \text{APC} = \text{tang. PZ cos. ZPC}\ \ldots\ .\ (1)$$

If we draw Zb perpendicular to PC, Pab will be an isosceles triangle, and we shall have, by Napier's rule,

$$\text{tang. P}c = \text{tang. P}a\ \cos.\ \text{APC} = \text{tang. PZ cos. ZPC}\ .\ (2)$$

Comparing equations (1) and (2), we see that the arc b is the complement of Pa, or the arc b is equal to the declination of the point a. If we produce the arcs PA and PB to meet the equator EQ, then aM or bN will represent the arc b.

But AM $=\delta$.

Therefore A$a=b-\delta$.

Also, BN $=\delta'=\delta-\pi$.

Therefore B$b=b-(\delta-\pi)=b-\delta+\pi$.

Also, by Spherical Trigonometry, Art. 215,

sin. ZaA : sin. ZA :: sin. AZa : sin. Aa,

and sin. ZB : sin. ZbB :: sin. Bb : sin. BZb.

Therefore, since Pab is an isosceles triangle, we have

$$\sin. \text{ZB} : \sin. \text{ZA} :: \sin. \text{B}b : \sin. \text{A}a,$$

or

$$\frac{\sin. z'}{\sin. z} = \frac{\sin. (b-\delta+\pi)}{\sin. (b-\delta)}.$$

This equation will be employed in Art. 218, page 200.

Ex. 1. Find the moon's parallax in declination for the High School Observatory, Philadelphia, when the horizontal parallax of the place is 59′ 36″.8, the moon's Dec. 24° 5′ 11″.6 N., the moon's hour angle 61° 10′ 47″.4, and the parallax in right ascension 44′ 17″.1.

Solution.—By formula (1), page 191,

$$\begin{aligned}
\sin. p &= 8.2390478 \\
\sin. \phi' &= 9.8059193 \\
\text{cosec. } \delta &= 10.3892161 \\
.02717585 &= 8.4341832 \\
.97282415 &= 9.9880344 \\
\sin. h' &= 9.9456035 \\
\text{tang. } \delta &= 9.6503464 \\
\text{cosec. } h &= 10.0574279 \\
\delta' = 23^\circ\ 39'\ 1''.50 \text{ tang.} &= 9.6414122
\end{aligned}$$

Therefore $\pi = 24^\circ\ 5'\ 11''.6 - 23^\circ\ 39'\ 1''.50 = 26'\ 10''.1$.

By formula (4), page 193,

$$\begin{aligned}
\cos. (h+\tfrac{1}{2}\Pi) &= 9.677980 \\
\cot. \phi' &= 0.079834 \\
\sec. \tfrac{1}{2}\Pi &= 0.000009 \\
b = 60^\circ\ 12'\ 22''.2 \text{ cot.} &= 9.757823 \\
\delta &= 24^\circ\ \ 5'\ 11''.6 \\
b-\delta &= 36^\circ\ \ 7'\ 10''.6
\end{aligned}$$

$$\begin{array}{rrr}
\sin. p = 8.239048 & & \\
\sin. \phi' = 9.805919 & & \\
\text{cosec. } b = 0.061571 & c^2 = 6.2131 & c^3 = 4.320 \\
c = 8.106538 & \sin. 2(b-\delta) = 9.9788 & \sin. 3(b-\delta) = 9.977 \\
\sin. (b-\delta) = 9.770464 & \text{cosec. } 2'' = 5.0134 & \text{cosec. } 3'' = 4.837 \\
\text{cosec. } 1'' = 5.314425 & 16''.04 = 1.2053 & 0''.14 = \bar{9}.134 \\
1553''.91 = 3.191427 & &
\end{array}$$

$$\pi = 1553''.91 + 16''.04 + 0''.14 = 26'\ 10''.1.$$

Therefore the moon's apparent declination is

$$\begin{aligned}
&24^\circ\ \ 5'\ 11''.6 - 26'\ 10''.1 \\
&= 23^\circ\ 39'\ \ 1''.5 \text{ N.}
\end{aligned}$$

With this declination the parallax may be computed by formula (2), page 192, thus:

$$\begin{aligned} c&=8.106538\\ \sin.\ (b-\delta')&=9.774963\\ \sin.\ 26'\ 10''.1&=\overline{7.881501} \end{aligned}$$

Ex. 2. Find the moon's parallax in declination for the High School Observatory, Philadelphia, when the horizontal parallax of the place is 57′ 7″.5, the moon's Dec. 26° 23′ 3″.6 N., the moon's hour angle 32° 39′ 49″.5, and the parallax in right ascension 26′ 46″.5.

Solution.—By formula (4),

$$\begin{aligned} \cos.\ (h+\tfrac{1}{2}\Pi)&=9.924147\\ \cot.\ \phi'&=0.079834\\ \sec.\ \tfrac{1}{2}\Pi&=0.000003\\ b=44^\circ\ 44'\ 13''.7\ \cot.&=\overline{0.003984}\\ \delta&=26^\circ\ 23'\ \ 3''.6\\ b-\delta&=\overline{18^\circ\ 21'\ 10''.1} \end{aligned}$$

$$\begin{aligned} \sin.\ p&=8.220532\\ \sin.\ \phi'&=9.805919\\ \operatorname{cosec.}\ b&=0.152517 \end{aligned}$$

$$\begin{array}{rlrlrl} c=&\overline{8.178968} & c^2=&6.3579 & c^3=&4.537\\ \sin.\ (b-\delta)=&9.498128 & \sin.2(b-\delta)=&9.7765 & \sin.3(b-\delta)=&9.914\\ \operatorname{cosec.}\ 1''=&5.314425 & \operatorname{cosec.}\ 2''=&5.0134 & \operatorname{cosec.}\ 3''=&4.837\\ 980''.67=&\overline{2.991521} & 14''.06=&\overline{1.1478} & 0''.19=&\overline{9.288} \end{array}$$

$$\pi=980''.67+14''.06+0''.19=16'\ 34''.92.$$

Therefore the moon's apparent declination is

$$\begin{aligned} &26^\circ\ 23'\ \ 3''.6-16'\ 34''.9\\ &=26^\circ\ \ 6'\ 28''.7\ \text{N.} \end{aligned}$$

With this declination the parallax may be computed by formula (2), thus:

$$\begin{aligned} c&=8.178968\\ \sin.\ (b-\delta')&=9.504392\\ \sin.\ 16'\ 34''.92&=\overline{7.683360} \end{aligned}$$

Ex. 3. Find the moon's parallax in declination for Western Reserve College, Ohio, when the horizontal parallax of the place is 59′ 36″.5, the moon's Dec. 24° 4′ 41″.7 N., and hour angle 68° 9′ 51″.9; and the parallax in right ascension 45′ 56″.5.

Ans. 29′ 17″.9.

Ex. 4. Find the moon's parallax in declination for Western Reserve College, Ohio, when the horizontal parallax of the place is 57′ 7″.7, the moon's Dec. 26° 24′ 31″.5 N.; the hour angle is 23° 13′ 12″.0, and the parallax in right ascension 19′ 12″.6.

Ans. 16′ 15″.8.

The effect of parallax is always to increase the hour angle, or the angular distance of the moon from the meridian; hence, when the moon is on the eastern side of the meridian, the parallax in right ascension increases the true right ascension of the moon; but when the moon is on the western side of the meridian, the parallax diminishes the right ascension. The parallax in declination increases the distance of the moon from the north pole in both situations.

In the computation of occultations of stars by the moon, it is convenient to know the *change* which the parallaxes undergo in a given interval of time, as, for example, in one hour. This may be effected by differentiating the expressions already obtained for the parallaxes.

PROBLEM.

(215.) *To find the hourly variation of the parallax in right ascension.*

Equation (1), of Art. 211, is

$$\sin. \Pi = \frac{\sin. p \cos. \phi' \sin. h'}{\cos. \delta}.$$

Since the arcs Π and p are in all cases small, they will differ but little from their sines, and sin. h' differs but little from sin. h; we will therefore employ the more convenient formula,

$$\Pi = \frac{p \cos. \phi' \sin. h}{\cos. \delta}.$$

In this formula h is the only quantity which, by its rapid variation, has any important influence on the quantity sought. Hence, regarding h as the only variable, we obtain

$$d\Pi = \frac{p \cos. \phi' \cos. h dh}{\cos. \delta}.$$

The differential of h must be taken in parts of radius. If the variation is required for one hour, dh will represent the arc of 15°, which is .2617994, radius being unity.

Ex. 1. Find the hourly variation of the moon's parallax in right ascension for Cambridge Observatory, whose geocentric latitude is 42° 11′ 21″, when the horizontal parallax of the place is 57′, the moon's Dec. 25°, and the hour angle 50°.

Solution.

$$\begin{aligned} p=57'=3420''&=3.534026\\ \cos.\ \phi'&=9.869778\\ \cos.\ h&=9.808067\\ dh=.2617994&=9.417969\\ \sec.\ \delta&=0.042724\\ 470''.5&=\overline{2.672564} \end{aligned}$$

Ex. 2. Find the hourly variation of the moon's parallax in right ascension for Cambridge Observatory, when the horizontal parallax of the place is 61′, the moon's Dec. 20°, and the hour angle 15°. *Ans.* 729″.8.

PROBLEM.

(216.) *To find the hourly variation of the parallax in declination.*

Equation (C), of Art. 213, is

$$\sin.\ \pi=\sin.\ p\ \sin.\ \phi'\ \cos.\ \delta'-\sin.\ p\ \cos.\ \phi'\ \cos.\ h\ \sec.\ \tfrac{1}{2}\Pi\ \sin.\ \delta'.$$

Substituting the arcs π and p for their sines, and using δ in place of δ', we obtain the following more convenient formula, which affords an approximate value of π,

$$\pi=p\ \sin.\ \phi'\ \cos.\ \delta-p\ \cos.\ \phi'\ \cos.\ h\ \sin.\ \delta.$$

Differentiating this formula, regarding h as the only variable, we obtain

$$d\pi=p\ \cos.\ \phi'\ \sin.\ \delta\ \sin.\ hdh.$$

Ex. 1. Find the hourly variation of the moon's parallax in declination for Cambridge Observatory, when the horizontal parallax of the place is 57′ the moon's Dec. 25°, and the hour angle 50°.

Solution.

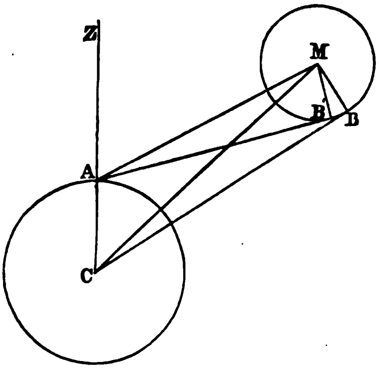
$$\begin{aligned} p&=3.534026\\ \cos.\ \phi'&=9.869778\\ \sin.\ \delta&=9.625948\\ \sin.\ h&=9.884254\\ dh&=9.417969\\ 214''.8&=\overline{2.331975} \end{aligned}$$

Ex. 2. Find the hourly variation of the moon's parallax in declination for Cambridge Observatory, when the horizontal parallax of the place is 61′, the moon's Dec. 20°, and the hour angle 15°. *Ans.* 62″.8.

(217.) The apparent diameter of the moon is the angle which its disk subtends. This angle is not the same for all points of the earth, on account of their different distances from the moon. As the moon rises above the horizon (if we suppose its distance from the centre of the earth to remain constant), its distance from the place of observation must diminish while its altitude increases, and, consequently, its apparent diameter must increase.

PROBLEM.

To find the augmentation of the moon's semi-diameter on account of its altitude above the horizon.



Let C and M be the centres of the earth and moon, and A a point on the earth's surface. The semi-diameter of the moon, as seen from C, is the angle BCM; but the semi-diameter, as seen from A, is the angle B′AM.

Represent the angle BCM by *S*; the angle B′AM by *S′*; the angle ZCM by Z, and the angle ZAM by Z′.

Then, in the right-angled triangle BCM, we have

$$\sin.\ \mathrm{BCM} = \sin.\ S = \frac{\mathrm{BM}}{\mathrm{CM}}.$$

Also, in the triangle B′AM, we have

$$\sin.\ \mathrm{B'AM} = \sin.\ S' = \frac{\mathrm{B'M}}{\mathrm{AM}}.$$

Hence $\sin.\ S : \sin.\ S' :: \frac{\mathrm{BM}}{\mathrm{CM}} : \frac{\mathrm{B'M}}{\mathrm{AM}} :: \mathrm{AM} : \mathrm{CM}.$

But in the triangle CAM we have

$$\mathrm{AM} : \mathrm{CM} :: \sin.\ \mathrm{ACM} : \sin.\ \mathrm{CAM} :: \sin.\ \mathrm{Z} : \sin.\ \mathrm{Z'}.$$

Therefore $\sin.\ S : \sin.\ S' :: \sin.\ \mathrm{Z} : \sin.\ \mathrm{Z'},$

or
$$\text{sin. } S' = \frac{\text{sin. } S \text{ sin. Z}'}{\text{sin. Z}} \quad . \; . \; . \; . \; . \; . \; . \; \text{(A)}$$

But since S never amounts to 17′, we may substitute the arc for its sine, and we obtain

$$S' = \frac{S\,.\,\text{sin. Z}'}{\text{sin. Z}}.$$

Hence
$$S' - S = S\,.\,\frac{\text{sin. Z}' - \text{sin. Z}}{\text{sin. Z}},$$

which represents the augmentation of the moon's semi-diameter, as seen from a point on the earth's surface instead of its centre; Z being the zenith distance of the moon viewed from the centre, and Z′ the zenith distance as seen from the surface.

But by Trig., Art. 75,

$$\text{sin. Z}' - \text{sin. Z} = 2 \text{ sin. } \tfrac{1}{2}(\text{Z}' - \text{Z}) \text{ cos. } \tfrac{1}{2}(\text{Z}' + \text{Z}).$$

Hence

$$x = S' - S = \frac{2S}{\text{sin. Z}}\,.\,\text{sin. } \tfrac{1}{2}(\text{Z}' - \text{Z}) \text{ cos. } \tfrac{1}{2}(\text{Z}' + \text{Z}).$$

If we represent the parallax in altitude by q, we shall have

$$\text{Z} = \text{Z}' - q.$$

Hence
$$x = \frac{2S}{\text{sin. } (\text{Z}' - q)} \text{ sin. } \tfrac{1}{2}\, q \text{ cos. } (\text{Z}' - \tfrac{1}{2}q).$$

But since the angle q is always small, we may, without sensible error, put q for sin. q, and make cos. q equal to unity. Hence

$$x = \frac{S\,.\,q \text{ cos. } (\text{Z}' - \tfrac{1}{2}q)}{\text{sin. } (\text{Z}' - q)}.$$

But by Trig., Art. 72,

$$\text{cos. } (\text{Z}' - \tfrac{1}{2}q) = \text{cos. Z}' \text{ cos. } \tfrac{1}{2}q + \text{sin. Z}' \text{ sin. } \tfrac{1}{2}q.$$

Also, $\text{sin. } (\text{Z}' - q) = \text{sin. Z}' \text{ cos. } q - \text{cos. Z}' \text{ sin. } q.$

Hence
$$x = \frac{S\,.\,q\,(\text{cos. Z}' + \tfrac{1}{2}q \text{ sin. Z}')}{\text{sin. Z}' - q \text{ cos. Z}'}.$$

But, according to Burckhardt's Tables of the Moon, we have

$$S : p :: 1 : 3.6697.$$

If we represent 3.6697 by k, then

$$p = k\,.\,\text{S}.$$

And by Art. 204,

$$q = k\,.\,S \text{ sin. Z}'.$$

Therefore

$$x = \frac{k\,.\,S^2 \text{ sin. Z}' \,(\text{cos. Z}' + \tfrac{1}{2}k\,.\,S \text{ sin. }^2\text{Z}')}{\text{sin. Z}' - k\,.\,S \text{ sin. Z}' \text{ cos. Z}'},$$

or $$x=\frac{k.S^2(\text{cos. Z}'+\tfrac{1}{2}k.S \text{ sin. }^2\text{Z}')}{1-k.S \text{ cos. Z}'}.$$

Dividing the numerator by the denominator in order to develop this expression into a series, we obtain

$$x=k.S^2 \text{ cos. Z}'+\tfrac{1}{2}k^2S^3+\tfrac{1}{2}k^2.S^3 \text{ cos. }^2\text{Z}'+\text{, etc.}$$

If we put $\text{A}=k \text{ sin. } 1''=0.00001779$, we shall have for the augmentation expressed in seconds,

$$x=\text{A}S^2 \text{ cos. Z}'+\tfrac{1}{2}\text{A}^2S^3+\tfrac{1}{2}\text{A}^2S^3 \text{ cos. }^2\text{Z}'+\text{, etc. } . \quad (1)$$

By this formula was computed Table XIII., by which the augmentation of the moon's semi-diameter may be obtained by inspection.

(218.) When the parallax in declination has been previously computed, the following method is preferable:

By Art. 214, page 194,

$$\frac{\text{sin. Z}'}{\text{sin. Z}}=\frac{\text{sin. }(b-\delta+\pi)}{\text{sin. }(b-\delta)}.$$

Hence, from equation (A), page 199,

$$\text{sin. }S'=\frac{\text{sin. }S \text{ sin. }(b-\delta+\pi)}{\text{sin. }(b-\delta)},$$

and $$\text{sin. }S'-\text{sin. }S=\frac{\text{sin. }S\{\text{sin. }(b-\delta+\pi)-\text{sin. }(b-\delta)\}}{\text{sin. }(b-\delta)}.$$

But by Trig., Art. 75,

$$\text{sin. }(b-\delta+\pi)-\text{sin. }(b-\delta)=2 \text{ sin. } \tfrac{1}{2}\pi \text{ cos. }(b-\delta+\tfrac{1}{2}\pi).$$

Therefore

$$\text{sin. }S'-\text{sin. }S=\frac{\text{sin. }S.2 \text{ sin. } \tfrac{1}{2}\pi \text{ cos. }(b-\delta+\tfrac{1}{2}\pi)}{\text{sin. }(b-\delta)}.$$

But since the arcs S, S', and π are very small, we may put

$$S=\text{sin. }S, \text{ and } 2 \text{ sin. } \tfrac{1}{2}\pi=\text{sin. }\pi.$$

Hence $$x=S'-S=\frac{S \text{ sin. } \pi \text{ cos. }(b-\delta+\tfrac{1}{2}\pi)}{\text{sin. }(b-\delta)}$$

But by Trig., Art. 72,

$$\text{cos. }(b-\delta+\tfrac{1}{2}\pi)=\text{cos. }(b-\delta) \text{ cos. } \tfrac{1}{2}\pi-\text{sin. }(b-\delta) \text{ sin. } \tfrac{1}{2}\pi.$$

Hence

$$x=\frac{S.\text{sin. }\pi\,\{\text{cos. }(b-\delta) \text{ cos. } \tfrac{1}{2}\pi-\text{sin. }(b-\delta) \text{ sin. } \tfrac{1}{2}\pi\}}{\text{sin. }(b-\delta)},$$

or $$x=S \text{ sin. } \pi \text{ cot. }(b-\delta) \text{ cos. } \tfrac{1}{2}\pi-S \text{ sin. } \pi \text{ sin. } \tfrac{1}{2}\pi.$$

If we assume cos. $\tfrac{1}{2}\pi$ equal to unity, and sin. $\tfrac{1}{2}\pi$ equal to $\tfrac{1}{2}$ sin. π, we shall have

$$x=S \text{ sin. } \pi \text{ cot. }(b-\delta)-\tfrac{1}{2}S \text{ sin. }^2\pi \quad . \; . \; . \; . \quad (2)$$

When we know the moon's apparent altitude, we may compute its apparent diameter by equation (1), or take it directly from Table XIII.; but when the parallax in declination has been computed, it is better to employ equation (2.) The value of $(b-\delta)$ is obtained by Art. 213, page 192.

Ex. 1. Calculate the augmentation of the moon's semi-diameter when its true semi-diameter is 16′ 30″, and its apparent altitude 66°.

Solution.—By equation (1), page 200,

$$\begin{array}{rl@{\qquad}rl}
16'\,30''=990''= & 2.99564 & S^3= & 8.9869\\
 & 2 & \mathrm{A}^2= & 0.5004\\
S^2= & \overline{5.99128} & 0.5= & 9.6990\\
\mathrm{A}= & 5.25021 & 0''.15= & \overline{9.1863}\\
\text{cos. } \mathrm{Z}'= & 9.96073 & \text{cos. } {}^2\mathrm{Z}'= & 9.9215\\
15''.93= & \overline{1.20222} & 0''.13= & \overline{9.1078}
\end{array}$$

Hence $15''.93+0''.15+0''.13=16''.21$, the augmentation, the same as given in Table XIII.

Ex. 2. Calculate the augmentation of the moon's semi-diameter in *Ex.* 1, Art. 214, when the horizontal semi-diameter is 16′ 16″.0.

Solution.—By equation (2), page 200,

$$\begin{array}{rl@{\qquad}rl}
16'\,16''=976''= & 2.98945 & 488''= & 2.688\\
\pi=26'\,10''.1\quad \text{sin.}= & 7.88150 & \text{sin. } {}^2\pi= & 5.763\\
b-\delta=36^\circ\,7'\,11''\ \ \text{cot.}= & 0.13683 & 0''.03= & \overline{8.451}\\
10''.18= & \overline{1.00778} & &
\end{array}$$

Hence the augmentation $=10''.18-0''.03=10''.15$.

Ex. 3. Calculate the augmentation of the moon's semi-diameter in *Ex.* 2, Art. 214, when the horizontal semi-diameter is 15′ 37″.1. *Ans.* 13″.6.

When the moon's hour angle is known, its altitude may be taken from a celestial globe with sufficient precision to furnish the augmentation of its semi-diameter within one or two tenths of a second, by means of Table XIII.

Ex. 4. Calculate the augmentation of the moon's semi-diameter in *Ex.* 3, Art. 214, when the horizontal semi-diameter is 16′ 16″.0. *Ans.* 8″.84.

CHAPTER IX.

MISCELLANEOUS PROBLEMS.

INTERPOLATION BY DIFFERENCES.

(219.) IT is frequently required, from a series of equidistant terms following any law whatever, to deduce some intermediate term. Thus, in the Nautical Almanac, we have given the moon's right ascension for every hour of the day, and from these data it may be required to determine its right ascension for some intermediate instant. This is effected by interpolation. The quantities upon which the values of the given magnitudes depend are called *Arguments.* Time is generally the argument in astronomical tables.

Let $\qquad a_{,,,}\ a_{,,}\ a_{,}\ a_0\ a'\ a''\ a'''$

be the given places of the moon, corresponding to the times

$$\mathrm{T}-3h,\ \mathrm{T}-2h,\ \mathrm{T}-h,\ \mathrm{T},\ \mathrm{T}+h,\ \mathrm{T}+2h,\ \mathrm{T}+3h,$$

where h may represent any interval of time at pleasure. These places may be right ascensions or declinations, longitudes or latitudes, or magnitudes of any other kind. Subtract the first term of the series from the second, the second from the third, and so on, giving to each remainder the sign which results from the rules of algebra; and let the first order of differences be represented by $b_{,,,}$ $b_{,,}$ $b_{,}$, etc. Subtract each of these first differences from the one next below it, for a second order of differences, paying attention to the signs, and represent these differences by $c_{,,,}$ $c_{,,}$ $c_{,}$, etc., and proceed in the same manner for the third, fourth, etc., orders of differences, as represented in the following table:

Time or Argument.	Quantities.	1st Diff.	2d Diff.	3d Diff.	4th Diff.	5th Diff.	6th Diff.
T−3h	$a_{\prime\prime\prime}$						
		$b_{\prime\prime\prime}$					
T−2h	$a_{\prime\prime}$		$c_{\prime\prime\prime}$				
		$b_{\prime\prime}$		$d_{\prime\prime\prime}$			
T− h	$a_{\prime}$		$c_{\prime\prime}$		$e_{\prime\prime\prime}$		
		$b_{\prime}$		$d_{\prime\prime}$		$f_{\prime\prime\prime}$	
T	a_0		$c_{\prime}$		$e_{\prime\prime}$		$g_{\prime\prime\prime}$
		b_0		$d_{\prime}$		$f_{\prime\prime}$	
T+ h	a'		c_0		$e_{\prime}$		$g_{\prime\prime}$
		b'		d_0		$f_{\prime}$	
T+2h	a''		c'		e_0		
		b''		d'			
T+3h	a'''		c''				
		b'''					
T+4h	a''''						

If we put $a^{(t)}$ to represent that term of the series which follows a_0 at the interval t, then, as shown in Algebra, Art. 297, we shall have

$$a^{(t)}=a_0+t\,.\,b_0+\frac{t(t-1)}{2}\,.\,c_0+\frac{t(t-1)\,(t-2)}{2\,.\,3}\,.\,d_0+,\text{ etc. } \quad \text{(A)}$$

Ex. 1. Given the moon's right ascension as follows:

Date.	Right Ascension.	1st Difference.	2d Difference.
1855. h.	h. m. s.	m. s.	s.
February 1, 0	8 34 36.65		
		+2 5.65	
1	8 36 42.30		−0.21
		+2 5.44	
2	8 38 47.74		−0.21
		+2 5.23	
3	8 40 52.97		−0.23
		+2 5.00	
4	8 42 57.97		

Required the moon's right ascension at February 1, 0h. 15m.

Here a_0=8h. 34m. 36.65s.; b_0=+2m. 5.65s.; c_0=−0.21s.; and t=15m.=.25 in parts of an hour. Therefore

$$a^{(t)}=\text{8h. 34m. 36.65s.}+125.65\times.25+\frac{0.21}{2}\times.25\times.75$$
$$=\text{8h. 34m. 36.65s.}+\text{31.41s.}+\text{0.02s.}$$
$$=\text{8h. 35m. 8.08s.}$$

Table XXIII., page 393, gives the coefficients of each order of differences for every hundredth part of the unit of time elapsing between the given terms of the series. In the preceding example the second differences are sensibly constant. In the following example the numbers appear more irregular.

Ex. 2. Given the right ascension of the moon's limb for the upper and lower transit at Washington, as follows:

Date.	Right Ascens.	1st Diff.	2d Diff.	3d Diff	4th Diff.	5th Diff.
1855.	h. m. s.	m. s.	s.	s.	s.	s.
July 3, L. T.	22 37 59.54					
		+27 57.01				
U. T.	23 5 56.55		−52.95			
		+27 4.06		+10 27		
4, L. T.	23 33 0.61		−42.68		+0.64	
		+26 21 38		+10 91		−0.63
U. T.	23 59 21.99		−31.77		+0.01	
		+25 49.61		+10.92		−0.40
5, L. T.	0 25 11.60		−20.85		−0.39	
		+25 28.76		+10.53		−0.17
U. T.	0 50 40.36		−10.32		−0.56	
		+25 18.44		+ 9.97		
6, L. T.	1 15 58.80		− 0.35			
		+25 18.09				
U. T.	1 41 16 89					

to find the moon's right ascension, July 3, at its transit over a place one hour west of Washington.

$$a^{(t)}=22\text{h. }37\text{m. }59.54\text{s.}+\frac{1677.01\text{s.}}{12}+\frac{52.95\text{s.}\times 11}{2\times 12\times 12}+\frac{10.27\text{s.}\times 11\times 23}{6\times 12\times 12\times 12}$$
$$-\frac{0.64\text{s.}\times 11\times 23\times 35}{24\times 12\times 12\times 12\times 12}-\frac{0.63\text{s.}\times 11\times 23\times 35\times 47}{120\times 12\times 12\times 12\times 12\times 12},$$

or

$$a^{(t)}=22\text{h. }37\text{m. }59.54\text{s.}+139.75\text{s.}+2.06+0.25-0.01-0.01$$
$$=22\text{h. }40\text{m. }21.58\text{s.}$$

(220.) It will generally be found more convenient in practice to take the coefficients for the several orders of differences directly from Table XXIII. It will be observed that the coefficients of the second and fourth differences are negative, while those of the odd differences are positive. Hence the corrections for the odd differences will have the *same* sign as those differences; but the corrections for the even differences will have a sign *contrary* to those differences. Hence, in the above example, the corrections for the first, second, and third differences are positive, while the other two corrections are negative.

(221.) Formula (A) proceeds from values which belong to a less argument, to those which belong to a greater argument;

but we are at liberty to proceed in the reverse order. Conceive the times and quantities given on page 203 to be written in an inverted order, so that the table shall begin with the last value, a'''', and end with the first, $a_{,,,}$. The first differences would then be $a'''-a''''=-b'''$; $a''-a'''=-b''$, etc. That is, the first differences would be the same as given in the preceding table, but with contrary signs. The second differences would be $-b''-(-b''')=b'''-b''=+c''$; that is, the second differences would retain the same signs as before. The third differences, $c'-c''=-d'$, etc., change their signs, while the fourth differences remain unchanged, and so on; that is, the differences of an odd order have their signs reversed.

Suppose now that t is a proper fraction, representing the distance of the term $a^{(t)}$ from $a_{\circ}$; then the first, second, third, fourth, etc., differences corresponding will be $-b_{\circ}$; $+c_{,}$; $-d_{,,}$; $+e_{,,,}$, etc.; and consequently,

$$a^{(t)}=a'-(1-t)b_{\circ}+\frac{(1-t)(1-t-1)}{2}c_{,}$$

$$-\frac{(1-t)(1-t-1)(1-t-2)}{2.3}d_{,,}+,\text{ etc.},$$

or

$$a^{(t)}=a'+(t-1)b_{\circ}+\frac{t(t-1)}{2}c_{,}+\frac{t(t-1)(t+1)}{2.3}d_{,,}$$

$$+\frac{t(t-1)(t+1)(t+2)}{2.3.4}e_{,,,}+,\text{ etc.}\quad\ldots\ldots\ldots\quad(\text{B})$$

(222.) Equations (A) and (B) are each of them only approximate; but when the error of one is positive, the error of the other will generally be negative, so that we shall obtain a more accurate expression if we take the half sum of both (A) and (B), and we shall have

$$a^{(t)}=\frac{a_{\circ}+a'}{2}+t.b_{\circ}-\frac{b_{\circ}}{2}+\frac{t(t-1)}{2}\left(\frac{c_{\circ}+c_{,}}{2}\right)$$

$$+\frac{t(t-1)}{6}\left\{\frac{(t-2)d_{\circ}+(t+1)d_{,,}}{2}\right\}+,\text{ etc.}$$

But $$a'-b_{\circ}=a_{\circ};$$

and consequently,

$$\frac{a_{\circ}+a'}{2}-\frac{b_{\circ}}{2}=a_{\circ}.$$

Also, $$d_{\circ}=d_{,}+e_{,};\ d_{,,}=d_{,}-e_{,,},\text{ etc.}$$

Substituting these values in the preceding equation, we obtain

$$a^{(t)}=a_{\circ}+t\,.\,b_{\circ}+\frac{t(t-1)}{2}\left(\frac{c_{\circ}+c_{,}}{2}\right)+\frac{t(t-1)\,(t-\frac{1}{2})}{2\,.\,3}d_{,}$$
$$+\frac{(t+1)t(t-1)\,(t-2)}{2\,.\,3\,.\,4}\left(\frac{e_{,}+e_{,,}}{2}\right)+\text{, etc. } \ldots\ldots \quad \text{(C)}$$

Since t is supposed to be included between 0 and 1, it is plain that the coefficients of the third, fourth, etc., differences in formula (C) are smaller than in formulas (A) and (B); that is, this series converges most rapidly.

It will readily be perceived that in formula (A) the first, second, third, etc., differences in the table on page 203 lie in a diagonal, which starts from between $a_{\circ}$ and a' and inclines downward. In formula (B), on the contrary, they lie in a diagonal which inclines upward; while in formula (C) the odd differences are intersected by a horizontal line, which starts from between $a_{\circ}$ and a'; but for the even differences we employ the half sum of that which lies above and that which is below the horizontal line.

(223.) If we represent the coefficients of the several orders of differences by B, C, D, etc., formula (C) may be written

$$a^{(t)}=a_{\circ}+\mathrm{B}b+\mathrm{C}c+\mathrm{D}d+\mathrm{E}e+\mathrm{F}f\text{, etc. } \ldots \quad \text{(D)}$$

where we put $b=b_{\circ}$

$$c=\frac{c_{\circ}+c_{,}}{2}$$
$$d=d_{,}$$
$$e=\frac{e_{,}+e_{,,}}{2}$$
$$f=f_{,,}$$
$$\mathrm{B}=t$$
$$\mathrm{C}=\frac{t(t-1)}{2}$$
$$\mathrm{D}=\frac{t(t-1)\,(t-\frac{1}{2})}{2\,.\,3}$$
$$\mathrm{E}=\frac{(t+1)t(t-1)\,(t-2)}{2\,.\,3\,.\,4}$$
$$\mathrm{F}=\frac{(t+1)t(t-1)\,(t-2)\,(t-\frac{1}{2})}{2\,.\,3\,.\,4\,.\,5}\text{, etc.}$$

This formula is the one recommended by Professor Bessel.

Table XXIII., page 392, gives the values of the preceding coefficients for every hundredth part of the unit of time. It will be observed that the coefficient of the second differences is invariably negative; but for values of t less than .50, the coefficients of the third and fourth differences are positive, and the fifth negative; while for values of t greater than .50, the coefficients of the third differences are negative, but the fourth and fifth are positive.

Example. Required the moon's right ascension for July 10, 1855, at 8h. mean time.

Take from the Almanac three places of the moon preceding and three places following the proposed time, and find their differences as in the following table:

Date.	R. A.	1st Diff.	2d Diff.	3d Diff.	4th Diff.	5th Diff.
h.	*h. m. s.*	*m. s.*	*s.*	*s.*	*s.*	*s.*
July 9, 0	3 20 56.61					
		+26 5.41				
12	3 47 2.02		+27.57			
		+26 32.98		−1.89		
" 10, 0	4 13 35.00		+25.68		−2.19	
		+26 58.66		−4.08		−0.06
12	4 40 33.66		+21.60		−2.25	
		+27 20.26		−6.33		
" 11, 0	5 7 53.92		+15.27			
		+27 35.53				
12	5 35 29.45					

For July 10, at 0h., which is the date next preceding the one proposed, we find

$$a_{\circ}=4\text{h. }13\text{m. }35.00\text{s.};\ b=26\text{m. }58.66\text{s.}$$

$$c_{\circ}=21.60\text{s.};\ c_{,}=25.68\text{s.};\ \because c=\tfrac{1}{2}(c_{\circ}+c_{,})=23.64\text{s.}$$

$$d=-4.08\text{s.};\ f=-0.06\text{s.}$$

$$e_{,}=-2.25\text{s.};\ e_{,,}=-2.19\text{s.};\ \because e=\tfrac{1}{2}(e_{,}+e_{,,})=-2.22\text{s.}$$

The difference between the proposed time and July 10 at 0h. is 8h., which is two thirds of the interval between the dates in the table. Therefore we have

$$\begin{aligned}
\text{B}b &= \tfrac{2}{3}(26\text{m. }58.66\text{s.}) = +17\text{m. }59.107\text{s.}\\
\text{C}c &= -\tfrac{1}{9}(23.64\text{s.}) = -2.627\text{s.}\\
\text{D}d &= -.00617\times-4.08\text{s.} = +\ .025\text{s.}\\
\text{E}e &= +.02057\times-2.22\text{s.} = -\ .046\text{s.}\\
\text{F}f &= +.00069\times-0.06\text{s.} = .000\text{s.}\\
\hline
\text{B}b+\text{C}c+\text{D}d+\text{E}e+\text{F}f &= +17\text{m. }56.46\text{s.}\\
a_{\circ} &= 4\text{h. }13\text{m. }35.00\text{s.}\\
\hline
\text{Moon's R. A. at 8h.} &= 4\text{h. }31\text{m. }31.46\text{s.}
\end{aligned}$$

(224.) Most of the numbers in the Nautical Almanac are computed for intervals of either 12 or 24 hours. The right ascension of the moon's bright limb is given for both the upper and lower culminations, that is, for intervals of 12 hours of longitude. If we wish to interpolate for any other meridian, we must consider 12 hours as the unit of time, and it is desirable to have the coefficients computed for convenient fractions of 12 hours. When the computation is performed by logarithms, it is convenient to have the logarithmic coefficients arranged in a table. This has accordingly been done in Table XXIV., which furnishes the logarithms of Bessel's coefficients for every five minutes throughout 12 hours. If the numbers between which we wish to interpolate are given for intervals of 24 hours, as the sun's places in the Nautical Almanac, we may avail ourselves of the same table of coefficients by simply doubling each of the numbers in column first. Thus, when the interval is 12 hours, the coefficients for an argument of one hour will be the same as for an argument of two hours when the interval is 24 hours.

The preceding example is most conveniently solved by the use of these coefficients. Taking the logarithms of the coefficients B, C, D, E, and F from the table, and the logarithms of b, c, d, e, and f as given on the preceding page, we have

$$\begin{array}{lll}
\text{log. B}=9.8239087\,; & \text{log. C}=9.04576n\,; & \text{log. D}=7.79048n\,; \\
\text{log. }b=\underline{3.2091556} & \text{log. }c=\underline{1.37365} & \text{log. }d=\underline{0.61066n} \\
\qquad 3.0330643 & \qquad 0.41941n & \qquad 8.40114 \\
\text{Nat. num.}+1079.107\text{s.} & \qquad -2.627\text{s.} & \qquad +0.025\text{s.}
\end{array}$$

$$\begin{array}{ll}
\text{log. E}=8.31336\,; & \text{log. F}=6.83624 \\
\text{log. }e=\underline{0.34635n} & \text{log. }f=\underline{8.77815n} \\
\qquad 8.65971n & \qquad 5.61439n \\
\text{Nat. num.}-0.046\text{s.} & \qquad .000
\end{array}$$

Adding to log. B, log. C, etc., the factors log. b, log. c, etc., we obtain for the several corrections

$$\begin{array}{lr}
\text{B}b = & +17\text{m. }59.107\text{s.} \\
\text{C}c = & -2.627\text{s.} \\
\text{D}d = & +0.025\text{s.} \\
\text{E}e = & -0.046\text{s.} \\
\text{F}f = & \underline{0.000\text{s.}} \\
\text{Sum} = & +17\text{m. }56.46\text{s.}
\end{array}$$

the same as found on page 207.

If we neglect the third and following differences, that is, if we suppose the second differences constant, formula C becomes

$$a^{(t)}=a_{\circ}+t\,.\,b_{\circ}+\frac{t(t-1)}{2}\,\frac{(c_{\circ}+c_{\prime})}{2}.$$

We accordingly take from the Nautical Almanac four consecutive arcs, such that the arc sought may fall between the two middle ones, and for the second difference we employ the mean of the two second differences thus obtained.

In the example on page 207, if we regard only first and second differences, we shall obtain the moon's right ascension,

4h. 31m. 31.48s.
instead of 4h. 31m. 31.46s.

The error arising from neglecting the third and following differences amounts, therefore, only to 0.02s.

PROBLEM.

(225.) *To find the time of conjunction or opposition of the moon with the sun.*

The right ascension of the moon is given in the Nautical Almanac for every hour of the day, and the right ascension of the sun is given for noon of each day. An inspection of these columns will readily show between what hours conjunction or opposition takes place. Take out four successive right ascensions of the moon, such that the phase sought shall fall between the second and third of the hours, and find by interpolation the corresponding right ascension of the sun. For each hour subtract the right ascension of the sun from that of the moon; the differences will represent the distances of the moon from the sun. Then, if only an approximate result is desired, we may determine by a simple proportion when the difference of right ascension amounts to zero, or twelve hours. But if a more accurate result is desired, we must take account of the second differences.

Example.

Required the Washington mean time of opposition in right ascension of the sun and moon, October 24, 1855.

We readily discover by an inspection of the ephemeris that opposition takes place between 19h. and 20h., Greenwich time. We then take from the Nautical Almanac the right ascension of the sun and moon for four successive hours, and find their differences, neglecting the 12 hours, as follows:

Date.	Sun's R. A.	Moon's R. A.	Moon—Sun.	1st Diff.	2d Diff.
h.	h. m. s.	h. m. s.	m. s.	m. s.	s.
18	13 56 30.45	1 53 48.67	−2 41.78		
				+2 3.78	
19	13 56 40.01	1 56 2.01	− 38.00		+0.16
				+2 3.94	
20	13 56 49.57	1 58 15.51	+1 25.94		+0.15
				+2 4.09	
21	13 56 59.13	2 0 29.16	+3 30.03		

If we neglect the second differences, the time of opposition may be found by the proportion

2m. 3.94s. : 3600s. :: 38.00s. : 18m. 23.8s.;

that is, opposition takes place at 19h. 18m. 23.8s., and this result is within half a second of the truth. If greater accuracy is required, we must take account of the second differences, which may be done as follows:

In the formula of interpolation, page 206,

$$a^{(t)}=a_0+t\,.\,b+\frac{t(t-1)}{2}c+, \text{ etc.},$$

t must be regarded as the unknown quantity, all the others being known.

Developing this formula, we have

$$a^{(t)}=a_0+t\,.\,b-\frac{tc}{2}+\frac{t^2c}{2},$$

or

$$a^{(t)}-a_0=t\left(b-\frac{c}{2}+\frac{tc}{2}\right).$$

Now the approximate value of t is $\frac{a^{(t)}-a_0}{b}$. Substituting this value above, we obtain

$$t=\frac{a^{(t)}-a_0}{b-\frac{c}{2}+\frac{c}{2}\cdot\frac{a^{(t)}-a_0}{b}},$$

which is a more accurate value of t. In the present case $a^{(t)}$ becomes zero, because we wish to determine when the difference of right ascension between the sun and moon is zero (neglecting the 12 hours); hence we have

$$t=\frac{-a_0}{b-\frac{c}{2}-\frac{c}{2}\cdot\frac{a_0}{b}},$$

where we must be careful to preserve the proper sign of each term.

Substituting the values of these letters, we have

$$t=\frac{38.00}{123.94-.08+.03}=\frac{38.00}{123.89},$$

where t is expressed in parts of an hour.

Multiplying by 3600 to reduce the result to seconds, we obtain 1104.2s., or 18m. 24.2s.; whence the corrected time of opposition is

19h. 18m. 24.2s. Greenwich mean time,
or 14h. 10m. 13.0s. Washington mean time.

PROBLEM.

(226.) *To find the hourly motion of the moon in right ascension, etc.*

The moon's hourly motion may be found very nearly by taking the difference between two successive numbers in the Nautical Almanac, the one before and the other after the time for which the hourly motion is wanted. If the proposed instant does not fall midway between the two dates in the Almanac, we must apply a correction by taking a proportional part of the second difference.

Example 1.

Required the moon's hourly motion in right ascension, October 24, 1855, at 19h. 18m. 24.2s., Greenwich mean time.

We take from the Nautical Almanac the following numbers:

Date.	Moon's R. A.	1st Diff.	2d Diff.
h.	h. m. s	m. s.	s.
18	1 53 48.67		
		+2 13.34	
19	1 56 2.01		+0.16
		+2 13.50	
20	1 58 15.51		

The change of the moon's right ascension from 18h. to 19h. is 2m. 13.34s., which may be regarded as the hourly motion for 18h. 30m. In the same manner, the hourly motion for 19h. 30m. is 2m. 13.50s. In order to obtain the hourly motion for 19h. 18m. 24.2s., we state the proportion

60m. : 0.16s. :: 48m. 24.2s. : 0.13s.,

which, being added to 2m. 13.34s., gives the hourly motion for 19h. 18m. 24.2s., equal to 2m. 13.47s. in time, or 33′ 22″.05 in arc.

In calculating eclipses, it is necessary to know the hourly motion of the moon from the sun. This is obtained by finding the sun's hourly motion in the manner already explained, and subtracting it from the moon's hourly motion. Thus, if the sun's hourly motion, October 24, was 9.56s., then the hourly motion of the moon from the sun was 2m. 13.47s.−9.56s.=2m. 3.91s. in time, or 30′ 58″.65 in arc.

(227.) The preceding method is slightly inaccurate in principle, and when the interval between the moon's places amounts to 12 hours, the error can not be neglected. An accurate formula for the hourly motion may be obtained by differentiating equation D, page 206. We thus find

$$\frac{d[a^{(t)}]}{dt}=b+\frac{2t-1}{2}c+\frac{3t^2-3t+\frac{1}{2}}{2.3}d+\frac{4t^3-6t^2-2t+2}{2.3.4}e+, \text{ etc.}$$

If the moon's places are given for intervals of 12 hours, and we assume successively $t=0$, $=\frac{1}{12}$, $=\frac{2}{12}$, etc., we shall obtain the hourly motion corresponding to each hour of the interval for which b, c, d, etc., are computed. If we wish the hourly motion for the instant midway between two values of b, we must make $t=\frac{1}{2}$; in which case the above coefficient of c becomes 0,

" d " $-\frac{1}{24}$,
" e " 0,

and we have the hourly motion equal to

$$\tfrac{1}{12}\left(b-\frac{d}{24}\right),$$

where b and d represent the first and third differences corresponding to the instant for which the hourly motion is required.

Ex. 2. Required the variation of the moon's right ascension in one hour of longitude for the instant of the Greenwich transit, January 5, 1854, from the following data:

Date.	Moon's R. A.	1st Diff.	2d Diff.	3d Diff.
	h. m. s.	m. s.	s.	s.
January 4, U. T.	23 55 36.86			
		24 21.19		
" L. T.	0 19 58.05		−32.06	
		23 49.13		+9.68
" 5, U. T.	0 43 47.18		−22.38	
		23 26.75		+9.23
" L. T.	1 7 13.93		−13.15	
		23 13.60		
" 6, U. T.	1 30 27.53			

The difference, 23m. 49.13s., corresponds to the instant midway between the lower transit, January 4th, and the upper transit, January 5th. By interpolation in the usual manner, we find the first difference corresponding to the upper transit, January 5th, to be

$$b=23\text{m. } 36.76\text{s.},$$

and the third difference for the same instant is

$$d=+9.45\text{s.}$$

Subtracting $\frac{1}{24}$th of d from b, we have

$$23\text{m. } 36.37\text{s.},$$

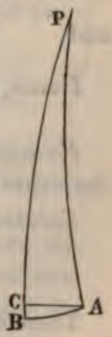
which, divided by 12, gives

$$118.03\text{s.},$$

which is the motion in right ascension for one hour of longitude, corresponding to the instant of Greenwich transit, January 5th.

PROBLEM.

(228.) *Two hour circles*, PA, PB, *make with each other a small angle at* P, *and from any point*, A, *in one of them, an arc*, AC, *of a great circle is let fall perpendicularly on the other; it is required to find the difference of declination of the points* A *and* C.



Let P be the pole of the earth, AB a parallel of declination, and AC an arc of a great circle perpendicular to PB.

Then, in the triangle APC,

$$\text{R. cos. APC}=\text{tang. PC cot. PA},$$

or

$$\text{cot. PC}=\frac{\text{cot. PA}}{\text{cos. APC}}.$$

If we put δ = the declination of the point A, δ' = the declination of the point C, and a the difference of right ascension of A and C, we shall have

$$\text{tang. } \delta' = \frac{\text{tang. } \delta}{\text{cos. } a} \quad \ldots\ldots\ldots\ldots \quad (1)$$

from which the declination of the point C may be computed; but the computation requires a table of tangents extending to single seconds, and seven places of decimals. We may obtain a more convenient formula as follows:

By Trig., Art. 77,

$$\text{tang. } (a-b) = \frac{\text{tang. } a - \text{tang. } b}{1 + \text{tang. } a \text{ tang. } b}.$$

Hence

$$\text{tang. } (\delta'-\delta) = \frac{\dfrac{\text{tang. } \delta}{\text{cos. } a} - \text{tang. } \delta}{1 + \dfrac{\text{tang.}^2 \delta}{\text{cos. } a}}$$

$$= \frac{\text{tang. } \delta (1 - \text{cos. } a)}{\text{cos. } a + \text{tang.}^2 \delta}.$$

Since a is supposed to be a small arc, we will put cos. a in the denominator equal to unity, and it becomes

$$\text{tang. } (\delta'-\delta) = \frac{\text{tang. } \delta (1 - \text{cos. } a)}{\text{sec.}^2 \delta} = \text{tang. } \delta \text{ cos.}^2 \delta (1 - \text{cos. } a)$$

But $\text{tang. } \delta \text{ cos.}^2 \delta = \text{sin. } \delta \text{ cos. } \delta = \frac{\text{sin. } 2\delta}{2}$, by Trig., Art. 73.

And by Trig., Art. 74,

$$1 - \text{cos. } a = 2 \text{ sin.}^2 \tfrac{1}{2}a.$$

Hence $\quad \text{tang. } (\delta'-\delta) = \text{sin. } 2\delta \text{ sin.}^2 \tfrac{1}{2}a.$

If we suppose a to be expressed in minutes, and $\delta' - \delta$ in seconds, we may put

$$\text{tang. } (\delta'-\delta) = (\delta'-\delta) \text{ sin. } 1'',$$

and

$$\text{sin. } \frac{a}{2} = \frac{a}{2} \text{ sin. } 1' = a(30 \text{ sin. } 1'').$$

Hence, by substitution, we obtain

$$\delta' - \delta = a^2(900 \text{ sin. } 1'' \text{ sin. } 2\delta) \quad \ldots\ldots \quad (2)$$

Example. Suppose $\delta = 29°$, and $a = 90'$; it is required to find the value of $\delta' - \delta$.

Solution.—By formula (1), $\quad \text{tang. } 29° = 9.7437520$

$\text{cos. } 90' = 9.9998512$

$\text{tang. } 29° \ 0' \ 30''.0 = 9.7439008$

Therefore $\quad \delta' - \delta = 30''.0.$

By formula (2),

$$\begin{aligned} 900 &= 2.9542 \\ \sin.\ 1'' &= 4.6856 \\ \sin.\ 58^\circ &= 9.9284 \\ 90^2 &= 3.9085 \\ \delta' - \delta = 30''.0 &= \overline{1.4767} \end{aligned}$$

In this manner was computed Table XVIII. It will be observed that the declination of the point C is always greater than that of A, whether it be north or south of the equator. This correction is applied to the moon's declination in computing eclipses and occultations.

CATALOGUES OF THE FIXED STARS.

(229.) The first individual who constructed a catalogue of the stars was Hipparchus, who flourished between 160 and 135 years B.C. This catalogue contains the longitude and latitude of 1028 stars, and is preserved in Ptolemy's Almagest.

The next catalogue of stars is that of the Tartar prince Ulugh Beigh. This catalogue contains the places of 1019 stars, derived from observations made at Samarcand. The epoch is the year 1437 A.D.

The next catalogue is that of Tycho Brahe, containing the places of 777 stars, for the year 1600. Kepler subsequently enlarged this catalogue, from Tycho Brahe's observations, to 1005 stars, and published it in the year 1627.

Halley's catalogue of southern stars contains the places of 341 stars, derived from observations made at St. Helena. The epoch is 1677. This was the first catalogue constructed from observations made by the use of telescopic sights.

The catalogue of Hevelius contained 1564 stars, and was published in 1690. The epoch is 1660.

Flamsteed's British Catalogue was published in 1725. It contains the places of 2935 stars, reduced to the year 1689.

The observations made by Bradley between the years 1750 and 1762 were published by Bessel in 1818. The number of stars in this catalogue is 3112. The epoch is 1750.

Lacaille's catalogue of stars in the southern hemisphere, observed at the Cape of Good Hope, contains 9766 stars, reduced to the year 1750. This catalogue was printed in 1845 at the expense of the British government.

Mayer is the author of a catalogue of 998 zodiacal stars, published in 1775.

(230.) Toward the close of the last century M. L. Lalande, at Paris, undertook to determine the positions of all the stars in the northern hemisphere down to the ninth magnitude. These observations have recently been reduced at the expense of the British Association, and were published in 1847. This catalogue contains the places of 47,390 stars, reduced to the year 1800.

In 1814 Piazzi published a catalogue of 7646 stars, from observations made at Palermo from 1792 to 1813. This was the most important catalogue of stars hitherto published. Every star was observed several times, and a mean of all the results taken as the final place of the star.

In 1806 De Zach published a catalogue of 1830 zodiacal stars, from observations made by him at Seeberg, in Saxe Gotha.

In 1838 was published Groombridge's catalogue of 4243 circumpolar stars, reduced to the year 1810. It contains the places of all the stars down to the eighth magnitude, situated within 50° of the north pole, derived from observations made between the years 1806 and 1816.

Brisbane's catalogue, edited by W. Richardson, contains the places of 7385 stars, chiefly in the southern hemisphere, observed at Paramatta, in New South Wales.

Taylor's catalogue contains 11,015 stars, observed at Madras, India, and reduced to 1835.

Bessel's catalogues contain 31,085 stars between $-15°$ and $+15°$ dec., and 31,445 stars between $+15°$ and $+45°$ dec. All observed at Königsberg, and reduced to 1825 by Weisse.

Argelander's catalogues contain 110,984 stars between $-2°$ and $+20°$ dec.; 105.075 stars between $+20°$ and $+41°$ dec.; and 108,129 stars between $+41°$ and $+90°$ dec. All of these stars were observed at Bonn, and are reduced to 1855.

Rumker's catalogues contain the places of 15,104 stars, observed at Hamburg, Germany.

Cooper's catalogue contains the places of 60,066 stars, observed at Markree, Ireland, in the years 1848 to 1856.

Lamont's catalogues contain 3571 stars between $+15°$ and $+9°$ dec.; 6323 stars between $+9°$ and $+3°$ dec.; 9412 stars between $+3°$ and $-3°$ dec.; 4793 stars between $-3°$ and

−9° dec.; 4093 stars between −9° and −15° dec.; and 5563 stars north of +15° or south of −15°. All of these stars were observed at Munich, Bavaria, and are reduced to 1850.

The Radcliffe catalogues contain the places of 8703 stars, chiefly circumpolar, observed at Oxford, England.

Schjellerup's catalogue contains 10,000 stars between −15° and +15° dec., observed at Copenhagen.

Airy's catalogues contain the places of about 10,000 stars, observed at Greenwich, England.

The Washington catalogue, by Prof. Yarnall, contains the places of 10,658 stars, reduced to 1860, derived from observations made at the United States Naval Observatory, from 1845 to 1871.

Bond's catalogues contain the places of 5500 stars between the equator and +0° 20′ dec.; 4484 stars between +0° 20′ and +0° 40′ dec.; and 6100 stars between +0° 40′ and +1° 0′ dec. All observed at Cambridge, Mass.

Mr. E. J. Stone has constructed a catalogue of all stars down to the seventh magnitude between N. P. D. 120° and N. P. D. 180°, and all Lacaille's stars beyond these limits. The catalogue contains 12,400 stars, almost all of which have been observed three times. The printing of this catalogue is now (1881) in progress.

The catalogue published by the British Association in 1845 contains the places of 8377 stars, reduced to 1850, derived from the best authorities, and furnishes the constants for deducing the apparent from the mean places with the greatest facility. This is the most valuable catalogue for general use which has yet been published

In 1852, Professor Struve, of St. Petersburgh, published a catalogue giving the places of 2874 stars for the year 1830, being chiefly double stars observed at Dorpat from 1832 to 1843.

The catalogue appended to this volume contains the mean places of all stars as large as the fifth magnitude, reduced to the year 1850. The mean places of such as were found in Airy's twelve-year catalogue were taken from that catalogue; the others were taken from the catalogue of the British Association, which also furnished the constants for reduction.

Determination of the apparent places of the fixed stars.

(232.) The positions of the stars given in the catalogues are called their mean places; and as these places vary from year to year in consequence of precession, the epoch for which the places are given should always be stated. In the accompanying catalogue, the mean places of the stars are given for January 1, 1850. In our observations, however, we do not find the stars in the positions here given; but their places are altered by the amount of their precession since January 1, 1850, and are also affected by aberration and nutation. The mean places must therefore be reduced to the apparent before they can be compared with observation.

The algebraic expressions for these corrections have been reduced by Bessel to the following form:

$$\text{Correction in R. A.} = \mathrm{A}a + \mathrm{B}b + \mathrm{C}c + \mathrm{D}d;$$

$$\text{Correction in N. P. D.} = \mathrm{A}a' + \mathrm{B}b' + \mathrm{C}c' + \mathrm{D}d';$$

where $\mathrm{A} = -18''.732 \text{ cos. } \odot,$

$\mathrm{B} = -20''.420 \text{ sin. } \odot,$

$\mathrm{C} = t - 0.025 \text{ sin. } 2\odot - 0.343 \text{ sin. } \text{☊} + 0.004 \text{ sin. } 2\text{☊},$

$\mathrm{D} = -0''.545 \text{ cos. } 2\odot - 9''.250 \text{ cos. } \text{☊} + 0''.090 \text{ cos. } 2\text{☊},$

$a = +\text{cos. } \alpha \text{ sec. } \delta,$

$b = +\text{sin. } \alpha \text{ sec. } \delta,$

$c = +3.0706\text{s.} + 1.3370\text{s. sin. } \alpha \text{ tang. } \delta,$

$d = +\text{cos. } \alpha \text{ tang. } \delta,$

$a' = -\text{tang. } \omega \text{ cos. } \delta + \text{sin. } \alpha \text{ sin. } \delta,$

$b' = -\text{cos. } \alpha \text{ sin. } \delta,$

$c' = -20''.055 \text{ cos. } \alpha,$

$d' = +\text{sin. } \alpha.$

Also,

t = the time from the beginning of the year, expressed in fractional parts of a year.

$\odot$ = the sun's true longitude,

☊ = the mean longitude of the moon's ascending node,

α = the mean right ascension of the star,

δ = the mean declination of the star,

ω = the obliquity of the ecliptic.

The factors A, B, C, D are independent of the star's places, and are the same for all the stars, but vary with the time. They are given for every day of the year in the English Nautical Almanac, on pages 295–302, and in the American Nautical Almanac, on pages 281–4. The coefficients which are denoted by A, B, C, and D in the English Almanac are denoted by C, D, A, and B in the American Almanac, so that for the American Almanac we shall have

$$\text{Correction in R. A.} = \text{C}a+\text{D}b+\text{A}c+\text{B}d;$$

$$\text{Correction in N. P. D.} = \text{C}a'+\text{D}b'+\text{A}c'+\text{B}d'.$$

The factors $a, b, c, d, a', b', c', d'$, depend only on the places of the stars, and are sensibly constant for a long period of years. They are accordingly calculated, and their logarithms are entered opposite each star in the accompanying catalogue.

(233.) In order to obtain the correction to the mean place of a star, we have only to take from the catalogue, and opposite to the given star, the logarithms of a, b, c, d, and a', b', c', d', with their proper signs; and to write down under them respectively, from the Nautical Almanac, opposite the given day, the logarithms of A, B, C, D, with their proper signs, remembering that the signs prefixed to the logarithms affect only the natural numbers. We then add each pair together, and find the natural number corresponding to the sum. The sum of the four natural numbers thus obtained (regard being had to their signs) will be the total correction required in right ascension and polar distance on the given day. This correction, applied to the mean place of the star at the beginning of the year, will give the apparent place of the star on the day required.

The mean right ascension and polar distance for the epoch of the catalogue is reduced to that of the current year, by adding as many times the annual precession in right ascension and polar distance as the number of whole years elapsed since the given epoch.

Example. Required the apparent right ascension and north polar distance of γ Orionis, on December 5, 1882, for midnight, at Greenwich.

	h. m. s.		° ′ ″
Mean R. A., January 1, 1850 . . .	5 17 5.31	Mean P. D., January 1, 1850 . . .	83 47 27.7
32 years' precession and proper motion	1 43.04	32 years' precession and proper motion	−1 59.0
Mean R. A., January 1, 1882 . . .	5 18 48.35	Mean P. D., January 1, 1882 . . .	83 45 28.7

	Logarithms.	Nat. Nos.		Logarithms.	Nat. Nos.
a	+8.0963		a'	−9.5120	
A	+0.7184	s.	A	+0.7184	″
Aa	+8.8147	+0.065	Aa'	−0.2304	−1.70
b	+8.8188		b'	−8.3039	
B	+1.2930		B	+1.2930	
Bb	+0.1118	+1.294	Bb'	−9.5969	−0.40
c	+0.5070		c'	−0.5721	
C	+0.0718		C	+0.0718	
Cc	+0.5788	+3.791	Cc'	−0.6439	−4.40
d	+7.1304		d'	+9.9923	
D	+0.8105		D	+0.8105	
Dd	+7.9409	+0.009	Dd'	+0.8028	+6.35
Correction of R. A. =		+5.159	Correction of P. D. =		−0.15

Hence the apparent right ascension of γ Orionis,

$$=5\text{h. } 18\text{m. } 48.35\text{s.} + 5.16\text{s.} = 5\text{h. } 18\text{m. } 53.51\text{s.};$$

and the apparent north polar distance,

$$=83^\circ\ 45'\ 28''.7 - 0''.1 = 83^\circ\ 45'\ 28''.6.$$

The mean right ascension on the 1st of January of the year of observation may be found by applying to the apparent observed right ascension the above correction with the contrary sign.

DIURNAL ABERRATION OF LIGHT.

(234.) The diurnal aberration of light is a phenomenon resulting from the movement of light, combined with the rotation of the earth on its axis; and it differs from the annual aberration merely in consequence of the difference between the velocity of the earth on its axis, and its velocity in its orbit.

The velocity of rotation of a point on the equator is to the velocity of the earth in its orbit as 1 to 65.82; and, since the annual aberration is 20″.445, the diurnal aberration will be

$0''.3107$ in arc,

or 0.0207s. in sidereal time.

(235.) This is the diurnal aberration of a star on the equator for a place situated on the equator; but for a place in latitude ϕ, the circle of diurnal rotation being less than at the equator, in the ratio of radius to the cosine of the latitude, the aberration will be equal to

$$0.0207\text{s. } \cos. \phi.$$

If the star be not in the equator, this expression denotes only the aberration on the parallel of the star's declination; and in order to reduce it to the celestial equator, or to the value of the aberration in right ascension, it must be multiplied by the secant of the star's declination. Hence, if ϕ denote the latitude of the place, and δ the declination of the star, then the correction in time for the *upper* transit, on account of daily aberration, is

$$+0.0207\text{s. } \cos. \phi \sec. \delta;$$

and for the *lower* transit,

$$-0.0207\text{s. } \cos. \phi \sec. \delta.$$

METHOD OF SOLVING EQUATIONS OF CONDITION.

(236.) In astronomical researches it is frequently required to determine the values of several quantities from a large number of simple equations, which are called *equations of condition;* and when the number of independent equations is greater than the number of unknown quantities, these equations can not be perfectly satisfied, and we can only obtain more or less probable values of the unknown quantities. The given equations may be combined in a variety of ways, and each mode of combination will furnish different values of the unknown quantities. Hence it is a question of the highest importance to determine in what manner these equations should be combined, so as to furnish the most probable values of the unknown quantities. Thus, for example, if our observations have furnished the four equations,

$$x-\ y+2z=3 \quad \dots\dots\dots \quad (1)$$
$$3x+2y-5z=5 \quad \dots\dots\dots \quad (2)$$
$$4x+\ y+4z=21 \quad \dots\dots\dots \quad (3)$$
$$-x+3y+3z=14 \quad \dots\dots\dots \quad (4)$$

and it is required to find the values of x, y, and z, we may pursue various methods, and we shall obtain various results for these quantities.

(237.) A method frequently practiced consists in rendering the coefficients of one of the unknown quantities, as x, positive in all the equations, and then, by adding all the equations together, obtaining a new equation, in which the coefficient of x is the greatest possible. In a similar manner, by rendering the coefficients of y positive in all the equations, and then adding all the equations together, we obtain a new equation, in which the coefficient of y is the greatest possible. Proceeding in the same manner with each of the other unknown quantities, we shall have as many new equations as there are unknown quantities, and these equations may be readily solved by the ordinary rules of algebra. Thus, by changing the signs of all the terms in equation (4), and adding the equations together, we obtain

$$9x-y-2z=15 \quad \ldots\ldots\ldots \quad (5)$$

Changing the signs in equation (1), we obtain, in the same manner,

$$5x+7y=37 \quad \ldots\ldots\ldots \quad (6)$$

Changing the signs in equation (2), we obtain, in a similar manner,

$$x+y+14z=33 \quad \ldots\ldots\ldots \quad (7)$$

From equations (5), (6), and (7), by the usual method of elimination, we obtain the values

$$x=2.4853;\ y=3.5105;\ z=1.9289.$$

The method practiced by Tobias Mayer consisted in combining the given equations by addition, subtraction, etc., in such a manner that one of the unknown quantities, as x, should have a very large coefficient in the resulting equation, and the other unknown quantities should have small coefficients. Another combination would furnish a final equation, in which only y should have a large coefficient; and so for each of the unknown quantities.

(238.) Methods similar to the preceding are frequently used by astronomers, on account of their convenience; but Legendre has demonstrated that the *most probable* values of the unknown quantities are those which render the sum of the squares of all the errors the least possible. This method is accordingly called *the method of least squares.*

If we substitute in equation (1) the values of x, y, and z, above given, we shall find that the second member of the equation reduces to 2.8326 instead of 3, showing that these values do not perfectly satisfy the equation. A similar remark applies to equations (2), (3), and (4). If, then, we transpose all the terms to one member of the equation, the sum of the terms will not reduce to zero, but will be equal to a small quantity, e. If these equations were deduced from observation, e may be regarded as the error of one of the observations. The equations may therefore be expressed under the form

$$\begin{aligned} e &= a + bx + cy + dz \\ e' &= a' + b'x + c'y + d'z \\ e'' &= a'' + b''x + c''y + d''z \\ &\text{etc.} \qquad \text{etc.} \end{aligned}$$

There will be as many of these equations as there are observations. For convenience, let us denote all the terms of the second members of the equations which are independent of x, by M, M′, etc., and we shall have

$$\begin{aligned} e &= bx + \mathrm{M} \\ e' &= b'x + \mathrm{M}' \\ e'' &= b''x + \mathrm{M}'' \\ &\text{etc.} \qquad \text{etc.} \end{aligned}$$

Taking the sum of the squares of these equations, we shall have

$$e^2 + e'^2 + e''^2 +, \text{etc.} = (bx+\mathrm{M})^2 + (b'x+\mathrm{M}')^2 + (b''x+\mathrm{M}'')^2 +, \text{etc.}$$

(239.) According to the principle above stated, this quantity must be made a minimum, which is done by putting its first differential coefficient equal to zero. If we consider only the unknown quantity x, we shall have, after differentiating and dividing by $2dx$,

$$0 = b(bx+\mathrm{M}) + b'(b'x+\mathrm{M}') + b''(b''x+\mathrm{M}''), \text{etc.};$$

that is, *to form the equation that gives a minimum for any one of the unknown quantities, as* x, *we must multiply each equation of condition by the coefficient of* x *in that equation, taken with its proper sign, and put the sum of all these products equal to zero.* We must proceed in the same manner for y, z, etc., and we shall obtain as many equations of the first degree as there are unknown quantities, whose values may then be obtained by the usual mode of elimination.

To apply this method to the example, on page 221, we must multiply equation (1) by 1; equation (2) by 3; equation (3) by 4; and equation (4) by -1, and we shall obtain

$$x-y+2z-3=0$$
$$9x+6y-15z-15=0$$
$$16x+4y+16z-84=0$$
$$x-3y-3z+14=0$$

Putting the sum of these equations equal to zero, we obtain

$$27x+6y-88=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

We must now multiply equation (1) by -1; equation (2) by 2; equation (3) by 1; and equation (4) by 3, and we shall obtain

$$-x+y-2z+3=0$$
$$6x+4y-10z-10=0$$
$$4x+y+4z-21=0$$
$$-3x+9y+9z-42=0$$

Putting the sum of these equations equal to zero, we obtain

$$6x+15y+z-70=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; (9)$$

We must also multiply equation (1) by 2; equation (2) by -5; equation (3) by 4; and equation (4) by 3, and we shall obtain

$$2x-2y+4z-6=0$$
$$-15x-10y+25z+25=0$$
$$16x+4y+16z-84=0$$
$$-3x+9y+9z-42=0$$

Putting the sum of these equations equal to zero, we obtain

$$y+54z-107=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; (10)$$

By comparing equations (8), (9), and (10) in the usual mode of elimination, we obtain the values

$$x=2.4702;\; y=3.5509;\; z=1.9157.$$

If we substitute in equations (1), (2), (3), and (4), of page 221, the values found in Art. 237, we shall find the errors of these several equations to be

$$-.1674,\; -.1676,\; +.1673,\; -.1671;$$

the sum of whose squares is

0.1120.

If we substitute in the same equations the values last found, we shall find the errors of these several equations to be

$$-.2493,\; -.0661,\; +.0945,\; -.0704;$$

the sum of whose squares is

0.0804.

The sum of the squares of the errors resulting from employing the last obtained values of x, y, and z, is less than that resulting from the former values; and hence, according to the principle of Legendre, the last values have a greater probability in their favor than the former.

An example of the application of this method will be found on page 334.

EXAMPLES.

1. Given the moon's right ascension, as follows:

1859, August 25,	0h.	R. A. =	7h.	43m.	7.05s.
	12	"	8	14	12.80
26,	0	"	8	44	34.81
	12	"	9	14	9.29
27,	0	"	9	42	56.19
	12	"	10	10	58.55

Required the moon's right ascension August 26, at 5h.

2. Required the apparent right ascension and north polar distance of α Lyræ (see page 446) August 19, 1859.

3. Given the equations

$$3x+4y=16,$$
$$5x-3y=14,$$
$$7x-6y=17,$$
$$2x+9y=19,$$

to find the most probable values of x and y.

4. Required the mean time of opposition in right ascension of the sun and moon Feb. 16, 1859, from the following data:

Date.		Sun's Right Ascension.			Moon's Right Ascension.		
	h.	h.	m.	s.	h.	m.	s.
Feb. 16,	21	22	1	32.87	9	58	3.57
	22	22	1	42.54	10	0	21.89
	23	22	1	52.21	10	2	39.85
	24	22	2	1.88	10	4	57.45

5. Required the hourly motion of the moon from the sun February 16, 1859, at 22h. 37m. 42s., according to the preceding data.

CHAPTER X.

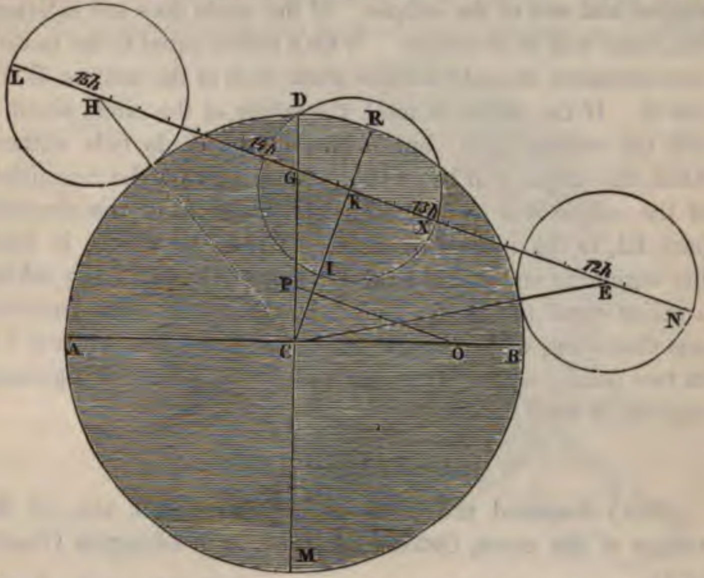
ECLIPSES OF THE MOON.

(240.) The time of beginning or end of a lunar eclipse at any place may be found by adding its longitude to the times given in the Nautical Almanac for the meridian of Washington when the longitude is east, or subtracting the longitude when it is west. The times given in the Nautical Almanac may be deduced from the right ascensions, declinations, etc., of the sun and moon, by the following method.

An eclipse of the moon can only happen at the time of full moon. If the moon at that time is within about 12 degrees of one of its nodes, there may be an eclipse. To find whether there will be one, and to calculate the times and phases, proceed as follows:

(241.) Find the Washington mean time of opposition in right ascension by the Nautical Almanac, in the manner explained in Art. 225. For this time compute the declination, horizontal parallax, and semi-diameter, both of the sun and moon; also the hourly motion of the moon from the sun, both in right ascension and declination, as explained in Art. 226.

Let C represent the centre of the earth's shadow (see figure on page opposite), whose right ascension is the same as that of the sun, increased by 12 hours, and its declination is the declination of the sun, with a contrary sign. Let DCM be a meridian passing through the centre of the shadow, and ACB a great circle perpendicular to it. Select a convenient scale of equal parts, and from it take CG, equal to the moon's declination, minus the declination of the centre of the shadow, and set it on CD, from C to G, above the line AB, if the centre of the moon is north of the centre of the shadow, but below if south. Take CO, equal to the hourly motion of the moon from the sun in right ascension, reduced to the arc of a great circle, and set it on the line CB, to the right of C. Take CP, equal to the moon's hourly motion from the sun in declination, and set it on the line CD, from C to P, above the line AB, if the moon is moving northward with respect to the shadow; below, if moving southward. Join the points O and P. The line OP will represent the hourly motion of the moon from the sun; and parallel to it, through G, draw NGL, which will represent the relative orbit of the moon, the earth's shadow being supposed stationary. On this line are to be marked the places of the moon before and after opposition, by means of the hourly motion OP, in such a manner that the moment of opposition may fall exactly on the point G.



(242.) The semi-diameter of the earth's shadow is equal to the horizontal parallax of the moon, plus that of the sun, minus the sun's semi-diameter; which result must be increased by $\frac{1}{60}$th part, on account of the earth's atmosphere. With this radius describe the circle ADB about the centre C. Add the moon's semi-diameter to the radius CB, and with this sum for a radius, describe about the centre C a circle, which, if there be an eclipse, will cut NL in two points, E and H representing respectively the places of the moon's centre at the beginning and end of the eclipse. Draw the line CKR perpendicular to LN, and cutting it in K. The hours and minutes marked on the line LN, at the points E, K, and H, will represent respectively the times of the beginning of the eclipse, middle of the eclipse, and end of the eclipse. If the circle does not intersect NL, there will be no eclipse. With a radius equal to the moon's semi-diameter, describe a circle about each of the centres, E, H, and K. If the eclipse is total, the whole of the circle about K will fall within ARB; but if part of the circle falls without ARB, the eclipse will be partial. In either case the magnitude of the eclipse will be represented by the ratio of the obscured part, RI, to the moon's diameter. When the eclipse is total, the beginning and end of total darkness may be found by taking a radius equal to CB, diminished by the moon's semi-diameter, and describing with it, round the centre C, a circle, cutting LN in two points, representing respectively the points of beginning and end of total darkness.

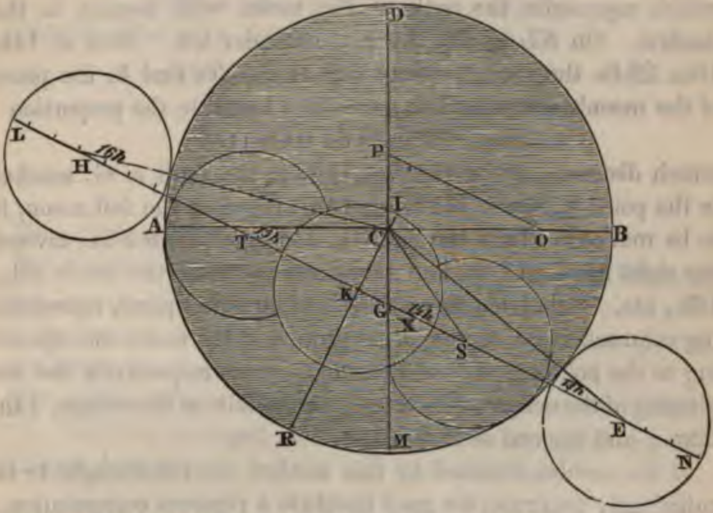
Example 1.

(243.) Required the times of beginning, end, etc., of the eclipse of the moon, October 24, 1855, at Washington Observatory.

By the Nautical Almanac, the Washington mean time of opposition in right ascension is, October 24, 14h. 10m. 29.6s., which result differs somewhat from that found on page 211. Corresponding to this time, the Nautical Almanac furnishes the following elements:

		°	′	″
Declination of the moon	N.	11	42	26.9
Declination of the earth's shadow	N.	11	56	48.0
Moon's equatorial horizontal parallax			59	45.8
Sun's horizontal parallax			0	8.6
Moon's semi-diameter			16	19.2
Sun's semi-diameter			16	7.9
Moon's hourly motion in right ascension			33	22.1
Sun's hourly motion in right ascension			2	23.4
Hourly motion of moon in declination	N.		15	39.8
Hourly motion of shadow in declination	N.		0	52.1

The figure of the earth being spheroidal, that of the shadow will deviate a little from a circle, so that to have a mean radius the horizontal parallax of the moon should be reduced to a mean latitude of 45°. This reduction, by Table XIV., is 5″.9; so that the moon's reduced parallax is 59′ 39″.9. Then, to obtain CB, the semi-diameter of the earth's shadow, we have 59′ 39″.9+8″.6−16′ 7″.9, which is equal to 43′ 40″.6. Increasing this by $\frac{1}{60}$th part of itself, or 43″.7, we have 44′ 24″.3=CB; to which adding the moon's semi-diameter, we obtain CE=60′ 43″.5. From the centre C, with a radius CB, taken from a convenient scale of equal parts, describe the circle ARB, representing the earth's shadow. Draw the line ACB to represent a parallel to the equator, and make CG perpendicular to it, equal to 14′ 21″.1, which is the moon's declination, minus the declination of the centre of the shadow; the point G being taken below C, because the centre of the moon is south of the centre of the shadow.



The hourly motion of the moon from the sun in right ascension is 30′ 58″.7, which must be reduced to the arc of a great circle by multiplying it by the cosine of the moon's declination, 11° 42′ 26″.9, Art. 72, thus:

$$
\begin{array}{rl}
30' \; 58''.7 = 1858''.7 = & 3.269209 \\
\text{cos. Dec.} = & \underline{9.990870} \\
\text{Reduced hourly motion} = 1820''.0 = & 3.260079
\end{array}
$$

Make CO equal to 1820″.0, and CP, perpendicular to it, equal to 14′ 47″.7, which is the hourly motion of the moon from the shadow in declination, the point P being placed above C, because the moon was moving northward with respect to the shadow. Join OP; and parallel to it, through G, draw the line NGL, which represents the path of the moon with respect to the shadow. On NL let fall the perpendicular CK. Now at 14h. 10m. 29.6s. the moon's centre was at G. To find X, the place of the moon's centre at 14h., we must institute the proportion

60m. : 10m. 29.6s. :: OP : GX;

which distance, set on the line GN, to the right of G, reaches to the point X, where the hour, 14h. preceding the full moon, is to be marked. Take the line OP, and lay it from 14h., toward the right hand, to 13h., and successively toward the left to 15h., 16h., etc. Subdivide these lines into 60 equal parts, representing minutes, if the scale will permit; and the times corresponding to the points E, K, and H will represent respectively the beginning of the eclipse, 12h. 36m.; the middle of the eclipse, 14h. 22m.; and the end of the eclipse, 16h. 7m.

If the results obtained by this method are not thought to be sufficiently accurate, we may institute a rigorous computation.

COMPUTATION OF ECLIPSE.

(244.) The phases of the eclipse may be accurately calculated in the following manner:

In the right-angled triangle OCP, we have given CO=1820″.0 and CP=887″.7, to find OP and the angle CPO, thus:

CP : R :: CO : tang. CPO.
CO=1820″.0=3.260079
CP= 887″.7=2.948266
CPO=63° 59′ 59″ tang.=0.311813

Also, sin. CPO : R :: CO : OP.
CO=3.260079
sin. CPO=9.953659
OP=2025″.0=3.306420

The angle CPO is equal to CGK, because GE and OP are parallel. Then, in the triangle CGE, we have the angle CGE =116° 0′ 1″; CG, the difference of declination between the moon and the centre of the shadow, =14′ 21″.1=861″.1; and the line CE=60′ 43″.5=3643″.5, to find the other parts of the triangle, thus:

CE : sin. CGE :: CG : sin. CEG.

CE *comp.* = 6.438481
sin. CGE = 9.953659
CG = 2.935054
CEG = 12° 15′ 51″ sin = $\overline{9.327194}$

Therefore the angle ECG = 51° 44′ 8″. Then

sin. CGE : CE :: sin. ECG : EG.

sin. CGE *comp.* = 0.046341
CE = 3.561519
sin. ECG = 9.894959
EG = 3182″.9 = $\overline{3.502819}$

Then, to find the time of describing EG, we say,

As OP (2025″.0) is to 1 hour, so is EG (3182″.9) to the time (5658.5s.) 1h. 34m. 18.5s., between the beginning of the eclipse and the time of opposition in right ascension, 14h. 10m. 29.6s., which gives the beginning of the eclipse 12h. 36m. 11.1s.

The middle of the eclipse is found by means of the triangle CGK, which is similar to CPO, in which the angles and hypothenuse are given to find CK and KG. We have

R : CG :: sin. CGK : CK :: cos. CGK : GK.

sin. CGK = 9.953659	cos. CGK = 9.641846
CG = 2.935054	CG = 2.935054
CK = 774″.0 = $\overline{2.888713}$	GK = 377″.5 = $\overline{2.576900}$

To find the time of describing GK, we form the proportion

2025″.0 : 3600s. :: 377″.5 : 671.1s. = 11m. 11.1s.;

which being added to 14h. 10m. 29.6s., because the point K falls to the left of G, gives the time of the middle of the eclipse, 14h. 21m. 40.7s. Subtract the time of beginning, 12h. 36m. 11.1s., from the time of middle, we obtain for half the duration of the eclipse 1h. 45m. 29.6s.; which, added to 14h. 21m. 40.7s., gives for the end of the eclipse 16h. 7m. 10.3s.

Subtracting CK, 12′ 54″.0, from CR, 44′ 24″.3, we have KR, 31′ 30″.3; to which adding KI, 16′ 19″.2, we obtain RI, 47′ 49″.5. Dividing this by the moon's diameter, 32′ 38″.4, we obtain the magnitude of the eclipse, 1.465 (the moon's diameter being unity); and the eclipse takes place on the moon's north limb.

(245.) The beginning and end of total darkness may be found in the same manner. With a radius equal to CB, diminished by the moon's semi-diameter (that is, 44′ 24″.3−16′ 19″.2, which equals 28′ 5″.1, or 1685″.1), describe about the centre C a circle, cutting LN in the points S and T, which will represent the points of beginning and end of total darkness.

In the triangle CGS, CG=861″.1, CS=1685″.1, and the angle CGS=116° 0′ 1″. Hence we have

CS : sin. CGS :: CG : sin. CSG.
CS *comp.*=6.773374
sin. CGS=9.953659
CG=2.935054
CSG=27° 20′ 29″ sin.=9.662087

Therefore the angle SCG=36° 39′ 30″. Then

sin. CGS : CS :: sin. SCG : SG.
sin. CGS *comp.*=0.046341
CS=3.226626
sin. SCG=9.776005
GS=1119″.4=3.048972

Then, to find the time of describing GS, we say,

2025″.0 : 3600s. :: 1119″.4 : 1990.0s.=33m. 10.0s.;

which, being subtracted from 14h. 10m. 29.6s., gives the beginning of total darkness, 13h. 37m. 19.6s. Subtracting this from the time of middle, we obtain, for half the duration of total darkness, 44m. 21.1s., which, added to 14h. 21m. 40.7s., gives, for the end of total darkness, 15h. 6m. 1.8s.

(246.) The contacts with the penumbra may be found in a similar manner. The semi-diameter of the penumbra is equal to the semi-diameter of the shadow, plus the sun's diameter, or 44′ 24″.3+32′ 15″.8=76′ 40″.1. If we take the circle ARB, in the figure on page 229, to represent the limits of the penumbra, CE will be equal to 76′ 40″.1+16′ 19″.2=92′ 59″.3. Then, in the triangle CGE, we have given the angle CGE =116° 0′ 1″, CG=861″.1, and CE=5579″.3, to find GE, thus:

CE : sin. CGE :: CG : sin. CEG.
CE *comp.*=6.253420
sin. CGE=9.953659
CG=2.935054
CEG=7° 58′ 25″ sin.=9.142133

Therefore the angle ECG=56° 1′ 34″. Then

sin. CGE : CE :: sin. ECG : EG.
sin. CGE *comp.* = 0.046341
CE = 3.746580
sin. ECG = 9.918708
EG = 5147″.9 = 3.711629

To find the time of describing EG, we say,

2025″.0 : 3600s. :: 5147″.9 : 9151.9s. = 2h. 32m. 31.9s.,

which, subtracted from 14h. 10m. 29.6s., gives the first contact with the penumbra at 11h. 37m. 57.7s. Subtracting the time of first contact from the middle of the eclipse, 14h. 21m. 40.7s., we have for half the duration, 2h. 43m. 43.0s.; which, added to 14h. 21m. 40.7s., gives, for the last contact with the penumbra, 17h. 5m. 23.7s.

The results thus obtained are as follows:

	h	m.	s.	
First contact with the penumbra at . .	11	37	58	Mean time at Washington.
First contact with the umbra	12	36	11	
Beginning of total eclipse	13	37	20	
Middle of the eclipse	14	21	41	
End of total eclipse	15	6	2	
Last contact with the umbra	16	7	10	
Last contact with the penumbra . . .	17	5	24	

Magnitude of the eclipse, 1.465 on the northern limb.

To obtain the time for any other place, we have only to add or subtract the longitude. For Cambridge Observatory, whose longitude is 23m. 41.5s. east of Washington, the times will accordingly be

	h.	m.	s.	
First contact with the penumbra at . .	12	1	39	Mean time at Cambridge.
First contact with the umbra	12	59	53	
Beginning of total eclipse	14	1	1	
Middle of the eclipse	14	45	22	
End of total eclipse	15	29	43	
Last contact with the umbra	16	30	52	
Last contact with the penumbra	17	29	5	

Ex. 2. Compute the phases of the eclipse of May 1, 1855, for Cambridge Observatory, Longitude 23m. 41.5s. east of Washington, from the following elements:

Washington mean time of opposition in right ascension	10h.	49m.	10.1s.
Declination of the moon	S. 15°	1′	24″.4.
Declination of the earth's shadow	S. 15	11	32 .0.
Moon's equatorial horizontal parallax		57	9 .4.
Sun's horizontal parallax		0	8 .5.
Moon's semi-diameter		15	35 .6.
Sun's semi-diameter		15	54 .1.
Moon's hourly motion in right ascension		31	34 .2.
Sun's hourly motion in right ascension		2	23 .2.
Hourly motion of moon in declination	S.	13	10 .1.
Hourly motion of shadow in declination	S.	0	45 .1.

Ans.

	h.	m.	s.	
First contact with the penumbra at	8	27	37	Mean time at Cambridge
First contact with the umbra	9	30	6	
Beginning of total eclipse	10	32	40	
Middle of the eclipse	11	20	49	
End of total eclipse	12	8	58	
Last contact with the umbra	13	11	32	
Last contact with the penumbra	14	14	0	

Magnitude of the eclipse, 1.549 on the southern limb.

CHAPTER XI.

ECLIPSES OF THE SUN AND OCCULTATIONS.

SECTION I.

METHOD OF PROJECTING SOLAR ECLIPSES.

(247.) IN order to ascertain whether a solar eclipse will be visible at a particular place, and if so, to determine its general appearance, we will suppose the spectator to be placed at the centre of the sun, to look down upon the earth, and see the moon passing across its disk. The earth, in that case, must appear to him like a flat circular disk, as the full moon does to us; and, on account of the obliquity of the ecliptic, the position of the pole, as well as the path described by each point on the earth's surface in consequence of the diurnal motion, must vary with the season of the year. At the time of the vernal equinox, the plane of the equator passes through the sun; the poles must therefore appear to be situated upon the margin of the disk, and the equator inclined $23\frac{1}{2}$ degrees to the ecliptic, as in *Fig.* 1, where AB represents the ecliptic, H, H′ the poles of the ecliptic, EQ the equator, P, P′ the poles of the equator, and DB, AF parallels of latitude, which appear to the spectator like straight lines. At the autumnal equinox the parallels of latitude will also appear as straight lines, but the poles of the earth will lie on the opposite side of the poles of the ecliptic, as represented in *Fig.* 2. At the summer solstice the north pole of the earth will

Fig. 1.

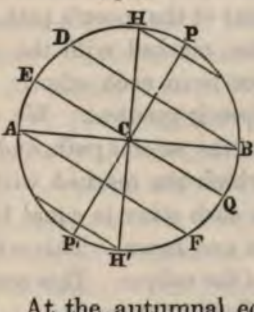
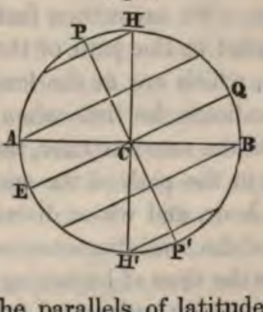


Fig. 2.



occupy the position indicated by P, in *Fig.* 3; the south pole of the earth will be invisible, the equator will occupy the position EMQ, and the parallels of latitude will all be projected into ellipses. At the winter solstice the south pole of the earth will be seen as represented at P′, in *Fig.* 4; the north pole will be invisible, and the equator will occupy the position EMQ. These different cases may all be readily illustrated by means of a terrestrial globe.

Fig. 3.

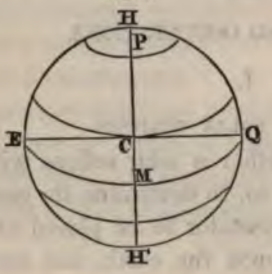
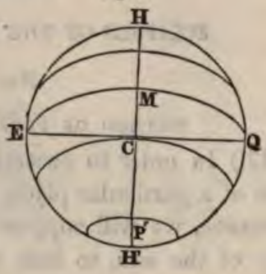


Fig. 4.



(248.) In order to project an eclipse of the sun, we must first represent the earth as it would appear to a spectator on the sun at the time proposed. We must then draw the parallel of latitude corresponding to the place for which the phases of the eclipse are to be determined, and mark upon this parallel the position of the given place for the different hours of the day. We must then draw the moon's apparent path across the earth's disk, and mark the points which it occupies at each hour of its transit. We must then find that point of the moon's path, and the point in the path of the spectator, marked with the same times, which are at the least distance from each other. This will indicate the time when the eclipse is greatest. We must find, in the same manner, that point of the moon's path, and that point in the path of the spectator, which are marked with the same hour, and whose distance from each other is equal to the sum of the semi-diameters of the sun and moon. This will indicate the time of beginning or end of the eclipse. This method will be easily understood from the following example:

Example.

(249.) Required the times and phases of the eclipse of the sun, May 26, 1854, at Boston, latitude 42° 21′ 28″ N., longitude 4h. 44m. 14s. W. of Greenwich.

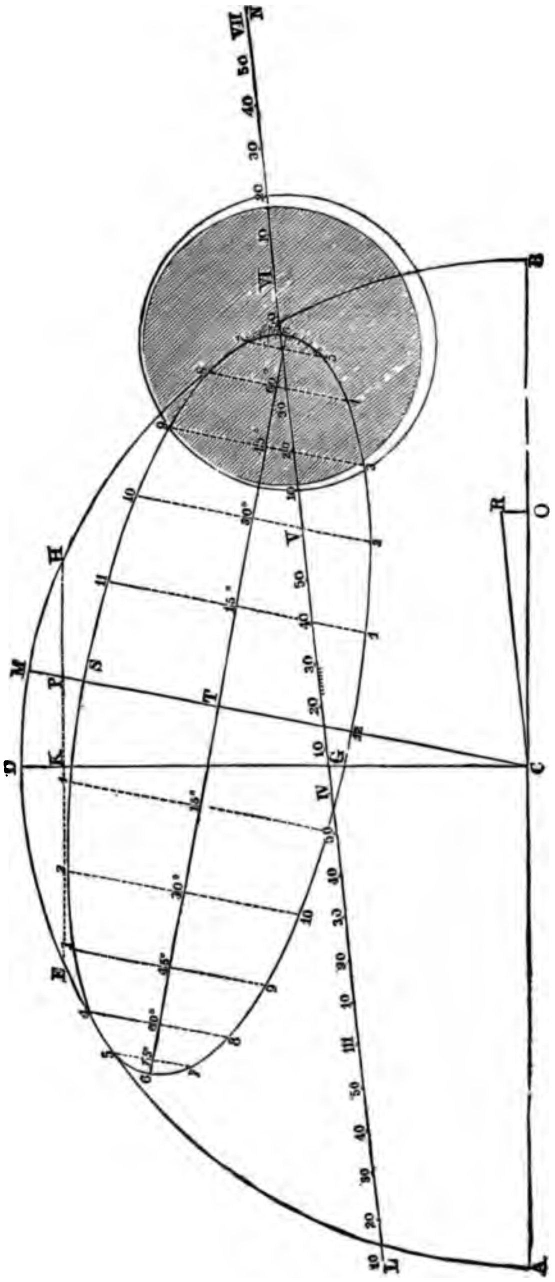
By the Nautical Almanac, the time of new moon at Greenwich is, May 26, 8h. 47.1m. mean time, corresponding to 4h. 2.9m. mean time at Boston; or 4h. 6.1m. apparent time, the equation of time being +3m. 15.4s.

For this time, the elements of the eclipse are as follows:

Sun's longitude	65° 12′ 32″.
Sun's declination	21° 11′ 17″ N.
Moon's latitude	21′ 30″=1290″ N.
Moon's hourly motion in longitude	1807″.
Sun's hourly motion in longitude	144″.
Moon's hourly motion in latitude	167″.
Moon's equatorial horizontal parallax . . .	54′ 32″.6.
Sun's equatorial horizontal parallax	8″.5.
Moon's true semi-diameter	14′ 53″.5.
Sun's true semi-diameter	15′ 48″.9.

The geocentric latitude of Boston, which is to be used in the following projection, is 42° 10′ 0″.

The relative positions of the sun and moon will be the same if we attribute to the moon the effect of the difference of their parallaxes, and suppose the sun to remain in his true position. This difference is, therefore, the relative parallax, or that which influences the relative position of the two bodies. The moon's equatorial horizontal parallax is 54′ 32″.6; its horizontal parallax for Boston (Art. 210) is 54′ 27″.6; and the relative parallax is 54′ 19″.1, or 3259″.1, which represents the apparent semi-diameter of the earth's disk, if seen at the distance of the moon from the earth; while 14′ 53″.5 represents the moon's apparent semi-diameter, seen from the same distance. These numbers will, therefore, represent their relative magnitude when seen at any distance. Take, therefore, AC (see figure on next page), equal to 3259, from any convenient scale of equal parts, and describe the semicircle ADB to represent the northern half of the earth's disk as seen from the sun, and draw CD perpendicular to AB for the axis of the ecliptic. Take the chord of 23° 28′


VII
N
50
40
30
20
10
VI
V
IV
III
L
A
B
C
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E
G
H
K
M
O
P
R
S
T
15°
30°
45°
60°
75°

(equal to the obliquity of the ecliptic), corresponding to the radius AC, and set it off on the circle ADB, upon each side of D, to E and H. In this and several subsequent cases, when a chord or sine is required corresponding to a particular radius, it is most conveniently obtained from a sector, but may be derived from any scale of chords or sines. Draw the line EH, cutting CD in K. By comparing figures 1, 2, 3, and 4, on pages 235 and 236, it will be perceived that the pole of the earth, as viewed from the sun, will appear to revolve with the seasons of the year through the line HKE; and since H is its position at the vernal equinox, its distance at any time from H will be equal to the versed sine of the sun's longitude; or its distance from the solstice, K, will be equal to the sine of the difference between the sun's longitude and 90°, or 270°. Take, then, the sine of $90° - 65° 12'.5$, that is, the sine of 24° 47′.5 to the radius EK, and set it off from K to P, which will be the place of the pole of the earth. Draw CP, and produce it to cut the circle ADB in M. The line CP represents the northern half of the earth's axis.

(250.) We wish now to represent the parallel of latitude of Boston, or the path of Boston on the earth's disk, as seen from the sun. If the latitude of the place were just equal to the sun's declination, the sun would be vertical at noon, and Boston would be seen precisely at the centre of the disk at C; but since the latitude exceeds the sun's declination by 20° 59′, Boston must be seen that distance north of the point where the sun is vertical, which, when projected on the disk, becomes the sine of the arc, measured from C on the axis CP. Take, then, the sine of 20° 59′ to the radius AC, and set it off from C to 12. This point will be the apparent position of Boston at noon.

If the earth were transparent, Boston would be seen at midnight somewhere upon the line CM, and north of the point 12. The point antipodal to that at which the sun is vertical, and which also would be seen at C, is as many degrees south of the equator as the sun's declination is north. Hence the distance of Boston from this point at midnight must be equal to the latitude of the place added to the sun's declination, which amounts to 63° 21′. With the radius AC, take the sine of 63° 21′, and set it off from C, upon the line CP, to S. The point S will represent the apparent place of Boston at midnight.

The line 12, S is the shortest diameter of the ellipse into which the parallel of latitude appears projected, from being seen obliquely. The point T, midway between 12 and S, is its centre; and the line 6, 6 drawn through T, perpendicular to CM, is its longest diameter. The line 6, 6 not being shortened by being seen obliquely, will appear of the length of the radius of the parallel, which is equal to the cosine of the latitude. The complement of the latitude of Boston is 47° 50′; and setting off its sine each way from T to 6 and 6, we find the extremities of the longest diameter, which must be the points on the disk where Boston will be seen at six o'clock in the morning and at six o'clock in the evening.

(251.) The position of Boston at any other hour of the day may be found as follows: With a radius equal to T, 6, take the sine of 15° (corresponding to one hour), and set it off on each side of the point T to the points marked 15°. In the same manner, set off the sines of 30°, 45°, 60°, and 75°. Through these points draw lines, as in the figure, parallel to CM. With a radius equal to ST, take the sine of 75°, and set it off on the line 1, 11, from the point marked 15°, above and below the line 6, 6. In the same manner, set off the sines of 60°, 45°, 30°, and 15°, from the points marked 30°, 45°, 60°, and 75°. The points 1, 2, 3, etc., obtained in this manner, will represent the situation of Boston at those hours, and an ellipse drawn through these points will represent its apparent path. The hours must be marked from noon toward the right, in succession, round the curve. The path touches the circle ADB in two points, representing the points of sunrising and sunsetting, which, in the present figure, are 4½ A.M. and 7½ P.M. These points divide the path into two parts, of which one represents the path by day and the other by night.

(252.) We wish now to represent the moon's apparent path across the earth's disk. From the same scale upon which AC was measured, take an interval equal to the moon's latitude, 1290″, and apply it on CD, from C to G, above the line ACB, because the moon's latitude is north. Take CO, equal to 1663″, the hourly motion of the moon from the sun in longitude, and set it on the line CB, from C to O. Draw OR perpendicular to CB, and make it equal to 167″, the moon's hourly motion in latitude, and set it above the line ACB, because the moon is going northward. Draw the line CR, which represents the hourly motion of the moon from the sun on the relative orbit; and parallel to this line, draw the relative orbit of the moon, LGN, on which are to be marked the places of the moon before and after the conjunction, by means of the hourly motion, CR, so that the moment of the new moon at Boston may fall exactly on the point G, where the new moon is at 4h. 6m. This may be done by instituting the proportion

60m. : the line CR :: 6m. : the line G, IV.

This distance is to be set off on the line GL, from G, toward the left, to the point IV, the place of the moon at four o'clock. Then the distance CR being taken in the compasses, and set from IV, both toward the right and left, as often as may be necessary, gives the places of the moon's centre at 3, 4, 5, 6, etc., o'clock, by apparent time. These hours may be divided into 60 equal parts, representing minutes, if the scale be taken sufficiently large.

(253.) Find, by trials with a pair of compasses, two points, one on the moon's path, and the other on the path of the spectator, both of which are marked with the same times, and which are at the least distance from each other. That time, which in the present case is 5h. 44m., is the instant when the eclipse is greatest.

The appearance of the moon, as projected upon the earth's disk at any hour, may be shown by taking its semi-diameter, 893″.5, and with this radius describing a circle, whose centre is the point where the moon's centre will be at the time proposed. The figure shows the appearance of the moon at 5h. 44m. If, with a radius equal to the sun's semi-diameter, 948″.9, we describe a circle whose centre is the position of Boston at the same instant, this circle will represent the sun's disk at the middle of the eclipse. The moon's semi-diameter being considerably less than the sun's, the eclipse is seen to be annular at Boston. Throughout the entire tract represented as covered by the moon's disk, the sun's centre must be invisible; that is, along the parallel of latitude of Boston, between the hours 3 and 7, which amounts to more than 60 degrees of longitude; and throughout a much larger area, some portion of the sun's disk will be concealed. The extent of this area may be determined by describing a circle with the same centre, and a radius equal to the sum of the radii of the sun and moon.

(254.) The eclipse must commence at Boston as soon as the disks of the sun and moon begin to interfere. Take, then, from the scale of equal parts, with a pair of compasses, an extent equal to the sum of the semi-diameters of the sun and moon, 1842″.4, and, beginning near L, set one foot on the moon's path and the other foot on the path of the spectator, and move them backward and forward till both the points fall into the same hour and minute in both paths. This will indicate the beginning of the eclipse, which, in the present case, is 4h. 30m. Do the same on the other side of the moon's path, and the end of the eclipse will be found, in the same manner, at 6h. 51m. We have thus obtained the following results for Boston:

	Apparent Time.		Mean Time.
	h. m.		*h. m.*
Beginning of eclipse	4 30	=	4 27 P.M.
Greatest obscuration	5 44	=	5 41
End of eclipse	6 51	=	6 48

The results are obtained in apparent time, because the points 1, 2, 3, etc., on the parallel of Boston, correspond to apparent time, and the places of the moon upon its relative orbit were also determined for apparent time.

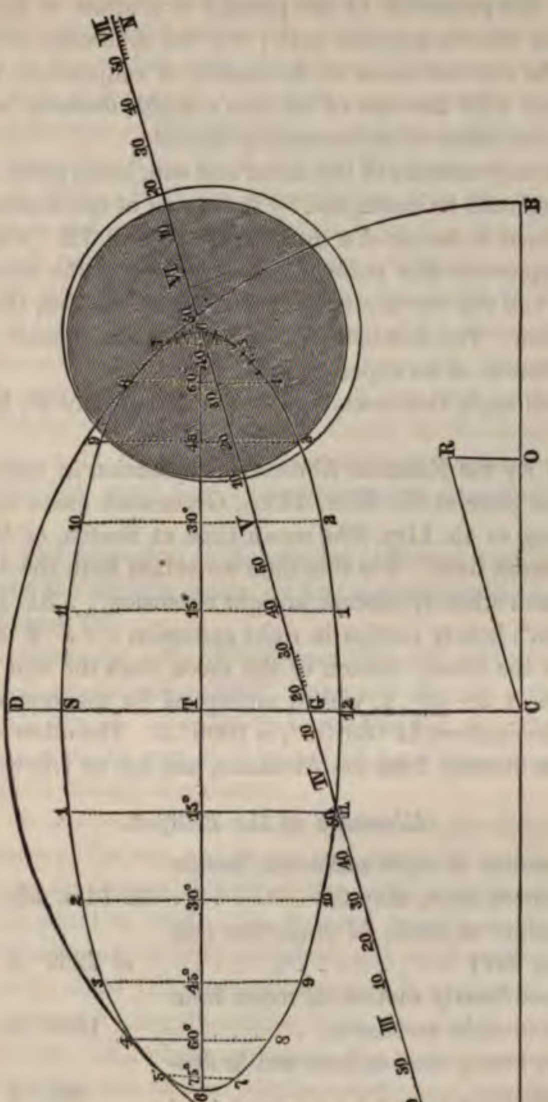
When this projection is carefully made, it will furnish the times of beginning and end within one or two minutes.

By drawing different parallels of latitude, we may determine the phases of the eclipse at any number of places required.

SECOND METHOD OF PROJECTION.

(255.) In the preceding projection we have employed the longitude and latitude of the sun and moon, as well as their hourly motions in longitude and latitude; but the projection may be made with about equal facility by employing the right ascension and declination of these bodies. This method differs from the preceding in only a few particulars.

In the figure on the opposite page, AC, the radius of the circle of projection, is the difference of the horizontal parallaxes of the sun and moon; CD is a meridian or circle of declination;


A
B
C
O
R
D
S
T
G
L
K
VII
VI
V
IV
III
15°
30°
45°
60°
75°
12
1
2
3
4
5
6
7
8
9
10
11
10
20
30
40
50

6, 12, 6 the projection of the parallel of latitude of the place; LGN the moon's apparent path; CG the difference of declination of the sun and moon at the instant of conjunction in right ascension; C12 the sine of the sun's zenith distance at noon; and 6T the radius of the parallel of latitude.

The hourly motions of the moon and sun, being given in right ascension, must be multiplied by the cosine of the declination to reduce them to an arc of a great circle, by Art. 72. CO, in the figure, represents this reduced hourly motion of the moon from the sun, and OR the hourly motion of the moon from the sun in declination. The distance CR represents the moon's relative hourly motion on its apparent path.

We will apply this method to the eclipse of May 26, 1854, for Boston.

(256.) By the Nautical Almanac, conjunction in right ascension takes place at 8h. 55m. 43.2s., Greenwich mean time, corresponding to 4h. 11m. 29s. mean time at Boston, or 4h. 14m. 44s. apparent time. For this time we obtain from the Almanac

the moon's hourly motion in right ascension . .	31′ 18″.9.
" sun's hourly motion in right ascension . . .	2′ 31″.8.

Hence the hourly motion of the moon from the sun in right ascension is 28′ 47″.1, which, multiplied by the cosine of the moon's declination, 21° 33′ 32″, is 1606″.3. The other elements are taken directly from the Almanac, and are as follows:

Elements of the Eclipse.

Conjunction in right ascension, Boston apparent time, May 26	4h. 14m. 44s. P.M.
R = radius of circle of projection (see page 237)	= 3259″.1.
Reduced hourly motion of moon from sun in right ascension	1606″.3.
Moon's hourly motion from sun in declination	461″.4.
Moon north of sun	1335″.0.
Sum of semi-diameters of sun and moon	1842″.4.
Difference of semi-diameters of sun and moon	55″.4.

Take AC, equal to 3259″, from any convenient scale of equal parts, and describe the semicircle ADB to represent the northern half of the earth's disk, and draw CD perpendicular to AB for the axis of the earth. Open the sector to the radius AC, and take the sine of $20° 59' = \phi' - \delta$, and set it off from C to 12, on the line CD. This point will be the apparent position of Boston at noon. With the same radius, take the sine of $63° 21' = \phi' + \delta$, and set it off from C to S, on the line CD. The point S will represent the apparent place of Boston at midnight. Bisect the line 12S in T, and through T draw 6, T, 6 perpendicular to CD. With the same opening of the sector as before, take the cosine of the latitude of the place, 42° 10′, and set it off each way from T to 6 and 6. These will be the points where Boston will be seen at six o'clock in the morning and six o'clock in the evening. The apparent path of Boston across the earth's disk must now be represented as described on page 240.

(257.) The projection of the parallel of Boston may be effected without the aid of a sector, by first computing the quantities C12, CS, and T6.

$$C12 = R \sin.(\phi' - \delta) = 3259''.1 \sin. 20° 58' 43'' = 1167''.$$
$$CS = R \sin.(\phi' + \delta) = 3259''.1 \sin. 63° 21' 17'' = 2913''.$$
$$T6 = R \cos.\phi' \quad = 3259''.1 \cos. 42° 10' \ 0'' = 2416''.$$

These quantities may now be set off from the same scale as AC, without the aid of a sector.

Take an interval equal to 1335″, which is the difference of declinations of the sun and moon, and set it off on CD from C to G, above the line ACB, because the moon is north of the sun. Take CO, equal to 1606″, the reduced hourly motion of the moon from the sun in right ascension, and set it on the line CB, from C to O; draw OR perpendicular to CB, and make it equal to 461, the moon's hourly motion from the sun in declination. Draw the line CR, which represents the hourly motion of the moon from the sun on the relative orbit; and parallel to this line draw the relative orbit of the moon, LGN. At the instant of conjunction in right ascension, the moon's centre will be at G.

(258.) Measure the distance CR on the scale, and say, as 60m. : CR :: the minutes of the time of conjunction : the distance from G to the first full hour point to the left. CR is found by the scale to be 1671″.

$$60\text{m.} : 1671 :: 14.7\text{m.} : 409''.$$

Take 409″ from the scale, and set it from G to IV. Take the distance CR, and set it from IV, along the moon's path, to V, VI, etc., and divide each hour into ten-minute spaces. Take from the scale the sum of the semi-diameters of the sun and moon, and running the *left* foot of the dividers along the moon's path, while the other is kept on the ellipse, notice when both stand on the same hour space. Subdivide that portion of the moon's orbit into single minute spaces, and that on the ellipse into 10 or 5 minute spaces. Do the same, keeping the *right* foot of the dividers on the moon's path, and subdivide the spaces in like manner. Also, notice what hour, or portion of an hour, on the moon's path is nearest to the corresponding hour on the ellipse, and subdivide these portions in the same way. Applying the dividers set to 1842″, we find that the feet stand on corresponding divisions when the left foot on the moon's path marks 4h. 29m. 40s., and also when the right foot marks 6h. 50m. 35s.; the former denoting the time of beginning, and the latter the time of ending of the eclipse at Boston.

Apply one side of a small square to the moon's path, and move it along until the other side cuts the same hour and minute on both lines. This is the moment of nearest approach of centres, which is at 5h. 44m. 10s. The distance between these corresponding points, measured on the scale, is 47″, which is the distance of the centres of the sun and moon at that time. This being less than 55″.4, the difference of the semi-diameters of the sun and moon, shows that the eclipse at Boston will be *annular*.

(259.) With a radius equal to 893″, from the point 5h. 44m. 10s. of the moon's path as a centre, describe a circle representing the moon's disk. With the corresponding point of the ellipse as a centre, and a radius equal to 949″, describe a second circle to represent the sun's disk. These circles will exhibit the phase of the eclipse at the moment of greatest obscuration. The figure represents the visible portion of the sun at this time as an unequal ring, extremely narrow on its northern side. With the dividers open to 55″, the times of formation and rupture of the ring may be determined in the same manner as the beginning and end of the eclipse.

The results of the projection are as follows:

	Apparent Time. h. m. s.		Mean Time. h. m. s.
Beginning of the eclipse at Boston, May 26, 1854	4 29 40	=	4 26 25 P.M.
Greatest obscuration	5 44 10	=	5 40 55
End of eclipse	6 50 35	=	6 47 20

In a working projection, for determining the phases of an eclipse for a particular place, it is not necessary to describe all the lines given in the figure. Thus, in the present example, it was only necessary to draw that portion of the path of Boston corresponding to the three hours which include the eclipse, viz., from 4 to 7 P.M.; but this part should be drawn with the utmost care. So, also, it is only necessary to draw the moon's path for the same hours; but the portions corresponding to the times of beginning, middle, and end of the eclipse should be subdivided as accurately as possible.

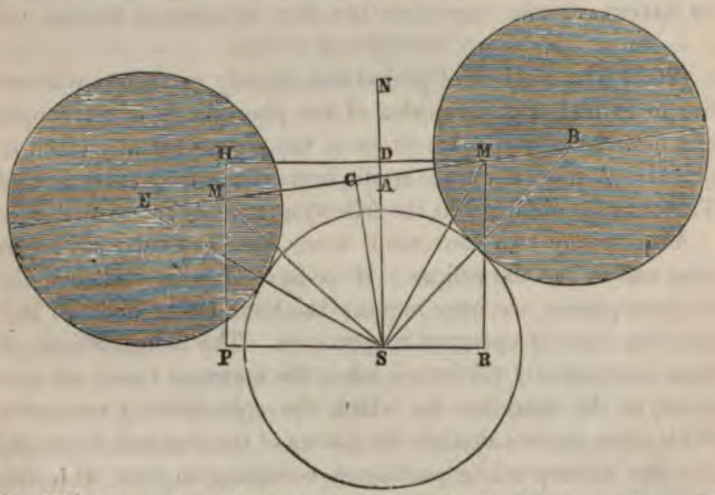
SECTION II.

TO CALCULATE THE BEGINNING AND END OF A SOLAR ECLIPSE FOR A PARTICULAR PLACE.

(260.) The method of projections already explained will suffice to furnish a general idea of the phenomena of an eclipse, and also the approximate times of the phases for any place required. A more accurate result, however, is frequently needed. This may be obtained in the following manner:

Assume any two convenient times near the supposed beginning and end of the eclipse. If we have no previous knowledge of these phases, we may assume the hour before and the hour after the time of apparent conjunction. The computations are most conveniently performed when the assumed times are even hours of the meridian for which the ephemeris is computed. With these times calculate the places of the sun and moon, and also the corresponding parallaxes, according to Arts. 211, 212. The relative positions of the sun and moon will be the same, if we attribute to the moon the effect of the difference of the parallaxes of the two bodies, and suppose the sun to remain in his true position. This difference is, therefore, the relative parallax, or that which influences the relative position of the two bodies. The difference between the equatorial parallaxes of the sun and moon must be multiplied by the radius of the earth for the place of observation, in order to obtain the parallax of the place, Art. 210. These parallaxes, applied to the right ascensions and declinations of the moon for the hours proposed, as given in the Nautical Almanac, will furnish its apparent right ascensions and declinations. Take the difference between the apparent places of the moon and sun, and reduce the differences of right ascension to seconds of arc of a great circle. These apparent positions of the moon with respect to the sun will furnish its apparent relative orbit; and the contact of limbs will evidently take place when the apparent distance of the centres becomes equal to the sum of the sun's semi-diameter and the augmented semi-diameter of the moon.

(261.) Let S represent the position of the sun's centre, which we will suppose to remain at rest throughout the eclipse; let SR and SP represent the apparent differences of right ascension of the sun and moon for the two selected hours, one preceding and the other following the apparent conjunction, and RM, PM′ the differences of declination. Draw the line MM′, and it will represent the relative direction of the moon's motion. Let SN



be the meridian passing through S, and suppose B and E to be the positions of the moon at the times of beginning and ending of the partial eclipse. Draw MDH perpendicular to SN, and SC perpendicular to BE.

In the triangle SDM, we have

SD : DM :: rad. : tang. DSM.

Also, sin. DSM : DM :: rad. : SM.

In the triangle HMM′, HM represents the hourly motion in right ascension, reduced to the arc of a great circle, and HM′ the hourly motion in declination, and we have

HM : HM′ :: rad. : tang. HMM′ or NSC.

Also, cos. HMM′ : rad. :: HM : MM′,

which is the hourly motion of the moon in its relative orbit.

The angle MSC = MSD + DSC.

Then, in the triangle MSC,

rad. : SM :: cos. MSC : SC.

In the triangle BSC, BS represents the sum of the radii of the sun and moon, and we have

BS : SC :: rad. : cos. BSC.

The angle BSM = BSC − MSC.

Also, ESM = BSC + MSC.

Now, in the triangles BSM and ESM, we have

sin. SBM : SM :: sin. BSM : BM,

and sin. SEM : SM :: sin. ESM : EM.

Also, the time of describing $\mathrm{BM} = \frac{\mathrm{BM}}{\mathrm{MM'}}$,

and the time of describing $\mathrm{EM} = \frac{\mathrm{EM}}{\mathrm{MM'}}$.

The time of describing BM being subtracted from the time when the moon's centre was at M, will furnish the instant of beginning of the eclipse; and the time of describing EM being added to the time when the moon was at M, will furnish the instant of ending.

(262.) *Ex.* 1. Required the time of the beginning and ending of the solar eclipse of July 28, 1851, at Cambridge, latitude 42° 22′ 48″ N., longitude 4h. 44m. 30s. W. of Greenwich.

The time of new moon, July 28, is 2h. 40m. Greenwich time; but as the sun at Cambridge is near the eastern horizon, the effect of parallax will be to accelerate the eclipse, and we will therefore select for our two hours of computation 1h. and 2h. Greenwich time. For these times we take out the right ascensions and declinations of the sun and moon from the Nautical Almanac. The moon's equatorial horizontal parallax at 1h. is 60′ 28″.6; the sun's horizontal parallax is 8″.4; difference, 60′ 20″.2; reduction to the latitude of Cambridge, 5″.5; making the relative horizontal parallax for Cambridge 60′ 14″.7. At 2h. we find it to be 60′ 15″.6.

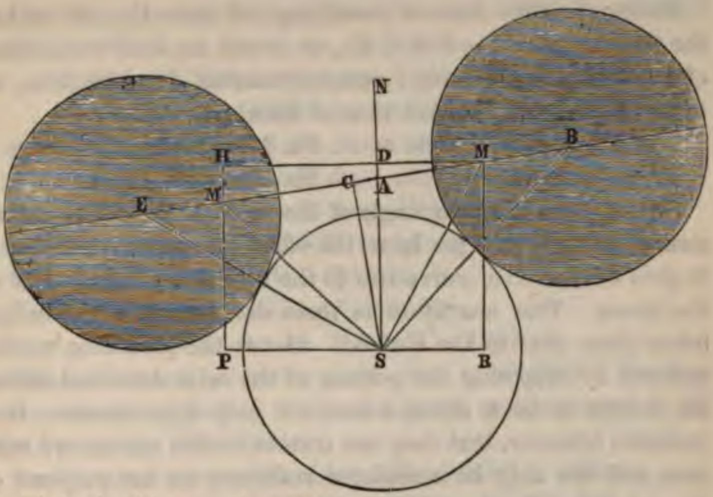
The moon's hour angle from the meridian is equal to the sidereal time, minus the moon's true right ascension. July 28, 1h. at Greenwich corresponds to July 27, 20h. 15m. 30s. mean time at Cambridge, which, converted into sidereal time by Art. 159, is 4h. 37m. 53.13s. Subtracting this from the moon's right ascension, we obtain the hour angle, 3h. 47m. 23.50s., or 56° 50′ 52″.5. In the same manner, the hour angle at 2h. is found to be 42° 27′ 26″.1. With these data we compute the parallaxes in right ascension and declination by Arts. 211 and 212. The differences of right ascension are reduced to seconds of arc of a great circle by multiplying them by 15 × cosine of the moon's apparent declination.

According to Art. 228, the declination of the point M is not exactly the same as that of D (MD being supposed to be a perpendicular let fall on the meridian NS). From Table XVIII. we find the correction to be added to the moon's declination at 1h., for a difference of right ascension of 37s. is 0″.3; and at 2h., for a difference of right ascension of 1m. 18s. is 1″.1.

Hence we obtain the following results:

	For 1h. Greenwich Time.		For 2h. Greenwich Time.	
	R. A.	Dec.	R. A.	Dec.
	h. m. s.	° ′ ″	h. m s.	° ′ ″
Moon's true place........	8 25 16.63	19 58 9.4 N.	8 27 52.72	19 52 40.0 N.
Moon's parallax	2 40.24	30 4.9	2 9.51	27 15.5
Moon's apparent place...	8 27 56.87	19 28 4.5	8 30 2.23	19 25 24.5
Sun's place	8 28 34.17	19 4 50.2	8 28 44.00	19 4 15.8
Difference	37.30	23 14.3	1 18.23	21 8.7
Reduced to seconds of arc	527.5	1394.6	1106.7	1269.8

Accordingly, we find SR represents 527″.5; RM, 1394″.6; PS, 1106″.7; PM′, 1269″.8. The hourly motion in declination is therefore 124″.8; and that in right ascension, reduced to an arc of a great circle, is 1634″.2.



Then, in the triangle SDM, we have

1394″.6 : 527″.5 :: 1 : tang. 20° 43′ 8″ = DSM.

Also, sin. DSM : 527″.5 :: 1 : SM = 1491″.0.

In the triangle HMM′, we have

1634″.2 : 124″.8 :: 1 : tang. HMM′ = 4° 22′ 1″.

Also,

cos. HMM′ : 1 :: 1634″.2 : MM′ = 1639″.0 = hourly motion of the moon in its relative orbit.

MSC = MSD + DSC = 25° 5′ 9″.

In the triangle MSC, we have

1 : 1491″.0 :: cos. MSC : SC = 1350″.4.

The moon's semi-diameter is not the same for the beginning and end of the eclipse; but for a first approximation we will suppose it to remain unchanged, and will compute it for the time 1h. 30m., which we find to be 16′ 28″.9. The augmentation for altitude, Art. 217, is 11″.8. The sun's semi-diameter is 15′ 46″.5, making SB = 32′ 27″.2 = 1947″.2. Then

1947″.2 : 1350″.4 :: 1 : cos. BSC = 46° 5′ 32″.

Therefore BSM = 21° 0′ 23″,

ESM = 71° 10′ 41″.

Then sin. SBM : 1491″.0 :: sin. BSM : BM = 770″.7,

sin. SEM : 1491″.0 :: sin. ESM : EM = 2035″.0.

The time of describing BM = 0h. 28m. 13s.

The time of describing EM = 1h. 14m. 30s.

Subtracting the time of describing BM from 1h., and adding the time of describing EM to 1h., we obtain the Greenwich times of beginning and ending; and, subtracting 4h. 44m. 30s., we obtain the results in mean time of Cambridge; viz.,

Beginning of the eclipse at . . 7h. 47m. 17s. } Cambridge
End of the eclipse . . . . 9h. 30m. 0s. } mean time.

(263.) Since the hour angle of the moon is subject to the variation of nearly 15° per hour, the effect produced by parallax is to give considerable curvature to the apparent relative orbit of the moon. This curvature is more decided when the eclipse takes place near to the horizon. Hence the preceding results, deduced by supposing the portion of the orbit described during the eclipse to be a straight line, are only approximate. It is probable, however, that they are correct within one or two minutes, and this may be considered sufficient for the purposes of the observer. If a more accurate determination is required, we must repeat the computation for the times here obtained; and it is better to conduct the computations for the beginning and end independently of each other, deriving the beginning of the eclipse from the assumed time near the beginning, and the end from the assumed time near the end. For convenience, we may omit the seconds, and repeat the computation for 0h. 32m. and 2h. 15m. Greenwich time. For these times we look out the places of the sun and moon from the Almanac. The relative horizontal parallax at 0h. 32m. is 60′ 14″.2; at 2h. 15m. is 60′ 15″.9. The moon's hour angle from the meridian for 0h. 32m., Greenwich time, is 63° 33′ 48″.4; for 2h. 15m. it is 38° 51′ 34″.2, from which we obtain the parallaxes as below. Proceeding as in the former case, we obtain the following results:

	For 0h. 32m. Greenwich Time.		For 2h. 15m. Greenwich Time.	
	R. A.	Dec.	R. A.	Dec.
	h. m. s.	° ′ ″	h. m. s.	° ′ ″
Moon's true place	8 24 3.76	20 0 40.1 N.	8 28 31.73	19 51 16.3 N.
Moon's parallax	2 51.16	31 35.5	2 0.43	26 39.4
Moon's apparent place . . .	8 26 54.92	19 29 4.6	8 30 32.16	19 24 36.9
Sun's place	8 28 29.59	19 5 6.2	8 28 46.46	19 4 7.1
Difference	1 34.67	23 58.4	1 45.70	20 29.8
Reduced to seconds of arc	1338.7	1439.8	1495.4	1231.5

The motion in declination for 1h. 43m. is 208″.3; hence the motion for 1h. is 121″.3; and in right ascension it is 1650″.9; which values differ a little from those found on page 250.

Let SR in the figure, page 251, represent 1338″.7; and RM 1439″.8. Then, as before, we shall have

1439″.8 : 1338″.7 :: 1 : tang. DSM = 42° 54′ 58″,
sin. DSM : 1338″.7 :: 1 : SM = 1966″.0.
Also, 1231″.5 : 1495″.4 :: 1 : tang. DSM′ = 50° 31′ 40″,
sin. DSM′ : 1495″.4 :: 1 : SM′ = 1937″.2.
1650″.9 : 121″.3 :: 1 : tang. HMM′ = 4° 12′ 8″,
cos. HMM′ : 1 :: 1650″.9 : 1655″.4 = hourly motion in orbit.

Therefore MSC = 47° 7′ 6″,
M′SC = 46° 19′ 32″,
1 : 1966″.0 :: cos. MSC : SC = 1337″.8.

The moon's semi-diameter at 0h. 32m. is 16′ 28″.6; the augmentation for altitude is 9″.3; the sun's semi-diameter is 15′ 46″.5; making SB = 1944″.4.

In the same manner we obtain SE = 1949″.1. Then

1944″.4 : 1337″.8 :: 1 : cos. BSC = 46° 31′ 33″.
Therefore BSM = 0° 35′ 33″.
Then sin. SBM : 1966″.0 :: sin. BSM : BM = 29″.6.
The time of describing BM = 64.3s.
Also, 1949″.1 : 1337″.8 :: 1 : cos. ESC = 46° 39′ 24″.
Therefore ESM′ = 0° 19′ 52″.
sin. SEM′ : 1937″.2 :: sin. ESM′ : EM′ = 16″.3.
The time of describing EM′ = 35.5s.

Hence the

Beginning of the eclipse is at . 7h. 48m. 34s. } Cambridge
End of the eclipse is at . . . . 9h. 31m. 5s. } mean time.

(264.) In observing the beginning of a solar eclipse, it is important for the accuracy of the observation that we should know on what part of the sun's limb the eclipse will begin. This is easily found by means of the diagram, page 251. The angle NSB is the angle of position of the moon's centre from the north toward the west, at the beginning of the eclipse; or, if we estimate the angle of position from the north toward the east, it will be 360° − NSB. Also, the angle of position from the north toward the east, at the end of the eclipse, is NSE.

But NSB = CSB − CSN = 46°.5 − 4°.2 = 42°.3,
and NSE = CSE + CSN = 46°.6 + 4°.2 = 50°.8.

Hence, at the beginning of the eclipse, the angle of the moon's centre from the north toward the east is 317°.7.

At the end, the angle of the moon's centre from the north toward the east is 50°.8.

(265.) The following formulæ embody the preceding principles in a form convenient for computation:

Put $x = \mathrm{SR} =$ the difference of apparent right ascension between the sun and moon in arc of a great circle, at an assumed instant;
$y = \mathrm{RM} =$ the difference of apparent declination at the same instant, corrected by Art. 228;
$z = \mathrm{SM}$;
$v = \mathrm{SC}$;
$\beta =$ the angle NSM;
$\iota =$ the angle $\mathrm{HMM'} = \mathrm{DSC}$;
$\gamma =$ the angle $\mathrm{BSC} = \mathrm{ESC}$;
$x_{,} = \mathrm{MH} =$ the hourly variation of x;
$y_{,} = \mathrm{HM'} =$ the hourly variation of y;
$\Delta = \mathrm{BS} =$ sum of the semi-diameters of sun and moon.

$$\text{tang. } \beta = \frac{x}{y},$$

$$z = \frac{x}{\sin.\,\beta} = \frac{y}{\cos.\,\beta},$$

$$\text{tang. } \iota = \frac{y_{,}}{x_{,}},$$

$$\frac{x_{,}}{\cos.\,\iota} = \text{hourly motion in relative orbit},$$

$$\mathrm{MSC} = \beta + \iota,$$

$$v = z \cos.\,(\beta + \iota),$$

$$\cos.\,\gamma = \frac{v}{\Delta}.$$

$$\text{Angle BSM} = \gamma - (\beta + \iota),$$

$$\mathrm{ESM} = \gamma + (\beta + \iota),$$

$$\mathrm{BM} = \frac{z}{\cos.\,\gamma} \sin.\{\gamma - (\beta + \iota)\},$$

$$\mathrm{EM} = \frac{z}{\cos.\,\gamma} \sin.\{\gamma + (\beta + \iota)\}.$$

$$\text{Time of describing BM} = \frac{\mathrm{BM} \cos.\,\iota}{x_{,}}.$$

$$\text{Time of describing EM} = \frac{\mathrm{EM} \cos.\,\iota}{x_{,}}.$$

After we have obtained the approximate times of beginning and ending, if the greatest accuracy is required, we must repeat the computation, with separate values of Δ for beginning and end, as was done in the last example.

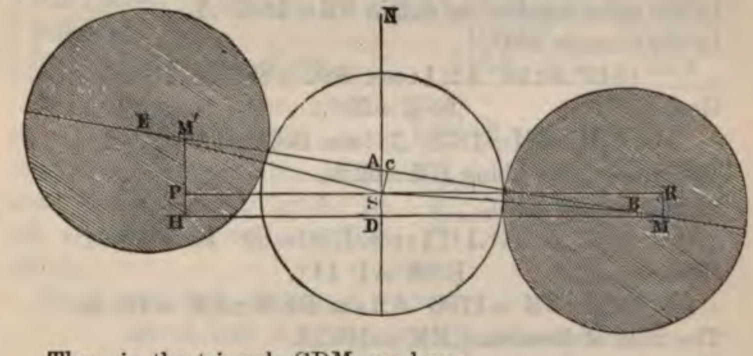
(266.) *Ex.* 2. Required the time of beginning and end of the solar eclipse of May 26, 1854, at Cambridge Observatory.

As we have already projected this eclipse, we shall avail ourselves of the approximate knowledge already obtained, and shall assume for our times of computation 9h. 10m., and 11h. 30m., Greenwich mean time. For these times we take the places of the sun and moon from the Nautical Almanac. The moon's equatorial horizontal parallax at 9h. 10m. is 54′ 32″.5; the sun's horizontal parallax is 8″.5; difference, 54′ 24″.0; which, reduced to the latitude of Cambridge, becomes 54′ 19″.1. At 11h. 30m. we find it to be 54′ 17″.3.

The sidereal time at Cambridge, corresponding to 9h. 10m. Greenwich mean time, is 8h. 41m. 55.21s. Hence the moon's hour angle is 4h. 28m. 17.98s., or 67° 4′ 29″.7. The hour angle at 11h. 30m. is 100° 57′ 1″.8. With these data we obtain the parallaxes as below. The following are the results:

	For 9h. 10m. Greenwich Time.		For 11h. 30m. Greenwich Time.	
	R. A. h. m. s.	Dec. ° ′ ″	R. A. h. m. s.	Dec. ° ′ ″
Moon's true place	4 13 37.23	21 35 27.7 N.	4 18 30.09	21 54 4.9 N.
Moon's parallax	2 40.23	28 27.5	2 49.83	36 50.0
Moon's apparent place ...	4 10 57.00	21 7 0.2	4 15 40.26	21 17 14.9
Sun's place	4 13 9.83	21 11 22.9	4 13 33.46	21 12 22.9
Difference	2 12.83	4 22.7	2 6.80	4 52.0
Reduced to seconds of arc	1858.7	259.4	1772.2	295.0

The hourly motion in declination is 237″.6, and that in right ascension 1556″.1.



Then, in the triangle SDM, we have

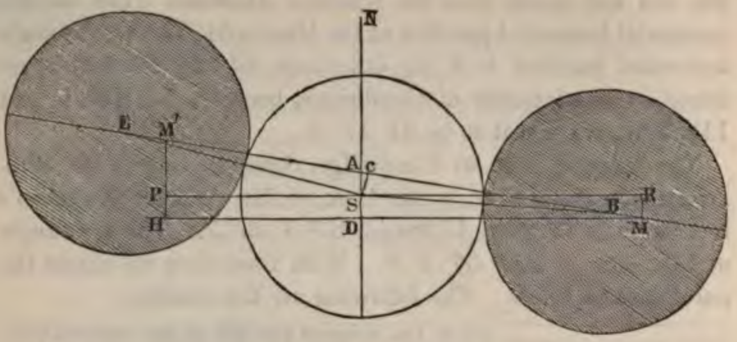
259″.4 : 1858″.7 :: 1 : tang. DSM = 82° 3′ 18″,
sin. DSM : 1858″.7 :: 1 : SM = 1876″.7.

Also, 295″.0 : 1772″.2 :: 1 : tang. PM′S = 80° 32′ 57″,
sin. PM′S : 1772″.2 :: 1 : SM′ = 1796″.6.

To avoid confusion, the lines SM, SM′ are omitted from the figure, but are to be supplied as on page 248.

In the triangle HMM′, we have

1556.″1 : 237″.6 :: 1 : tang. HMM′ = 8° 40′53″,
cos. HMM′ : 1 :: 1556″.1 : 1574″.1 = the hourly motion in orbit.



Hence MSC = 89° 15′ 49″,
M′SC = 89° 13′ 50″.

In the triangle MSC,

1 : SM = 1876″.7 :: cos. MSC : SC = 24″.1.

The moon's semi-diameter at 9h. 10m. is 14′ 53″.5; the augmentation for altitude is 7″.2; the sun's semi-diameter is 15′ 48″.9; making SB = 1849″.6.

In the same manner we obtain SE = 1843″.5.

In the triangle BSC,

1849″.6 : 24″.1 :: 1 : cos. BSC = 89° 15′ 10″.

Hence BSM = 39″.

sin. SBM : SM = 1876″.7 :: sin. BSM : BM = 27″.2.

The time of describing BM = 62.2s.

In the triangle ESC,

1843″.5 : 24″.1 :: 1 : cos. ESC = 89° 15′ 1″.

Hence ESM′ = 1′ 11″.

sin. SEM′ : SM′ = 1796″.6 :: sin. ESM′ : EM′ = 47″.3.

The time of describing EM′ = 108.1s.

Hence the eclipse begins at . . 4h. 26m. 32s. } Cambridge
" ends " . . 6h. 47m. 18s. } mean time.

At the beginning, the angle of the moon's centre from north toward east is 262° 4′.

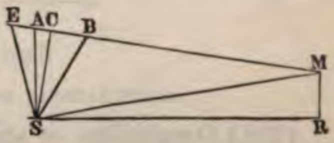
At the end, the angle of the moon's centre from north toward east is 80° 34′.

(267.) As the computation thus far indicates that this eclipse will be annular, it is important to determine precisely the time of formation, and also of the rupture of the ring. In doing this, we can not assume that the moon's path from 9h. 10m. to 11h.

30m. is a straight line. By inspecting Table XVI., we shall see that the parallax in right ascension increases with the hour angle until this angle becomes six hours; and after that it diminishes. Now at the middle of this eclipse, the moon's hour angle is very nearly 6 hours; so that the parallax in right ascension is greater for the middle of the eclipse than for either the beginning or end. We must, therefore, make an independent computation for a time near to the middle of the eclipse, which we will assume at 10h. 20m. Greenwich time.

Proceeding as heretofore, we find the moon's relative parallax, reduced to the latitude of Cambridge, to be 54′ 18″.2, and the moon's hour angle 84° 0′ 47″.5, whence we obtain the following results:

	For 10h. 20m. Greenwich Time.	
	R. A. h. m. s.	Dec. ° ′ ″
Moon's true place	4 16 3.54	21 44 50.5 N.
Moon's parallax	2 52.54	32 34.3
Moon's apparent place	4 13 11.00	21 12 16.2
Sun's place	4 13 21.64	21 11 52.9
Difference	10.64	23.3
Reduced to seconds of arc . .	148.8	23.3



In the annexed figure, let MA represent a portion of the moon's relative orbit on a much larger scale than the former figure; let SR represent 148″.8, and RM 23″.3.

Then, in the triangle SMR,

$$23''.3 : 148''.8 :: 1 : \text{tang. SMR} = 81^\circ\ 6'\ 1'',$$

$$\text{sin. SMR} : 148''.8 :: 1 : \text{SM} = 150''.6.$$

The angle $\text{MSC} = \text{SMR} - \text{ASC} = 72^\circ\ 25'\ 8''.$

$$1 : \text{SM} :: \text{cos. MSC} : \text{SC} = 45''.5,$$

$$1 : \text{SM} :: \text{sin. MSC} : \text{CM} = 143''.6.$$

The time of describing CM = 328.3s.

Hence the nearest approach of the centres of the sun and moon is at 5h. 40m. 58.3s. Cambridge mean time.

The semi-diameter of the sun is 15′ 48″.9; the augmented semi-diameter of the moon is 14′ 57″.6; difference, 51″.3. The least distance between the centres of the sun and moon is 45″.5

Hence the eclipse will be annular. To find the times of formation and rupture of the ring, with S as a centre, and a radius equal to $51''.3$, describe an arc, cutting the moon's path in the points B and E, which will represent the points required.

Then, in the triangle SCB,

$$\mathrm{SB} : 1 :: \mathrm{SC} : \cos.\ \mathrm{BSC} = 27^\circ\ 32',$$
$$1 : \mathrm{SB} :: \sin.\ \mathrm{BSC} : \mathrm{BC} = 23''.7.$$

The time of describing BC = 54.2s.

Hence the

Formation of the ring is at . . 5h. 40m. 4s. } Cambridge
Rupture of the ring is at . . . 5h. 41m. 52s. } mean time.

The preceding computations were all in type in 1853, but owing to the destruction of the stereotype plates by fire in December of that year, it became necessary to re-cast the entire volume, and thus its publication has been delayed until after the occurrence of the eclipse. The eclipse could not be observed at Cambridge on account of the interference of clouds. The instants of first and last contact observed at New York and Washington differed but a few seconds from the time computed from the Tables.

SECTION III.

OCCULTATIONS OF STARS BY THE MOON.

(268.) Occultations of stars by the moon may be computed in the same manner as eclipses of the sun, the only difference in the operation consisting in this, that the star has neither motion, parallax, nor semi-diameter. These circumstances render the computation of an occultation more simple than that of an eclipse.

Ex. 1. It is required to find the times of immersion and emersion of α Tauri, Jan. 23, 1850, at Cambridge Observatory, latitude 42° 22′ 48″, longitude 4h. 44m. 30s. W. of Greenwich.

The Greenwich mean time of apparent conjunction, according to the Nautical Almanac, is 12h. 41m. 49s. We will, therefore, select 12h. and 13h. as the two hours of computation for the first approximation.

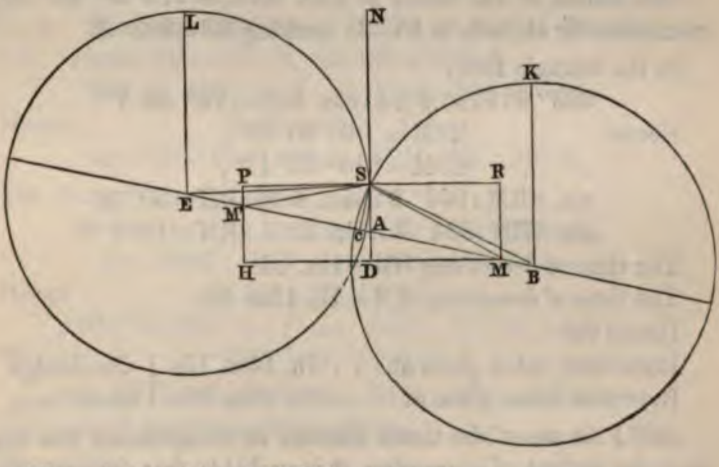
For these times we find the following data:

	For 12h. Greenwich Time.		For 13h. Greenwich Time.	
	R. A.	Dec.	R. A.	Dec.
	h. m. s.	° ′ ″	h. m. s.	° ′ ″
Moon's true place........	4 25 36.22	16 30 4.1 N.	4 28 4.53	16 36 33.2 N.
Moon's parallax	47.06	26 42.7	0.47	26 13.2
Moon's apparent place....	4 26 23.28	16 3 21.4	4 28 5.00	16 10 20.0
Star's place	4 27 19.54	16 12 3.4	4 27 19.54	16 12 3.4
Difference	56.26	8 42.0	45.46	1 43.4
Reduced to seconds of arc	811.0	521.6	654.9	103.1

The moon's horizontal parallax, reduced to the latitude of Cambridge at 12h., is 59′ 44″.6; at 13h. it is 59′ 46″.6. The moon's hour angle at 12h. is 14° 34′ 4″.8 east; at 13h. it is 0° 8′ 41″.5 east of the meridian, from which we compute the parallaxes as above.

The hourly motion in right ascension is 1465″.9, and that in declination 418″.5.

Let S represent the position of the star. Take SR = 811″.0, RM = 521″.6; then M will be the position of the moon's centre at 12h. Take SP = 654″.9, PM′ = 103″.1; then M′ will be the position of the moon at 13h., and MM′ is the moon's relative orbit.



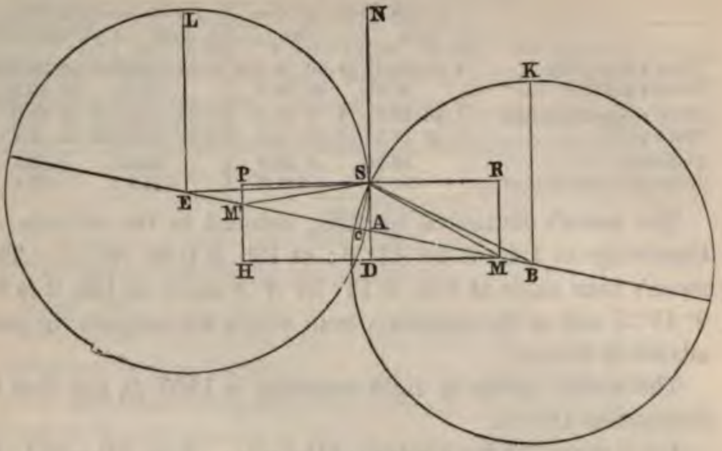
Then, in the triangle SDM, we have

521″.6 : 811″.0 :: 1 : tang. DSM = 57° 15′ 9″,
sin. DSM : 811″.0 :: 1 : SM = 964″.3.

In the triangle HMM′,

1465″.9 : 418″.5 :: 1 : tang. HMM′ = 15° 56′ 1″,
cos. HMM′ : 1 :: 1465″.9 : MM′ = 1524″.5,

the hourly motion of the moon in its orbit.



Hence $\text{MSC} = 73° \; 11' \; 10''$.

In the triangle MSC,

$$1 : 964''.3 :: \cos.\ \text{MSC} : \text{SC} = 278''.9.$$

The radius of the moon at 12h. 30m. is 978″.6; the augmentation for altitude is 15″.3; making BS=993″.9.

In the triangle BSC,

$$993''.9 : 278''.9 :: 1 : \cos.\ \text{BSC} = 73° \; 42' \; 7''.$$

Hence $\text{BSM} = 0° \; 30' \; 57''$,

$\text{ESM} = 146° \; 53' \; 17''$,

$$\sin.\ \text{SBM} : 964''.3 :: \sin.\ \text{BSM} : \text{BM} = 30''.9,$$

$$\sin.\ \text{SEM} : 964''.3 :: \sin.\ \text{ESM} : \text{EM} = 1876''.8.$$

The time of describing BM=1m. 13s.

The time of describing EM=1h. 13m. 52s.

Hence the

Immersion takes place at . . . 7h. 14m. 17s. } Cambridge
Emersion takes place at . . . 8h. 29m. 22s. } mean time.

(269.) As one of the times selected for computation was very near the instant of immersion, it is probable that the preceding result for immersion is pretty accurate. For the sake of verification, we will, however, repeat the entire computation for 12h., and 13h. 15m., Greenwich mean time.

	For 12h. Greenwich Time.		For 13h. 15m. Greenwich Time.	
	R. A.	Dec.	R. A.	Dec.
	h. m. s.	° ′ ″	h. m. s.	° ′ ″
Moon's true place........	4 25 36.22	16 30 4.1 N.	4 28 41.66	16 38 9.5 N
Moon's parallax	47.06	26 42.7	11.31	26 13.2
Moon's apparent place...	4 26 23.28	16 3 21.4	4 28 30.35	16 11 56.3
Star's place	4 27 19.54	16 12 3.4	4 27 19.54	16 12 3.4
Difference	56.26	8 42.0	1 10.81	7.1
Reduced to seconds of arc	811.0	521.6	1019.9	6.3

The moon's horizontal parallax, reduced to the latitude of Cambridge at 13h. 15m., is $59'\,47''.0$; and the moon's hour angle is $3^\circ\,27'\,38''.4$ west, from which we obtain the parallaxes as above. The hourly motion in right ascension is $1464''.7$, and in declination $412''.2$.

Hence, in the triangle HMM′,

$$1464''.7 : 412''.2 :: 1 : \text{tang. } HMM' = 15^\circ\,43'\,9'',$$

$$\text{cos. } HMM' : 1 :: 1464''.7 : MM' = 1521''.6,$$

the hourly motion of the moon in its orbit.

Hence $$MSC = 72^\circ\,58'\,18'',$$

$$1 : 964''.3 :: \text{cos. } MSC : SC = 282''.4.$$

The radius of the moon at 12h. is $978''.3$; at 13h. 15m. is $979''.0$. The augmentation at 12h. is $15''.1$; at 13h. 15m. is $15''.5$. Hence $SB = 993''.4$, and $SE = 994''.5$.

$$993''.4 : 282''.4 :: 1 : \text{cos. } BSC = 73^\circ\,29'\,6''.$$

Hence $$BSM = 0^\circ\,30'\,48'',$$

$$\text{sin. } SBM : 964''.3 :: \text{sin. } BSM : BM = 30''.4.$$

The time of describing $BM = 71.9\text{s}.$

$$6''.3 : 1019''.9 :: 1 : \text{tang. } DSM' = 89^\circ\,38'\,46'',$$

$$\text{sin. } DSM' : 1019''.9 :: 1 : SM' = 1019''.9.$$

Hence $$M'SC = 73^\circ\,55'\,37'',$$

$$994''.5 : 282''.4 :: 1 : \text{cos. } ESC = 73^\circ\,30'\,14''.$$

Hence $$ESM' = 0^\circ\,25'\,23'',$$

$$\text{sin. } SEM' : 1019''.9 :: \text{sin. } ESM' : EM' = 26''.5.$$

The time of describing $EM' = 62.7\text{s}.$

Hence the

Immersion takes place at . . . 7h. 14m. 18s. ⎫ Cambridge
Emersion takes place at . . . 8h. 29m. 27s. ⎭ mean time,

which results are almost identical with those first obtained.

The angle of position of the poin[illegible] the moon's centre at immersion, and [illegible] east, is KBS, which equals D[illegible]

The angle of position of the point S at emersion, measured from the north toward west, is LES, which equals DSE, or CSE +CSD. But if the angle be measured from north toward east, which is the usual method, it is 360° − DSE.

Hence at immersion the angle of position of the star is 58° from the north point of the moon's limb.

At emersion the angle of position of the star is 271° from the north point.

(270.) These results are doubtless correct within one or two seconds, according to the moon's places given in the Tables; but it is not to be supposed that these times are absolutely reliable to this degree of accuracy. Burckhardt's tables of the moon frequently exhibit errors of 15″, and occasionally of 30″. Now an error of 30″ in the moon's place would cause an error of more than one minute in the computed time of occultation. However accurately, therefore, the computations are performed, the result may be found erroneous by half a minute of time, and occasionally even more than a minute. For simple purposes of observation, therefore, there is little advantage in making the computations with the precision which is here attempted, and we may generally be content with the results of the first approximation. Indeed, if we take the parallaxes directly from a table, like Table XVI., and make a careful geometrical construction with scale and dividers, we may generally obtain the time of beginning and end of the occultation within a minute of the truth, which is quite sufficient to guide the astronomer in observing an immersion. For an emersion, it is desirable to know the time as accurately as possible, in order that the eye of the observer may not be fatigued by too long watching for the phenomenon.

Ex. 2. It is required to find the time of immersion and emersion of γ Virginis, January 9, 1855, at Washington Observatory, latitude 38° 53′ 39″ N., longitude 5h. 8m. 11s. W. of Greenwich, from the following data:

	Jan. 10, 0h. Gr. Mean Time.	Jan. 10, 1h. Gr. Mean Time.
Moon's right ascension . .	12h. 35m. 12.33s.	12h. 37m. 2.80s.
Moon's declination	0° 5′ 32″.3 S.	0° 19′ 36″.2 S.
Moon's equatorial hor. par.	55′ 32″.8	55′ 34″.4
Moon's true semi-diameter	15′ 10″.1	15′ 10″.6

Right ascension of γ Virginis, 12h. 34m. 18.43s.; declination, 0° 39′ 12″.1 south.

Sidereal time of mean noon at Washington, January 9, 19h. 14m. 40.33s.

Ans. Immersion, 18h. 17m. 34s. Washington mean time.
Emersion, 19h. 36m. 45s. " " "

Angle of position of star 116°, from north point toward east, at immersion.

Angle of position of star 322°, from north point toward east, at emersion.

SECTION IV.

BESSEL'S METHOD OF COMPUTING SOLAR ECLIPSES.

(271.) Bessel has developed the complete theory of eclipses in the second volume of his Astronomical Researches. We propose to exhibit the main points of this theory, together with its application to the determination of geographical longitudes.

Let S represent the centre of the sun, M that of the moon, E that of the earth, and O the place of the observer on the earth's surface. The limbs of the sun and moon will appear to be in contact when the point O is situated on the surface of the cone which circumscribes these two bodies. There are two such circumscribing cones. One of them, VTV′, has its vertex at T, between the centres of the sun and moon; the other, VT′V′, has its vertex, T′, in the prolongation of the line MS, on the side of

Fig. 1.

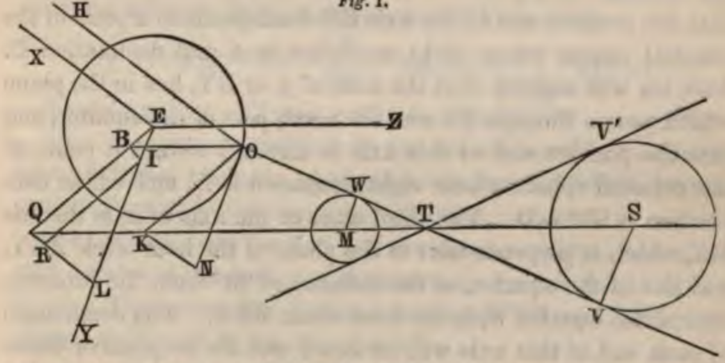
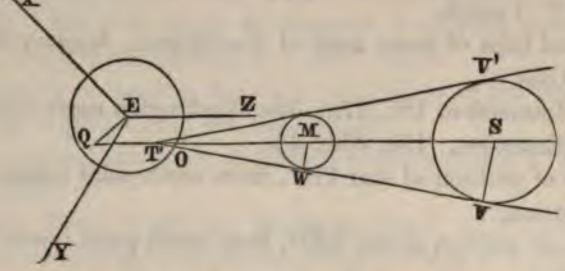


Fig. 2.



the moon which is opposite to the sun. If the point O is situated on the surface of the first cone, an observer at O will witness the external contact of the disks of the sun and moon; but if O is on the surface of the second cone, the observer will see the internal contact of the disks.

(272.) In order to obtain the equation of this conical surface, let us conceive a system of three rectangular axes, whose origin is at E, the centre of the earth. Let the axis of z, or EZ, be drawn parallel to the line MS, which joins the centres of the sun and moon. We will assume that the positive direction of this line is that which proceeds from the moon to the sun, and also that the positive end of the axis EZ corresponds to a point of the celestial sphere whose right ascension is A and declination D. Also, we will suppose that the axis of y, or EY, lies in the plane which passes through EZ and the north pole of the equator, and that the positive end of this axis is directed toward a point of the celestial sphere whose right ascension is A, and whose declination is $90° + \text{D}$. The third axis, or the axis of x, is the line EX, which is perpendicular to the plane of the hour circle ZEY, and lies in the equator, at the distance of 90° from the intersection of the equator with the hour circle ZEY. The declination of each end of this axis will be zero; and for its positive direction, we will assume that which corresponds to a right ascension of $90° + \text{A}$. We accordingly assume that, when referred to the centre of the earth,

The co-ordinates	*Determine the position*
x, y, and z,	of the centre, M, of the moon;
x', y', and z',	of the centre, S, of the sun;
ξ, η, and ζ,	of the point O, or the place of the observer.

Also, let G = the line MS, or the distance from the moon to the sun;

" f = the angle OTM, or OT′M, which the axis of the cone forms with its side;

" s = the perpendicular distance of the vertex of the cone from the plane YEX.

Since the axis EZ is parallel to the line MS, we have

$$x'=x,\ y'=y, \text{ and } z'=z+G \quad . \ . \ . \ . \ . \ . \ (1)$$

(273.) If now, from the vertex of the cone T, and from the point O, we draw the lines TQ and OB perpendicular to the plane XEY; also the lines QL and BI in this plane, perpendicular to the axis EY; and the line IR, parallel to the line BQ, we shall have

$$OB=\zeta,\ EI=\eta,\ BI=\xi;\ TQ=s,\ EL=y,\ QL=x;$$
$$IL=y-\eta, \text{ and } RL=QL-QR=QL-BI=x-\xi.$$

Draw the plane NOK parallel to the plane YEX, and passing through O, the place of the observer. In this plane draw the lines ON, OH parallel with the axes EY and EX; let the line MS produced meet the plane OHN in K, and draw KN perpendicular to ON. Then we shall have

$$KN=RL=x-\xi;\ ON=IL=y-\eta, \text{ and } OK=\sqrt{(x-\xi)^2+(y-\eta)^2}.$$

In the triangle TOK, right-angled at K, we have

$$\text{tang. } f=\text{tang. } OTK=\frac{OK}{TK};\ \text{ and } TK=TQ-KQ=s-\zeta.$$

$$\text{Therefore} \quad (x-\xi)^2+(y-\eta)^2=(s-\zeta)^2 \text{ tang.}^2 f \quad . \ . \ . \ . \ (2)$$

This equation corresponds to the conical surface in the case of an external contact. A similar one may be deduced for the conical shadow in the case of an internal contact.

(274.) Since both the sun and moon are sensibly spherical, we may represent the radius of the moon by k, and that of the sun by k'. Then, from the similar triangles, MTW and STV, right-angled at W and V, we shall have

$$ST : SV :: MT : MW.$$

Also, $$ST+TM=G.$$

And $$ST \,.\, \text{sin. } STV=SV,$$
$$MT \,.\, \text{sin. } MTW=MW.$$

But $STV=MTW=f;\ ST=z'-s;\ MT=s-z.$

Consequently,

$$z'-s : k' :: s-z : k;\ \text{ and } G \text{ sin. } f=k'+k.$$

Consider now the conical surface which corresponds to the internal contact, and whose vertex is at T′, *Fig.* 2. In this case we shall have

$$T'M = QM - QT' = z - s;\ T'S = z' - s, \text{ and } MS = T'S - T'M = G.$$

But

$$T'S \,.\, \sin. VT'S = SV = k';\ T'M \,.\, \sin. WT'M = MW = k.$$

Consequently, in the case of an internal contact, we shall have

$$z' - s : k' :: z - s : k; \text{ and } G \sin. f = k' - k.$$

Hence, by reduction, we obtain for an external contact,

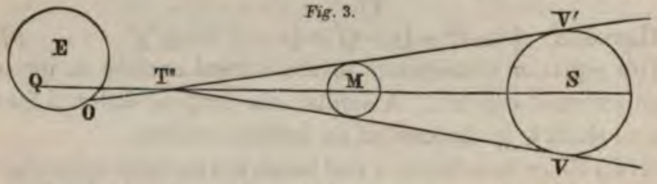
$$s = \frac{zk' + z'k}{k' + k}, \text{ and } \sin. f = \frac{k' + k}{G}.$$

Also, for an internal contact,

$$s = \frac{zk' - z'k}{k' - k}, \text{ and } \sin. f = \frac{k' - k}{G}.$$

(275.) It remains to consider in what cases the angle f is acute, and when obtuse. For an observer at the point O, on the earth's surface, the vertex of the conical shadow may be situated either on the same side of the heavens as the eclipsing body, or on the opposite side. The first case always happens at an external contact, and also at an internal contact in an annular eclipse. The vertex of the conical shadow is then found either at T, *Fig.* 1, or at T″, *Fig.* 3. The second case happens when the eclipse is total, and the contact an internal one; in which case T′, *Fig.* 2, is situated on the side of the observer, which is opposite to the sun. If, then, we reckon the angle f, which the axis of the cone forms with its side, always in the same direction, we shall have $f = OTQ$, *Fig.* 1, or $f = OT''Q$, both of which angles are acute; and $f = OT'Q$, *Fig.* 2, which is an obtuse angle. Hence we see that for an external contact the angle f is always acute, and also for an internal contact in annular eclipses; but for an internal contact in total eclipses this angle is obtuse.

Fig. 3.



(276.) We will now eliminate s and tang. f from equation (2), by employing the values just found.

$$\cos.^2 f = 1 - \sin.^2 f = \frac{G^2 - (k' \mp k)^2}{G^2},$$

$$\text{tang.}^2 f = \frac{\sin.^2 f}{\cos.^2 f} = \frac{(k' \mp k)^2}{G^2 - (k' \mp k)^2}.$$

Hence

$$(x-\xi)^2 + (y-\eta)^2 = \left(\frac{zk' \mp z'k}{k' \mp k} - \zeta\right)^2 \frac{(k' \mp k)^2}{G^2 - (k' \mp k)^2},$$

or $$(x-\xi)^2 + (y-\eta)^2 = \frac{[k'(z-\zeta) \mp k(z'-\zeta)]^2}{G^2 - (k' \mp k)^2} \quad . \; . \; . \; . \; . \; (3)$$

where the sign + belongs to the external, and − to the internal contacts.

For convenience, let us put

$$l = \frac{z(k' \mp k) \mp kG}{\mp\sqrt{G^2 - (k' \mp k)^2}} = z \text{ tang.} f \mp k \sec. f,$$

$$i = \frac{k' \mp k}{\mp\sqrt{G^2 - (k' \mp k)^2}} = \text{tang.} f \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

By substituting $z+G$ for z' in equation (3), we obtain

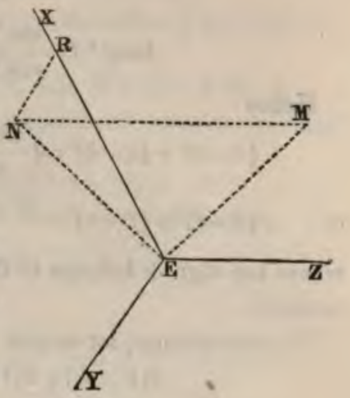
$$(x-\xi)^2 + (y-\eta)^2 = (l - i\zeta)^2 \quad . \; . \; . \; . \; . \; . \; . \; (5)$$

Comparing this equation with equation (2), we see that

$$l = s \text{ tang.} f;$$

and this represents the radius of the circle formed by the intersection of the conical shadow with the plane which passes through the centre of the earth, and perpendicular to the axis of z.

(277.) We will now show how the values of x, ξ, y, η, l, i, and ζ may be computed with the assistance of an ephemeris. For this purpose, conceive a new system of rectangular axes intersecting each other at the centre of the earth. Let EZ, the new axis of z, be directed toward the north pole of the equator; let EX, the new axis of x, be situated in the equator, and directed toward a point of the heavens whose right ascension, α', is equal to that of the sun from the earth. Let EY, the axis of y, be directed toward a point of the equator whose right ascension is $90° + \alpha'$; and also, let these directions correspond to the positive side of the co-ordinate axes. Let α, δ, and r represent the true right ascension, declination, and the distance of the moon's centre from that of the earth; and let α', δ', and R represent the same quantities for the sun's centre, where r and R are supposed to refer to the same unit of length.



Let M represent the centre of the moon, and from it let fall upon the plane XEY the perpendicular $MN=z$. From its extremity N, upon the line EX, let fall the perpendicular $NR=y$, and represent ER by x.

Then, in the triangle EMN, right-angled at N, the side $EM=r$; $MN=z$; and the angle MEN, which represents the inclination of the line EM to the equator, is $=\delta$.

Hence $\qquad EN=r\cos.\delta$; and $z=r\sin.\delta$.

Also, in the triangle ENR, right angled at R, the angle

$$NER=\alpha-\alpha'.$$

Hence

$$NR=y=EN\sin.(\alpha-\alpha')=r\cos.\delta\sin.(\alpha-\alpha');$$

and $\qquad ER=x=r\cos.\delta\cos.(\alpha-\alpha')$.

That is, we find the co-ordinates of the moon's centre,

parallel to the new axis of z, to be $r\sin.\delta$;
" " y, " $r\cos.\delta\sin.(\alpha-\alpha')$;
" " x, " $r\cos.\delta\cos.(\alpha-\alpha')$.

In the same manner, since the axis of x has the same right ascension as the sun's centre, the co-ordinates of the sun

parallel to the axis of z will be $R\sin.\delta'$;
" " y " 0;
" " x " $R\cos.\delta'$.

(278.) If we transfer the origin of co-ordinates to the centre, M, of the moon, so that the axis of z shall be directed toward the pole, the axis of x toward a point whose right ascension is α', and the axis of y toward a point whose right ascension is $90^\circ+\alpha'$, these co-ordinate axes will be parallel with those before mentioned, and we shall have for the co-ordinates of the centre of the sun referred to the moon,

parallel to the new axis of z, G sin. D;
" " y, G cos. D sin. $(A-a')$;
" " x, G cos. D cos. $(A-a')$,

since the right ascension of the sun's centre, seen from that of the moon, is A, its declination is D, and its distance is G (see page 264).

Hence we have

$$G \sin. D = R \sin. \delta' - r \sin. \delta,$$
$$G \cos. D \sin. (A-a') = -r \cos. \delta \sin. (a-a'),$$
$$G \cos. D \cos. (A-a') = R \cos. \delta' - r \cos. \delta \cos. (a-a').$$

Dividing each equation by R, and putting $\frac{G}{R}=g$, and $\frac{r}{R}=e$, we shall have

$$\left.\begin{aligned} g \sin. D &= \sin. \delta' - e \sin. \delta, \\ g \cos. D \sin. (A-a') &= -e \cos. \delta \sin. (a-a'), \\ g \cos. D \cos. (A-a') &= \cos. \delta' - e \cos. \delta \cos. (a-a'), \end{aligned}\right\} . \quad (6)$$

from which A, D, and g may be computed. Dividing the second of these equations by the third, we obtain

$$\text{tang. } (A-a') = -\frac{e \cos. \delta \sin. (a-a')}{\cos. \delta' - e \cos. \delta \cos. (a-a')}$$
$$= -\frac{e \cos. \delta \sec. \delta' \sin. (a-a')}{1 - e \cos. \delta \sec. \delta' \cos. (a-a')}.$$

Dividing the first equation by the third, we obtain

$$\text{tang. } D = \frac{(\sin. \delta' - e \sin. \delta) \cos. (A-a')}{\cos. \delta' - e \cos. \delta \cos. (a-a')}.$$

Also, from equation third,

$$g = \frac{\cos. \delta' - e \cos. \delta \cos. (a-a')}{\cos. D \cos. (A-a')}.$$

In solar eclipses the value of $A-a'$ never exceeds a few seconds, and its cosine differs from unity by a fraction which is inappreciable in the first seven decimal figures; and therefore the factor cos. $(A-a')$, in the last two formulas, may be suppressed.

(279.) The preceding expression for the value of D may be converted into an expression for the value of $D-\delta'$ by omitting the factor cos. $(a-a')$, which in a solar eclipse differs but little from unity.

By Trig., Art. 77, we have

$$\text{tang. } (A-B) = \frac{\text{tang. } A - \text{tang. } B}{1 + \text{tang. } A \text{ tang. } B}.$$

Hence

$$\text{tang.}\,(\mathrm{D}-\delta')=\frac{\dfrac{\sin.\,\delta'-e\sin.\,\delta}{\cos.\,\delta'-e\cos.\,\delta}-\text{tang.}\,\delta'}{1+\dfrac{\sin.\,\delta'\,\text{tang.}\,\delta'-e\sin.\,\delta\,\text{tang.}\,\delta'}{\cos.\,\delta'-e\cos.\,\delta}}$$

$$=\frac{\sin.\,\delta'-e\sin.\,\delta-\sin.\,\delta'+e\cos.\,\delta\,\text{tang.}\,\delta'}{\cos.\,\delta'-e\cos.\,\delta+\sin.\,\delta'\,\text{tang.}\,\delta'-e\sin.\,\delta\,\text{tang.}\,\delta'}$$

$$=\frac{-e\sin.\,\delta\cos.\,\delta'+e\cos.\,\delta\sin.\,\delta'}{\cos.^2\delta'-e\cos.\,\delta\cos.\,\delta'+\sin.^2\delta'-e\sin.\,\delta\sin.\,\delta'}.$$

That is, $$\text{tang.}\,(\mathrm{D}-\delta')=\frac{-e\sin.\,(\delta-\delta')}{1-e\cos.\,(\delta-\delta')}.$$

But since $\delta-\delta'$ is a small arc, we may, without material error, substitute the arc for its sine, and we may also use the arc $\mathrm{D}-\delta'$ instead of its tangent, and, neglecting the factor cos. $(\delta-\delta')$, we obtain

$$\mathrm{D}=\delta'-\frac{e(\delta-\delta')}{1-e}.$$

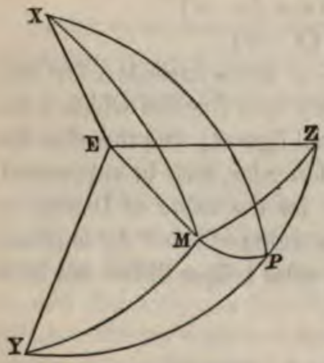
In the same manner, we obtain

$$\mathrm{A}=\alpha'-\frac{e\cos.\,\delta\sec.\,\delta'\,(\alpha-\alpha')}{1-e\cos.\,\delta\sec.\,\delta'}.$$

Also, $$g=\frac{1-e\cos.\,\delta\sec.\,\delta'}{\cos.\,\mathrm{D}\sec.\,\delta'},$$

or $$g=1-e,\text{ very nearly.}$$

(280.) In order to compute x, y, etc., we must return to our original system of co-ordinates, page 264. Conceive about the point E a sphere to be described with any radius at pleasure, and let M represent the moon's place upon this sphere. Let P represent the pole of the equator, and let Z, Y, X represent the points where this sphere is intersected by the positive ends of the above-mentioned axes. In this system, the point P will lie in the plane of the great circle ZY; and the points M, Z, Y, and X will be determined respectively by the right ascensions and declinations α and δ, A and D, A and 90°+D, 90°+A and 0°.



The co-ordinates z, y, and x of the point M, in respect to E, taken parallel to the above-mentioned axes, are equal to the projections of the line $\mathrm{EM}=r$ on these axes, or to the products of the line EM, by the cosines of the arcs ZM, YM, and XM. The cosines of these arcs may be derived from the spherical triangles ZPM, YPM, and XPM, in which the side $\mathrm{ZP}=90°-\mathrm{D}$, $\mathrm{MP}=90°-\delta$, $\mathrm{YP}=\mathrm{D}$, $\mathrm{XP}=90°$; also, the angle $\mathrm{ZPM}=\mathrm{A}-a$, $\mathrm{YPM}=180°-(\mathrm{A}-a)$, and $\mathrm{XPM}=90°+\mathrm{A}-a$.

Hence, by Spherical Trigonometry, Art. 225, we obtain

$$\left.\begin{aligned} z&=r\,[\sin.\ \mathrm{D}\ \sin.\ \delta+\cos.\ \mathrm{D}\ \cos.\ \delta\ \cos.\ (a-\mathrm{A})],\\ y&=r\,[\cos.\ \mathrm{D}\ \sin.\ \delta-\sin.\ \mathrm{D}\ \cos.\ \delta\ \cos.\ (a-\mathrm{A})],\\ x&=r\ \cos.\ \delta\ \sin.\ (a-\mathrm{A}). \end{aligned}\right\}\ \ldots\ (7)$$

The above expression for the value of y is subject to the inconvenience of furnishing y by means of the difference of two large numbers. We may, however, easily transform it into another which is free from this inconvenience.

Since $\cos.\ x=\cos.^2\ \tfrac{1}{2}x-\sin.^2\ \tfrac{1}{2}x$; that is, $\cos.\ (a-\mathrm{A})=\cos.^2\ \tfrac{1}{2}(a-\mathrm{A})-\sin.^2\ \tfrac{1}{2}(a-\mathrm{A})$, and $\sin.^2 x+\cos.^2 x=1$, by substitution and reduction we obtain

$$z=r\,[\cos.\ (\delta-\mathrm{D})\ \cos.^2\ \tfrac{1}{2}(a-\mathrm{A})-\cos.\ (\delta+\mathrm{D})\ \sin.^2\ \tfrac{1}{2}(a-\mathrm{A})],$$
$$y=r\,[\sin.\ (\delta-\mathrm{D})\ \cos.^2\ \tfrac{1}{2}(a-\mathrm{A})+\sin.\ (\delta+\mathrm{D})\ \sin.^2\ \tfrac{1}{2}(a-\mathrm{A})].$$

(281.) Having thus computed z, y, and x, we can find z', y', and x' by the following expressions:

$$z'=z+\mathrm{G},\ y'=y,\ \text{and}\ x'=x.$$

Conceive now that M in the preceding figure no longer represents the moon's centre, but the geocentric zenith of the observer; the declination of the point M will then be equal to ϕ', or the geocentric latitude of the place of observation; and its right ascension will be equal to μ, the sidereal time of the observer expressed in degrees. If, then, we represent the distance of the observer from the centre of the earth by ρ, we shall obtain the values of ζ, η, and ξ from equations (7), by substituting ρ, μ, and ϕ' in place of r, a, and δ. We thus obtain

$$\left.\begin{aligned} \zeta&=\rho\,[\sin.\ \mathrm{D}\ \sin.\ \phi'+\cos.\ \mathrm{D}\ \cos.\ \phi'\ \cos.\ (\mu-\mathrm{A})],\\ \eta&=\rho\,[\cos.\ \mathrm{D}\ \sin.\ \phi'-\sin.\ \mathrm{D}\ \cos.\ \phi'\ \cos.\ (\mu-\mathrm{A})],\\ \xi&=\rho\ \cos.\ \phi'\ \sin.\ (\mu-\mathrm{A}). \end{aligned}\right\}\ \ldots\ (8)$$

(282.) The unit to which the lengths of the lines r, R, and ρ are referred is entirely arbitrary. Bessel has chosen for this unit, as being most convenient for computation, the equatorial radius of the earth. If we represent the moon's equatorial horizontal parallax by π, the sun's mean horizontal parallax by π', and the distance from the centre of the earth to that of the sun by r', expressed as in the solar tables, where the mean distance of the earth from the sun is considered as unity; then, if the equatorial radius of the earth be taken as unity,

$$r = \frac{1}{\sin. \pi}, \text{ and } \mathrm{R} = \frac{r'}{\sin. \pi'}.$$

Let H represent the mean radius of the sun, or the apparent radius of the sun's disk at the distance $r'=1$; then the linear radius of the sun, or k', the equatorial radius of the earth being taken as unity, will be represented by

$$k' = \frac{\sin. \mathrm{H}}{\sin. \pi'} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

Consequently, for all eclipses of the sun we shall have

$$\left.\begin{array}{c} r = \dfrac{1}{\sin. \pi}; \; e = \dfrac{\sin. \pi'}{r' \sin. \pi}; \; g = \dfrac{\mathrm{G} . \sin. \pi'}{r'}, \\ \sin. f = \dfrac{\sin. \mathrm{H} \pm k \sin. \pi'}{r'g}, \\ s . \tan g. f = l = z \tan g. f \pm k \sec. f, \end{array}\right\} \; . \; . \; (10)$$

where the sign + applies to an external contact, and − to an internal contact.

(283.) The numerator of the expression for sin. f is constant for all eclipses of the sun. From the transits of Venus in the years 1761 and 1769, Encke has determined $\pi' = 8''.5776$; from Bessel's measurements at the transit of Mercury in the year 1832, H was determined $=959''.788$; and according to Burckhardt's tables of the moon, if we take the equatorial radius of the earth as unity, the linear radius of the moon, or k, will be equal to 0.2725. Hence we have generally

$$\left.\begin{array}{l} \log. \sin. \pi' = 5.6189407; \\ \log. (\sin. \mathrm{H} + k \sin. \pi') = 7.6688050; \\ \log. (\sin. \mathrm{H} - k \sin. \pi') = 7.6666896. \end{array}\right\} \; . \; . \; . \; (11)$$

(284.) Let ϕ represent the geographical latitude of a given place, ϕ' its geocentric latitude, and ω the east longitude of the place from the meridian of the ephemeris expressed in time.

The beginning and end of the eclipse can nowhere happen many hours before or after the middle of the eclipse, as given in the ephemeris. Let, then, T represent the mean solar time corresponding to the middle of the eclipse under the meridian for which the ephemeris is computed; $T+\omega$ will be the corresponding mean time of the middle under the meridian of the given place. If we represent the mean time of the beginning or end of the eclipse at the given place by $T+\omega+t$, we may be sure that t is a short interval of time. If we have not the use of an astronomical ephemeris, we may employ the solar and lunar tables, and may assume for T either the time of true conjunction, or, still better, the time of middle of the eclipse for the earth generally.

For the mean times T−1h., T, and T+1h., compute from the ephemeris the values of a, δ, and π for the moon; also a', δ', and r' for the sun. Compute from equations (6) the values of A, D, and g; and from equations (7) the values of z, y, and x. Also, compute the values of l and log. i from equations (4) and (10). Since the values of l and log. i change but slowly, when only an approximate computation is required, we may assume that these quantities remain constant throughout the entire duration of the eclipse.

(285.) We will now assume that for the mean times T−1h. and T+1h., under the meridian of the ephemeris, the co-ordinates x, y, and z have the values

$$p-p',\ q-q',\ b-b',\ \text{and}\ p+p',\ q+q'\ b+b';$$

which values will be general for all parts of the earth. But for the given place we must also compute the sidereal times which, under the meridian of the place, correspond to the instants when the mean times T−1h. and T+1h. occurred under the meridian of the ephemeris. We then compute from equations (8) the values of the co-ordinates ξ, η, and ζ for the given place; and we will assume that these co-ordinates for the two instants above mentioned are

$$u-u',\ v-v',\ w-w',\ \text{and}\ u+u',\ v+v',\ w+w'.$$

We may now assume approximately that at the time T, under the meridian of the ephemeris, the values of x, y, z, ξ, η, and ζ are equal to p, q, b, u, v, and w, and that the hourly variations of these values are represented by p', q', b', u', v', and w'; also that, during a moderate interval of time, the change of the preceding values is proportional to the time. We shall therefore find approximately for the mean time $T+t$, under the meridian of the ephemeris,

$$x=p+p't\,;\ y=q+q't\,;\ z=b+b't\,;$$
$$\xi=u+u't\,;\ \eta=v+v't\,;\ \zeta=w+w't\,;$$

where t is expressed in hours and fractions of an hour.

Substituting these values in equation (5), we obtain

$$[p-u+(p'-u')t]^2+[q-v+(q'-v')t]^2=(l-i\zeta)^2.$$

(286.) In order to facilitate the computation, we will assume

$$\left.\begin{array}{c} p-u=m \sin.\ \mathrm{M}\,;\ p'-u'=n \sin.\ \mathrm{N}, \\ q-v=m \cos.\ \mathrm{M}\,;\ q'-v'=n \cos.\ \mathrm{N}, \\ l-i\zeta=\mathrm{L}, \end{array}\right\} \ .\ .\ .\ (12)$$

where m and n are always to be considered positive. Substituting these values, we obtain

$$(m \sin.\ \mathrm{M}+nt \sin.\ \mathrm{N})^2+(m \cos.\ \mathrm{M}+nt \cos.\ \mathrm{N})^2=\mathrm{L}^2$$

By expanding this equation, we obtain

$$\left.\begin{array}{l} m^2 \sin.^2 \mathrm{M}+2mnt \sin.\ \mathrm{M} \sin.\ \mathrm{N}+n^2t^2 \sin.^2 \mathrm{N} \\ +m^2 \cos.^2 \mathrm{M}+2mnt \cos.\ \mathrm{M} \cos.\ \mathrm{N}+n^2t^2 \cos.^2 \mathrm{N} \end{array}\right\} =\mathrm{L}^2.$$

But since $\sin.^2 + \cos.^2=1$, we have

$$m^2+2mnt \cos.\ (\mathrm{M}-\mathrm{N})+n^2t^2=\mathrm{L}^2,$$

or

$$m^2 \sin.^2(\mathrm{M}-\mathrm{N})+m^2 \cos.^2(\mathrm{M}-\mathrm{N})+2mnt \cos.(\mathrm{M}-\mathrm{N})+n^2t^2=\mathrm{L}^2;$$

that is, $m^2 \sin.^2 (\mathrm{M}-\mathrm{N})+[m \cos.\ (\mathrm{M}-\mathrm{N})+nt]^2=\mathrm{L}^2.$

Let us assume

$$\frac{m \sin.\ (\mathrm{M}-\mathrm{N})}{\mathrm{L}}=\sin.\ \psi\ .\ .\ .\ .\ .\ .\ .\ (13)$$

then, if an eclipse actually takes place, it will always be possible to compute the angle ψ. Substituting $\sin.\ \psi$ in the last equation but one, we have

$$\mathrm{L}^2 \sin.^2 \psi+[m \cos.\ (\mathrm{M}-\mathrm{N})+nt]^2=\mathrm{L}^2,$$

or $\quad [m \cos.\ (\mathrm{M}-\mathrm{N})+nt]^2=\mathrm{L}^2(1-\sin.^2 \psi)=\mathrm{L}^2 \cos.^2 \psi.$

Extracting the square root,

$$m \cos.\ (\mathrm{M}-\mathrm{N})+nt=\pm \mathrm{L} \cos.\ \psi,$$

or

$$t=-\frac{m \cos.\ (\mathrm{M}-\mathrm{N})}{n}\pm\frac{\mathrm{L} \cos.\ \psi}{n},$$

where the unit to which t refers is the mean solar hour.

It is obvious that the greater of the two values of t, understood in a positive sense, must correspond to the end of the eclipse, and the least of the two to the beginning. Assuming the angle ψ to be taken in either the first or fourth quadrant, we find for the given place, in mean time of the place,

Beginning of the eclipse at $\mathrm{T}+\omega-\frac{m}{n}\cos.(\mathrm{M}-\mathrm{N})-\frac{\mathrm{L}}{n}\cos.\psi$,

End " " $\mathrm{T}+\omega-\frac{m}{n}\cos.(\mathrm{M}-\mathrm{N})+\frac{\mathrm{L}}{n}\cos.\psi$,

where we may employ the value of l instead of L without material error.

(287.) The hourly variations of ξ and η may be found by differentiating the values of ξ and η in equations (8). We thus obtain

$$\frac{d\xi}{d\mathrm{T}}=\rho\cos.\phi'\cos.(\mu-\mathrm{A})\frac{d(\mu-\mathrm{A})}{d\mathrm{T}},$$

$$\frac{d\eta}{d\mathrm{T}}=-\rho\sin.\phi'\sin.\mathrm{D}\frac{d\mathrm{D}}{d\mathrm{T}}-\rho\cos.\phi'\cos.\mathrm{D}\cos.(\mu-\mathrm{A})\frac{d\mathrm{D}}{d\mathrm{T}}$$
$$+\rho\cos.\phi'\sin.\mathrm{D}\sin.(\mu-\mathrm{A})\frac{d(\mu-\mathrm{A})}{d\mathrm{T}},$$

or

$$\frac{d\eta}{d\mathrm{T}}=\xi\sin.\mathrm{D}\frac{d(\mu-\mathrm{A})}{d\mathrm{T}}-\zeta\frac{d\mathrm{D}}{d\mathrm{T}}.$$

The unit of time is here taken at one hour, and the above values must be expressed in parts of radius.

(288.) In order to determine on what point of the sun's disk the first and last contacts will take place, conceive a line which passes through the place of the observer, parallel to the line which joins the centres of the sun and moon, and directed toward the positive side of the axis of z; the plane which passes through this line, and is parallel to the axis of y (see figure, page 263), makes, with the plane which passes through the former line and the apparent place of the moon, the angle KON, whose tangent is $\frac{\mathrm{KN}}{\mathrm{ON}}=\frac{x-\xi}{y-\eta}$. Represent this angle by Q. Since the sun is at a great distance from the earth and moon, the line which joins the centres of the sun and moon at the time of an eclipse forms a very small angle with that which passes through the place of the observer and the sun. We may therefore assume that the angle Q is the same as that which is formed at the sun's centre by the hour circle of the sun, and that circle which passes through the sun's centre and the point of the sun's limb where the first or last contact takes place. Consequently, we shall have

$$\text{tang. } Q = \frac{x-\xi}{y-\eta}.$$

But
$$x-\xi = p-u+(p'-u')t$$
$$= m \text{ sin. } M + nt \text{ sin. } N, \text{ by equation 12.}$$

Also,
$$nt = -m \text{ cos. } (M-N) \mp L \text{ cos. } \psi.$$

Hence

$$x-\xi = m \text{ sin. } M - m \text{ sin. } N \text{ cos. } (M-N) \mp L \text{ cos. } \psi \text{ sin. } N.$$

But

$$\begin{aligned}\text{sin } M - \text{sin. } N \text{ cos. } (M-N) &= \\ &= \text{sin. } M - \text{sin. } N \text{ cos. } M \text{ cos. } N - \text{sin. } N \text{ sin. } M \text{ sin. } N \\ &= \text{sin. } M(1-\text{sin.}^2 N) - \text{sin. } N \text{ cos. } M \text{ cos. } N \\ &= \text{sin. } M \text{ cos. } N \text{ cos. } N - \text{sin. } N \text{ cos. } M \text{ cos. } N \\ &= \text{cos. } N \text{ sin. } (M-N).\end{aligned}$$

That is,

$$x-\xi = m \text{ sin. } (M-N) \text{ cos. } N \mp L \text{ cos. } \psi \text{ sin. } N.$$

But
$$L = \frac{m \text{ sin. } (M-N)}{\text{sin. } \psi}.$$

Hence

$$x-\xi = \frac{m \text{ sin. } (M-N) \text{ cos. } N \text{ sin. } \psi \mp m \text{ cos. } \psi \text{ sin. } N \text{ sin. } (M-N)}{\text{sin. } \psi}$$
$$= \frac{m \text{ sin. } (M-N)}{\text{sin. } \psi} \left\{ \text{cos. } N \text{ sin. } \psi \mp \text{sin. } N \text{ cos. } \psi \right\}$$
$$= \mp L \text{ sin. } (N \mp \psi).$$

In the same manner, we find

$$y-\eta = \mp L \text{ cos. } (N \mp \psi).$$

Hence, for the first contact,

$$x-\xi = -L \text{ sin. } (N-\psi) = L \text{ sin. } (N+180^\circ-\psi),$$
$$y-\eta = -L \text{ cos. } (N-\psi) = L \text{ cos. } (N+180^\circ-\psi);$$

and for the last contact,

$$x-\xi = L \text{ sin. } (N+\psi),$$
$$y-\eta = L \text{ cos. } (N+\psi).$$

That is, for the first contact,

$$\text{tang. } Q = \text{tang. } (N+180^\circ-\psi),$$

or
$$Q = N+180^\circ-\psi;$$

and for the last contact,

$$\text{tang. } Q = \text{tang. } (N+\psi),$$

or
$$Q = N+\psi.$$

(289.) The angle Q is measured on the sun's limb, from his north point by the east, from 0° to 360°. If we conceive an hour circle drawn through the sun's centre, Q will represent the angle comprehended between the intersection of this hour circle with the sun's disk and the point of first or last contact. If the observer has an equatorial telescope, he may easily determine the north point of the sun's disk by the method explained in Art. 42. If the telescope is not equatorially mounted, we must refer the points of first and last contact to the vertex of the sun's disk; for which purpose we must compute the angle P, which is formed at the sun's centre by an hour circle and a vertical circle, as explained in Art. 145. The north point of the sun's disk will be situated to the right of the vertex if the sun is west of the meridian, but on the left of the vertex if it is east of the meridian.

(290.) The following is a recapitulation of the formulæ employed in this computation:

Let T represent a convenient assumed time near to the time of conjunction. Take from the ephemeris, for two or three full hours preceding and following T, the following quantities:

a = the moon's right ascension, a' = the sun's right ascension,
δ = the moon's declination, δ' = the sun's declination,
π = the moon's equ. hor. parallax, r' = the earth's radius vector.

Then compute the following quantities:

$$e=\frac{\sin.\,8''.5776}{r'\sin.\,\pi},\qquad r=\frac{1}{\sin.\,\pi},$$

$$\log.\,\sin.\,8''.5776=5.6189407,$$

$$\mathrm{A}=a'-\frac{e\cos.\,\delta\sec.\,\delta'(a-a')}{1-e\cos.\,\delta\sec.\,\delta'},$$

$$\mathrm{D}=\delta'-\frac{e(\delta-\delta')}{1-e},$$

$$g=\frac{1-e\cos.\,\delta\sec.\,\delta'}{\cos.\,\mathrm{D}\sec.\,\delta'},$$

$$x=r\cos.\,\delta\sin.\,(a-\mathrm{A}),$$

$$y=r\sin.\,(\delta-\mathrm{D})\cos.^2\tfrac{1}{2}\,(a-\mathrm{A})+r\sin.\,(\delta+\mathrm{D})\sin.^2\tfrac{1}{2}\,(a-\mathrm{A}),$$

$$z=r\cos.\,(\delta-\mathrm{D})\cos.^2\tfrac{1}{2}\,(a-\mathrm{A})-r\cos.\,(\delta+\mathrm{D})\sin.^2\tfrac{1}{2}\,(a-\mathrm{A}),$$

$$\sin.\,f=\frac{7.6688050}{r'g}\ \text{(for an external contact)},$$

$$\sin.\,f=\frac{7.6666896}{r'g}\ \text{(for an internal contact)},$$

$$i = \text{tang.}\, f,$$
$$k = 0.2725,$$
$$l = z\ \text{tang.}\, f \mp k\ \text{sec.}\, f.$$

Compute, also, the following quantities for the given place, where

μ = the sidereal time of the place of observation;
μ' = the same for the meridian of the ephemeris;
ω = the longitude of the place of observation; east longitudes being considered positive, west longitudes negative;
ϕ' = the geocentric latitude of the place of observation;
ρ = the earth's radius for the place of observation.

$$\xi = \rho\ \text{cos.}\ \phi'\ \text{sin.}\ (\mu - \text{A}),$$
$$\eta = \rho\ \text{sin.}\ \phi'\ \text{cos.}\ \text{D} - \rho\ \text{cos.}\ \phi'\ \text{sin.}\ \text{D}\ \text{cos.}\ (\mu - \text{A}),$$
$$\zeta = \rho\ \text{sin.}\ \phi'\ \text{sin.}\ \text{D} + \rho\ \text{cos.}\ \phi'\ \text{cos.}\ \text{D}\ \text{cos.}\ (\mu - \text{A}),$$
$$d\xi = \rho\ \text{cos.}\ \phi'\ \text{cos.}\ (\mu - \text{A})\ d(\mu - \text{A}),$$
$$d\eta = \xi\ \text{sin.}\ \text{D}\ d(\mu - \text{A}) - \zeta d\text{D},$$
$$m\ \text{sin.}\ \text{M} = x - \xi,$$
$$m\ \text{cos.}\ \text{M} = y - \eta,$$
$$n\ \text{sin.}\ \text{N} = x' - d\xi,$$
$$n\ \text{cos.}\ \text{N} = y' - d\eta,$$

x' = the hourly variation of x,
y' = the hourly variation of y.

m and n are always positive.

$$l - i\zeta = \text{L},$$
$$\text{sin.}\ \psi = \frac{m}{\text{L}}\ \text{sin.}\ (\text{M} - \text{N}).$$

ψ must be taken in the first or fourth quadrant

For beginning of eclipse, $t_1 = -\frac{m}{n}\ \text{cos.}\ (\text{M} - \text{N}) - \frac{\text{L}}{n}\ \text{cos.}\ \psi.$

For end of eclipse, $t_2 = -\frac{m}{n}\ \text{cos.}\ (\text{M} - \text{N}) + \frac{\text{L}}{n}\ \text{cos.}\ \psi.$

Time of beginning of eclipse, $= \text{T} + \omega + t_1.$
Time of end of eclipse, $= \text{T} + \omega + t_2.$
Angle from north point for beginning, $= 180° + \text{N} - \psi = \text{Q}_1.$
Angle from north point for end, $= \text{N} + \psi = \text{Q}_2.$
Angle from vertex, $= \text{Q} + \text{P} = \text{V}.$

(291.) *Example.* It is required to compute the time of beginning and end of the solar eclipse of July 28, 1851, for Cambridge

Observatory, latitude 42° 22′ 48″ north, longitude 4h. 44m. 30s. west of Greenwich.

The right ascension and declination of the moon are computed for the Nautical Almanac for each noon and midnight, examined by means of differences to the fourth order, and interpolated for every hour. The following places of the moon for several hours before and after conjunction have been interpolated from the computed places in the Nautical Almanac, regard being had to differences of the fifth order. The places of the sun have also been carefully interpolated.

For the Moon.

Greenwich mean Solar Time.	α=R. A.	δ=Dec.	π=Parallax.
h.	° ′ ″	° ′ ″	′ ″
July 28, 0	125 40 6.75	20 3 30.00	60 27.600
" 1	126 19 9.41	19 58 9.36	60 28.710
" 2	126 58 10.80	19 52 39.98	60 29.794
" 3	127 37 10.82	19 47 1.91	60 30.851
" 4	128 16 9.37	19 41 15.20	60 31.880
" 5	128 55 6.36	19 35 19.88	60 32.882

For the Sun.

Greenwich mean Solar Time.	α'=R. A.	δ'=Dec.	Log. r'=Log. Distance.	μ'=Greenwich Sider. Time, reduced to Arc.
h	° ′ ″	° ′ ″		° ′ ″
July 28, 0	127 6 5.25	19 5 24.70	0.0065782	125 33 19.05
" 1	127 8 32.63	4 50.23	65761	140 35 46.90
" 2	127 10 59.99	4 15.74	65739	155 38 14.74
" 3	127 13 27.34	3 41.21	65718	170 40 42.59
" 4	127 15 54.67	3 6.64	65697	185 43 10.44
" 5	127 18 21.99	2 32.04	65675	200 45 38.29

Computation of the quantities e, A, D, and g.

Greenwich mean Time.	July 28, 0h.	July 28, 1h.	July 28, 2h.	July 28, 3h.	July 28, 4h.	July 28, 5h.
log. π	3.5596194	3.5597523	3.5598820	3.5600084	3.5601315	3.5602512
log. sin. $\pi -$ log. π	4.6855525	4.6855525	4.6855525	4.6855524	4.6855524	4.6855524
log. sin. π	8.2451719	8.2453048	8.2454345	8.2455608	8.2456839	8.2458036
log. r'	0.0065782	0.0065761	0.0065739	0.0065718	0.0065697	0.0065675
log. r' sin. π	8.2517501	8.2518809	8.2520084	8.2521326	8.2522536	8.2523711
sin. $8''.5776$	5.6189407	5.6189407	5.6189407	5.6189407	5.6189407	5.6189407
log. e	7.3671906	7.3670598	7.3669323	7.3668081	7.3666871	7.3665696
$\alpha - \alpha'$	$-1^{\circ}\ 25'\ 58''.50$	$-49'\ 23''.22$	$-12'\ 49''.19$	$+23'\ 43''.48$	$+1^{\circ}\ 0'\ 14''.70$	$+1^{\circ}\ 36'\ 44''.37$
log. e	7.36719	7.36706	7.36693	7.36681	7.36669	7.36657
cos. δ	9.97282	9.97307	9.97332	9.97358	9.97384	9.97411
sec. δ'	0.02457	0.02454	0.02452	0.02449	0.02447	0.02444
log. e cos. δ sec. δ'	7.36458	7.36467	7.36477	7.36488	7.36500	7.36512
e cos. δ sec. δ'	.0023151	.0023156	.0023162	.0023168	.0023174	.0023180
$1 - e$ cos. δ sec. δ'	.9976849	.9976844	.9976838	.9976832	.9976826	.9976820
log. e cos. δ sec. δ'	7.36458	7.36467	7.36477	7.36488	7.36500	7.36512
log. $(\alpha - \alpha')$	3.71252n	3.47176n	2.88603n	3.15335	3.55807	3.76375
log. e cos. δ sec. δ' log. $(\alpha - \alpha')$	1.07710n	0.83643n	0.25080n	0.51823	0.92307	1.12887
log. $(1 - e$ cos. δ sec. $\delta')$	9.99899	9.99899	9.99899	9.99899	9.99899	9.99899
log. $(\mathrm{A} - \alpha')$	1.07811	0.83744	0.25181	0.51924n	0.92408n	1.12988n
$\mathrm{A} - \alpha'$	$+11''.97$	$+6''.88$	$+1''.79$	$-3''.31$	$-8''.40$	$-13''.48$
A	$127^{\circ}\ 6'\ 17''.22$	$127^{\circ}\ 8'\ 39''.51$	$127^{\circ}\ 11'\ 1''.78$	$127^{\circ}\ 13'\ 24''.03$	$127^{\circ}\ 15'\ 46''.27$	$127^{\circ}\ 18'\ 8''.51$

$\delta-\delta'$	$+58'\ 5''.30$	$+53'\ 19''.13$	$+48'\ 24''.24$	$+43'\ 20''.70$	$+38'\ 8''.56$	$+32'\ 47''.84$
e	.0023291	.0023284	.0023277	.0023271	.0023264	.0023258
$1-e$	.9976709	.9976716	.9976723	.9976729	.9976736	.9976742
log. e	7.36719	7.36706	7.36693	7.36681	7.36669	7.36657
log. $(\delta-\delta')$	3.54224	3.50503	3.46303	3.41509	3.35956	3.29399
log. $e\ (\delta-\delta')$	0.90943	0.87209	0.82996	0.78190	0.72625	0.66056
log. $(1-e)$	9.99899	9.99899	9.99899	9.99899	9.99899	9.99899
log. $(D-\delta')$	0.91044*n*	0.87310*n*	0.83097*n*	0.78291*n*	0.72726*n*	0.66157*n*
$D-\delta'$	$-8''.14$	$-7.''47$	$-6''.78$	$-6''.07$	$-5''.34$	$-4''.59$
D	19° 5′ 16″.56	19° 4′ 42.″76	19° 4′ 8″.96	19° 3′ 35″.14	19° 3′ 1″.30	19° 2′ 27″.45
cos. D	9.9754400	9.9754646	9.9754892	9.9755138	9.9755385	9.9755631
sec. δ'	0.0245659	0.0245408	0.0245157	0.0244906	0.0244654	0.0244403
cos. D sec. δ'	0.0000059	0.0000054	0.0000049	0.0000044	0.0000039	0.0000034
log. $(1-e$ cos. δ sec. $\delta')$	9.9989934	9.9989932	9.9989929	9.9989927	9.9989924	9.9989922
log. g	9.9989875	9.9989878	9.9989880	9.9989883	9.9989885	9.9989888

Computation of the co-ordinates x, y, and z

Greenwich mean Time.	July 28, 0h	July 28, 1h.	July 28, 2h.	July 28, 3h.	July 28, 4h.	July 28, 5h.
$a-\mathrm{A}$	−1° 26′ 10″.47	−49′ 30″.10	−12′ 50″.98	+23′ 46″.79	+1° 0′ 23″.10	+1° 36′ 57″.85
log. $(a-\mathrm{A})$	3.7135300n	3.4727711n	2.8870431n	3.1543601	3.5590803	3.7647626
log. sin. − log.	4.6855294	4.6855599	4.6855738	4.6855714	4.6855525	4.6855173
cos. δ	9.9728246	9.9730705	9.9733219	9.9735786	9.9738405	9.9741075
log. r	1.7548281	1.7546952	1.7545655	1.7544392	1.7543161	1.7541964
log. x	0.1267121n	9.8860967n	9.3005043n	9.5679493	9.9727894	0.1785838
x	−1.338789	−.769302	−.199758	+.369785	+.939268	+1.508633
$\delta+\mathrm{D}$	39° 8′ 46″.56	39° 2′ 52″.12	38° 56′ 48″.94	38° 50′ 37″.05	38° 44′ 16″.50	38° 37′ 47″.33
$\delta-\mathrm{D}$	0 58 13.44	0 53 26.60	0 48 31.02	0 43 26.77	0 38 13.90	0 32 52.43
cos. $\frac{1}{2}(a-\mathrm{A})$	9.9999659	9.9999887	9.9999992	9.9999974	9.9999832	9.9999568
cos. $\frac{1}{2}(a-\mathrm{A})$	9.9999659	9.9999887	9.9999992	9.9999974	9.9999832	9.9999568
log. $(\delta-\mathrm{D})$	3.5432533	3.5060448	3.4640452	3.4161027	3.3605745	3.2950016
log. sin. − log.	4.6855541	4.6855574	4.6855604	4.6855633	4.6855659	4.6855683
log. r	1.7548281	1.7546952	1.7545655	1.7544392	1.7543161	1.7541964
log. r sin. $(\delta-\mathrm{D})$ cos.^{2} $\frac{1}{2}(a-\mathrm{A})$	9.9835673	9.9462748	9.9041695	9.8561000	9.8004229	9.7346799
log. $\frac{1}{2}(a-\mathrm{A})$	3.41250	3.17174	2.58601	2.85333	3.25805	3.46373
log. sin. − log.	4.68556	4.68557	4.68557	4.68557	4.68557	4.68556
sin. $\frac{1}{2}(a-\mathrm{A})$	8.09806	7.85731	7.27158	7.53890	7.94362	8.14929
sin. $(\delta+\mathrm{D})$	9.80024	9.79932	9.79837	9.79740	9.79641	9.79538
log. r	1.75483	1.75470	1.75457	1.75444	1.75432	1.75420
log. r sin. $(\delta+\mathrm{D})$ sin.^{2} $\frac{1}{2}(a-\mathrm{A})$	7.75119	7.26864	6.09610	6.62964	7.43797	7.84816
r sin. $(\delta+\mathrm{D})$ sin.^{2} $\frac{1}{2}(a-\mathrm{A})$	.005639	.001856	.000125	.000426	.002741	.007050
r sin. $(\delta-\mathrm{D})$ cos.^{2} $\frac{1}{2}(a-\mathrm{A})$	.962869	.883639	.801991	.717960	.631572	.542850
y	.968508	.885495	.802116	.718386	.634313	.549900

$\cos.^2 \frac{1}{2}(a-A)$	9.9999318	9.9999774	9.9999984	9.9999948	9.9999664	9.9999136
$\cos. (\delta-D)$	9.9999377	9.9999475	9.9999567	9.9999653	9.9999731	9.9999802
$\log. r$	1.7548281	1.7546952	1.7545655	1.7544392	1.7543161	1.7541964
$\log. r \cos. (\delta-D) \cos.^2 \frac{1}{2}(a-A)$	1.7546976	1.7546201	1.7545206	1.7543993	1.7542556	1.7540902
$\sin.^2 \frac{1}{2}(a-A)$	6.19612	5.71462	4.54316	5.07780	5.88724	6.29858
$\cos. (\delta+D)$	9.88960	9.89021	9.89083	9.89146	9.89210	9.89276
$\log. r$	1.75483	1.75470	1.75457	1.75444	1.75432	1.75420
$\log. r \cos. (\delta+D) \sin.^2 \frac{1}{2}(a-A)$	7.84055	7.35953	6.18856	6.72370	7.53366	7.94554
$r \cos. (\delta-D) \cos.^2 \frac{1}{2}(a-A)$	56.84570	56.83555	56.82254	56.80667	56.78787	56.76625
$r \cos. (\delta+D) \sin.^2 \frac{1}{2}(a-A)$	0.00693	0.00229	0.00015	0.00053	0.00342	0.00882
z	56.83877	56.83326	56.82239	56.80614	56.78445	56.75743
$\log. z$	1.7546447	1.7546026	1.7545194	1.7543952	1.7542295	1.7540227

Computation of the quantities i and l.

For the external Contacts.

Greenwich mean Time.	July 28, 0h	July 28, 1h	July 28, 2h	July 28, 3h	July 28, 4h.	July 28, 5h.
$\log. r'$	0.0065782	0.0065761	0.0065739	0.0065718	0.0065697	0.0065675
g	9.9989875	9.9989878	9.9989880	9.9989883	9.9989885	9.9989888
$\log. r'g$	0.0055657	0.0055639	0.0055619	0.0055601	0.0055582	0.0055563
Constant	7.6688050	7.6688050	7.6688050	7.6688050	7.6688050	7.6688050
$\sin. f$	7.6632393	7.6632411	7.6632431	7.6632449	7.6632468	7.6632487
$\sec. f$	0.0000046	0.0000046	0.0000046	0.0000046	0.0000046	0.0000046
$\log. i = \text{tang.} f$	7.6632439	7.6632457	7.6632477	7.6632495	7.6632514	7.6632533
$\log. z$	1.7546447	1.7546026	1.7545194	1.7543952	1.7542295	1.7540227
$\log. z \text{ tang.} f$	9.4178886	9.4178483	9.4177671	9.4176447	9.4174809	9.4172760

Greenwich mean Time.	July 28, 0h.	July 28, 1h.	July 28, 2h.	July 28, 3h.	July 28, 4h.	July 28, 5h.
log. k	9.4353665					
sec. f	0.0000046					
log. k sec. f	9.4353711					
k sec. f	.272503	.272503	.272503	.272503	.272503	.272503
z tang. f	.261751	.261727	.261678	.261604	.261505	.261382
l	.534254	.534230	.534181	.534107	.534008	.533885
For the internal Contacts.						
Constant	7.6666896	7.6666896	7.6666896	7.6666896	7.6666896	7.6666896
log. $r'g$	0.0055657	0.0055639	0.0055619	0.0055601	0.0055582	0.0055563
sin. f	7.6611239	7.6611257	7.6611277	7.6611295	7.6611314	7.6611333
sec. f	0.0000046n	0.0000046n	0.0000046n	0.0000046n	0.0000046n	0.0000046n
log. $i=$tang. f	7.6611285n	7.6611303n	7.6611323n	7.6611341n	7.6611360n	7.6611379n
log. z	1.7546447	1.7546026	1.7545194	1.7543952	1.7542295	1.7540227
log. z tang. f	9.4157732n	9.4157329n	9.4156517n	9.4155293n	9.4153655n	9.4151606n
z tang. f	−.260479	−.260455	−.260406	−.260333	−.260235	−.260112
k sec. f	−.272503	−.272503	−.272503	−.272503	−.272503	−.272503
l	+.012024	+.012048	+.012097	+.012170	+.012268	+.012391

A portion of the labor of the preceding computation may be saved by the use of a table by Zech, which furnishes the logarithm of the sum, or difference of two numbers which are known only by their logarithms. This table is contained in Hülsse's Sammlung mathematischer Tafeln. Leipzig, 1849.

The following are the results for x, y, and z:

Hour.	x	x'	Diff.	y	y'	Diff.	Log. z	Diff.
0	−1.338789			+.968508			1.7546447	
		+.569487			−.083013			421
1	−0.769302		+57	+.885495			1.7546026	
		+.569544			−.083379	−366		832
2	−0.199758		−1	+.802116			1.7545194	
		+.569543			−.083730	−351		1242
3	+0.369785		−60	+.718386			1.7543952	
		+.569483			−.084073	−343		1657
4	+0.939268		−118	+.634313			1.7542295	
		+.569365			−.084413	−340		2068
5	+1.508633			+.549900			1.7540227	

(292.) The preceding quantities are independent of geographical position, and serve not only for calculating the times of beginning, etc., of the eclipse for any place at which it may be visible, but also for the calculations requisite to determine the longitude of a place from the observed time of beginning and end.

Computation of the beginning and end of the Eclipse for Cambridge, by Formulæ, page 278.

$\omega = -71° \; 7' \; 30''$; $\phi' = 42° \; 11' \; 21''.1$; log. $\rho = 9.9993429$.

For a first approximation we will assume T = 2h. Greenwich time.

$$\begin{aligned} \mu' &= \quad 155° \; 38' \; 15'' \\ \omega &= - \; 71° \;\; 7' \; 30'' \\ \mu = \mu' + \omega &= \overline{\quad 84° \; 30' \; 45''} \\ \text{A} &= \quad 127° \; 11' \;\; 2'' \\ \mu - \text{A} &= - \overline{\; 42° \; 40' \; 17''} \end{aligned} \qquad \begin{aligned} \rho \cos. \phi' &= 9.869121 \\ \sin. \text{D} &= 9.514161 \\ \cos. (\mu - \text{A}) &= \underline{9.866437} \\ .17771 &= 9.249719 \\ \eta = + \overline{.45606} \end{aligned}$$

$$\begin{aligned} \rho &= 9.999343 \\ \cos. \phi' &= 9.869778 \\ \sin. (\mu - \text{A}) &= \underline{9.831097n} \\ \xi = -.50144 &= 9.700218n \end{aligned} \qquad \begin{aligned} \rho \sin. \phi' &= 9.826441 \\ \sin. \text{D} &= \underline{9.514161} \\ .21908 &= 9.340602 \end{aligned}$$

$$\begin{aligned} \rho &= 9.999343 \\ \sin. \phi' &= 9.827098 \\ \text{D} = 19° \; 4' \; 9'' \; \cos. &= \underline{9.975489} \\ .63377 &= 9.801930 \end{aligned} \qquad \begin{aligned} \rho \cos. \phi' &= 9.869121 \\ \cos. \text{D} &= 9.975489 \\ \cos. (\mu - \text{A}) &= \underline{9.866437} \\ .51410 &= 9.711047 \\ \zeta = + \overline{.73318} \end{aligned}$$

The hourly variation of $\mu-\mathrm{A}$, that is,

$$d(\mu-\mathrm{A})=15^\circ\ 0'\ 5''.6,$$

which, in parts of radius, is

.2618265.

$$\begin{aligned}
\rho\ \cos.\ \phi' &= 9.869121\\
\cos.\ (\mu-\mathrm{A}) &= 9.866437\\
d(\mu-\mathrm{A}) &= 9.418014\\
d\xi=+.14242 &= \overline{9.153572}\\
\\
\log.\ \xi &= 9.700218n\\
\sin.\ \mathrm{D} &= 9.514161\\
d(\mu-\mathrm{A}) &= 9.418014\\
-.04289 &= \overline{8.632393n}\\
d\mathrm{D} &= -33''.8,
\end{aligned}$$

which, in parts of radius, is

.0001638.

$$\begin{aligned}
\log.\ \zeta &= 9.8652\\
d\mathrm{D} &= 6.2143n\\
-.00012 &= \overline{6.0795n}\\
d\eta &= -\overline{.04277}\\
\\
x &= -.19976\\
\xi &= -.50144\\
x-\xi &= +\overline{.30168}\\
y &= \ \ .80212\\
\eta &= \ \ .45606\\
y-\eta &= \ \ \overline{.34606}\\
\log.\ (x-\xi) &= 9.479546\\
\log.\ (y-\eta) &= 9.539151\\
\text{tang. M} &= \overline{9.940395}\\
\mathrm{M} &= 41^\circ\ 4'\ 50''\\
\log.\ (x-\xi) &= 9.479546\\
\sin.\ \mathrm{M} &= 9.817644\\
m &= \overline{9.661902}
\end{aligned}$$

$$\begin{aligned}
x' &= +.56954\\
d\xi &= +.14242\\
x'-d\xi &= +\overline{.42712}\\
y' &= -.08355\\
d\eta &= -.04277\\
y'-d\eta &= -\overline{.04078}\\
\log.\ (x'-d\xi) &= 9.630550\\
\log.\ (y'-d\eta) &= 8.610447n\\
\text{tang. N} &= \overline{1.020103n}\\
\mathrm{N} &= 95^\circ\ 27'\ 14''\\
\\
\log.\ (x'-d\xi) &= 9.630550\\
\sin.\ \mathrm{N} &= 9.998029\\
n &= \overline{9.632521}\\
\\
\log.\ \zeta &= 9.8652\\
i &= 7.6632\\
i\zeta=.00338 &= \overline{7.5284}\\
l &= .53418\\
\mathrm{L} &= \overline{.53080}\\
\\
\mathrm{M} &= \ \ 41^\circ\ \ 4'\ 50''\\
\mathrm{N} &= \ \ 95^\circ\ 27'\ 14''\\
\mathrm{M}-\mathrm{N} &= 305^\circ\ 37\ \ 36''\\
\sin.\ (\mathrm{M}-\mathrm{N}) &= 9.910000n\\
m &= 9.661902\\
\text{L comp.} &= 0.275069\\
\sin.\ \psi &= \overline{9.846971n}\\
\psi &= 315^\circ\ 19'\ 47''\\
\\
\cos.\ (\mathrm{M}-\mathrm{N}) &= 9.765297\\
m &= 9.661902\\
\text{comp. } n &= 0.367479\\
+.62327 &= \overline{9.794678}\\
\\
\cos.\ \psi &= 9.851970\\
\mathrm{L} &= 9.724931\\
\text{comp. } n &= 0.367479\\
+.87979 &= \overline{9.944380}
\end{aligned}$$

Hence $t_1 = -1.50306$h.; $t_2 = +0.25652$h.

Beginning . . . . . $= T + t_1 = 0.49694$h. Greenwich time,
End . . . . $= T + t_2 = 2.25652$h. " "

which are only to be considered as approximate values.

For a second approximation we will assume 0.5h. for the beginning, and 2.25h. for the end.

	Beginning.	End.
T	0.5h. Gr. m. t.	2.25h. Gr. m. t.
μ'	$133^\circ\ 4'\ 32''.97$	$159^\circ\ 23'\ 51''.7$
ω	$-\ 71\ \ 7\ 30.$	$-\ 71\ \ 7\ 30.$
$\mu = \mu' + \omega$	$61\ 57\ \ 2.97$	$88\ 16\ 21.7$
A	$127\ \ 7\ 28.36$	$127\ 11\ 37.34$
$\mu - A$	$-\ 65\ 10\ 25.39$	$-\ 38\ 55\ 15.64$
D	$19\ \ 4\ 59.66$	$19\ \ 4\ \ 0.50$
ρ cos. ϕ'	9.8691208	9.8691208
sin. $(\mu - A)$	9.9578873*n*	9.7981314*n*
log. ξ	9.8270081*n*	9.6672522*n*
ξ	−.671441	−.464785
ρ sin. ϕ'	9.8264412	9.8264412
cos. D	9.9754523	9.9754953
log. ρ sin. ϕ' cos. D	9.8018935	9.8019365
ρ cos. ϕ'	9.8691208	9.8691208
sin. D	9.5144700	9.5141097
cos. $(\mu - A)$	9.6231132	9.8909867
log. ρ cos. ϕ' sin. D cos. $(\mu - A)$	9.0067040	9.2742172
ρ sin. ϕ' cos. D	.633714	.633777
ρ cos. ϕ' sin. D cos. $(\mu - A)$	.101556	.188026
η	+.532158	+.445751
ρ sin. ϕ'	9.8264412	9.8264412
sin. D	9.5144700	9.5141097
log. ρ sin. ϕ' sin. D	9.3409112	9.3405509
ρ cos. ϕ'	9.8691208	9.8691208
cos. D	9.9754523	9.9754953
cos. $(\mu - A)$	9.6231132	9.8909867
log. ρ cos. ϕ' cos. D cos. $(\mu - A)$	9.4676863	9.7356028
ρ sin. ϕ' sin. D	.219236	.219054
ρ cos. ϕ' cos. D cos. $(\mu - A)$	.293553	.544005
ζ	+.512789	+.763059
ρ cos. ϕ'	9.8691208	9.8691208
cos. $(\mu - A)$	9.6231132	9.8909867
$d(\mu - A)$	9.4180136	9.4180136
log. $d\xi$	8.9102476	9.1781211
$d\xi$	+.081329	+.150703

	Beginning.	End.
log. ξ	9.8270081*n*	9.6672522*n*
sin. D	9.5144700	9.5141097
$d(\mu-\mathrm{A})$	9.4180136	9.4180136
log. ξ sin. $\mathrm{D}d(\mu-\mathrm{A})$	8.7594917*n*	8.5993755*n*
log. ζ	9.7099	9.8826
$d\mathrm{D}$	6.2143*n*	6.2143*n*
log. $\zeta d\mathrm{D}$	5.9242*n*	6.0969*n*
ξ sin. $\mathrm{D}d(\mu-\mathrm{A})$	− .057477	− .039754
$\zeta d\mathrm{D}$	− .000084	− .000125
$d\eta$	− .057393	− .039629
x	−1.054056	−0.057375
ξ	−0.671441	−0.464785
$x-\xi$	−0.382615	+0.407410
y	+0.927049	+0.781216
η	+0.532158	+0.445751
$y-\eta$	+0.394891	+0.335465
log. $(x-\xi)$	9.5827620*n*	9.6100317
log. $(y-\eta)$	9.5964772	9.5256472
tang. M	9.9862848*n*	0.0843845
M	315° 54′ 16″.4	50° 31′ 54″.0
log. $(x-\xi)$	9.5827620*n*	9.6100317
sin. M	9.8425192*n*	9.8876038
m	9.7402428	9.7224279
x'	+0.569487	+0.569543
$d\xi$	+0.081329	+0.150703
$x'-d\xi$	+0.488158	+0.418840
y'	−0.083013	−0.083643
$d\eta$	−0.057393	−0.039629
$y'-d\eta$	−0.025620	−0.044014
log. $(x'-d\xi)$	9.6885604	9.6220482
log. $(y'-\delta\eta)$	8.4085791*n*	8.6435908*n*
tang. N	1.2799813*n*	0.9784574*n*
N	93° 0′ 15″.5	95° 59′ 56″.2
log. $(x'-d\xi)$	9.6885604	9.6220482
sin. N	9.9994027	9.9976152
n	9.6891577	9.6244330
log. ζ	9.70994	9.88256
i	7.66324	7.66325
log. $i\zeta$	7.37318	7.54581
$i\zeta$	.002361	.003514
l	.534242	.534162
$\mathrm{L}=l-i\zeta$	.531881	.530648
M−N	222° 54′ 0″.9	314° 31′ 57″.8

	Beginning.	End.
sin. (M−N)	9.8329711n	9.8529982n
m	9.7402428	9.7224279
comp. L	0.2741855	0.2751934
sin. ψ	9.8473994n	9.8506195n
ψ	315° 16′ 25″.8	314° 50′ 59″.8
cos. (M−N)	9.8648313n	9.8459141
m	9.7402428	9.7224279
comp. n	0.3108423	0.3755670
	9.9159164n	9.9439090
	−.823979	+.878838
cos. ψ	9.8515508	9.8483445
L	9.7258145	9.7248066
comp. n	0.3108423	0.3755670
	9.8882076	9.9487181
	+.773050	+.888624
t	+.050929	+.009786
	Beginning of Eclipse.	End of Eclipse
T+t	0.550929h., or	2.259786h., or
Greenwich mean time	0h. 33m. 3.3s.	2h. 15m. 35.2s.
ω	4h. 44m. 30s., or	4h. 44m. 30s., or
Cambridge mean time	7h. 48m. 33.3s.	9h. 31m. 5.2s.
N	93° 0′	96° 0′
ψ	315° 16′	314° 51′
Q	$180°+N-\psi$ $=317°\ 44'$	$N+\psi$ $=50°\ 51'$

These results agree well with those found on page 253.

(293.) We may obtain a check upon the accuracy of our computations in the following manner:

Equation (5), page 267, is

$$(x-\xi)^2+(y-\eta)^2=(l-i\zeta)^2=L^2;$$

all the quantities being supposed to be computed for the instant of first or last contact of the limbs of the sun and moon. If these quantities have been computed for a time, T, which differs from the instant of contact by a small interval, t, they may be reduced to the instant of contact by means of the quantities x', y', $d\xi$, and $d\eta$, which represent the hourly variations of x, y, ξ, and η. In this case we shall have

$$\{x-\xi+(x'-d\xi)t\}^2+\{y-\eta+(y'-d\eta)t\}^2=L^2.$$

Thus, in the preceding example, for the beginning of the eclipse, $t=+.050929$.

$$\begin{aligned} x-\xi &= -0.382615\\ (x'-d\xi)t &= \underline{+0.024861}\\ \text{Sum} &= -0.357754, \text{ whose square is } .127988. \end{aligned}$$

Also,

$$\begin{aligned} y-\eta &= +0.394891\\ (y'-d\eta)t &= \underline{-0.001305}\\ \text{Sum} &= +0.393586, \text{ whose square is } .154910. \end{aligned}$$

The sum of these two squares is .282898, which is the square of .531881, the value of L.

For the end of the eclipse, $t=+.009786$.

$$\begin{aligned} x-\xi &= +0.407410\\ (x'-d\xi)t &= \underline{+0.004099}\\ \text{Sum} &= +0.411509, \text{ whose square is } .169340. \end{aligned}$$

Also,

$$\begin{aligned} y-\eta &= +0.335465\\ (y'-d\eta)t &= \underline{-0.000431}\\ \text{Sum} &= 0.335034, \text{ whose square is } .112248. \end{aligned}$$

The sum of these two squares is .281588, which is the square of .530648, the value of L.

(294.) When the highest accuracy is not required, the labor of the preceding computations may be diminished by substituting approximate formulæ for some of those here used. The expressions for A, D, and g, given on page 277, may be simplified without greatly diminishing their accuracy. Since e is always a small quantity, the denominators of the expressions for A and D are nearly equal to unity, and may be omitted. Moreover, at the time of an eclipse, δ, δ', and D are very nearly equal to each other; hence the following expressions will afford a good approximation to the values of A, D, and g.

$$\begin{aligned} \mathrm{A} &= \alpha'-e(\alpha-\alpha'),\\ \mathrm{D} &= \delta'-e(\delta-\delta'),\\ g &= 1-e. \end{aligned}$$

These formulæ will furnish the values of A and D within a small fraction of a second.

For the remaining computations we must proceed according to the formulæ on pages 277–8.

SECTION V.

BESSEL'S METHOD OF COMPUTING OCCULTATIONS OF STARS.

(295.) The formulas required for the computation of occultations of stars by the moon are easily deduced from those already given for solar eclipses, since the distance of the fixed stars is such that they have no diurnal parallax, and the rays of light which emanate from them and touch the moon's disk may be considered as forming the surface of a cylinder. Hence, for occultations, the quantities f and i, as well as the horizontal parallax of the star, become each equal to zero; also, $a'=\text{A}$, $\delta'=\text{D}$, and $l=k$. It is unnecessary to compute either z or ζ. Since A, the right ascension of the star, is invariable, $d(\mu-\text{A})$ becomes $d\mu$. But the variation of μ in one solar hour is 1h. 0m. 9.8565s., or 15° 2′ 27″.85, which, in parts of radius, is .2625162, whose logarithm is 9.4191561. For a solar eclipse, the angle Q was referred to the sun's limb, but in an occultation of a star this angle is referred to the moon's limb, and in the latter case the angle Q will differ 180° from the angle Q in the former case. Hence we have the following formulæ for the computation of occultations:

T = any convenient assumed time near conjunction;

a = the moon's right ascension;

δ = the moon's declination;

π = the moon's equatorial horizontal parallax;

A = the star's right ascension;

D = the star's declination;

μ = the sidereal time of the place of observation;

μ' = the same for the meridian of the ephemeris;

ω = the longitude of the place of observation; east longitudes positive, west longitudes negative;

ϕ' = the geocentric latitude of the place of observation;

ρ = the earth's radius for the place of observation;

$$x=\frac{\cos.\ \delta\ \sin.\ (a-\text{A})}{\sin.\ \pi};$$

$$y=\frac{\sin.\ (\delta-\text{D})\cos.^2\tfrac{1}{2}(a-\text{A})+\sin.\ (\delta+\text{D})\sin.^2\tfrac{1}{2}(a-\text{A})}{\sin.\ \pi}$$

$$\xi=\rho\cos.\ \phi'\sin.\ (\mu-\text{A});$$

$$\eta=\rho\sin.\ \phi'\cos.\ \text{D}-\rho\cos.\ \phi'\sin.\ \text{D}\cos.\ (\mu-\text{A});$$

$$d\xi=\rho\cos.\ \phi'\cos.\ (\mu-\text{A})d\mu;$$

$d\eta = \xi \sin. \mathrm{D} d\mu;$

$\log. d\mu = 9.4191561;$

$m \sin. \mathrm{M} = x - \xi;$

$m \cos. \mathrm{M} = y - \eta;$

$n \sin. \mathrm{N} = x' - d\xi;$

$n \cos. \mathrm{N} = y' - d\eta;$

x' = the hourly variation of x;

y' = the hourly variation of y;

m and n are always positive;

$$\sin. \psi = \frac{m}{k} \sin. (\mathrm{M} - \mathrm{N});$$

ψ must be taken in the first or fourth quadrant;

$\log. k = 9.4353665;$

$$t_1 = -\frac{m}{n} \cos. (\mathrm{M} - \mathrm{N}) - \frac{k}{n} \cos. \psi;$$

$$t_2 = -\frac{m}{n} \cos. (\mathrm{M} - \mathrm{N}) + \frac{k}{n} \cos. \psi.$$

Time of immersion $= \mathrm{T} + \omega + t_1$.

Time of emersion $= \mathrm{T} + \omega + t_2$.

For immersion, angle from north point toward east,

$= \mathrm{Q}_1 = \mathrm{N} - \psi$.

For emersion, angle from north point toward east,

$= \mathrm{Q}_2 = 180^\circ + \mathrm{N} + \psi$.

Angle from vertex $= \mathrm{V} = \mathrm{Q} + \mathrm{P}$.

(296.) *Ex.* 1. Required the time of occultation of α Tauri, January 23, 1850, for Cambridge Observatory.

We find that the apparent conjunction takes place at about 13 hours Greenwich mean time. We therefore interpolate, from the computed places in the Nautical Almanac, the moon's places for several hours before and after conjunction, regard being had to differences of the fifth order, as on page 279, and obtain the following results:

Greenwich mean Time.	α.	δ.	π.
h.	° ′ ″	° ′ ″	′ ″
11	65 47 3.70	+16 23 28.91	59 48.046
12	66 24 3.30	16 30 4.10	59 50.000
13	67 1 7.95	16 36 33.22	59 51.942
14	67 38 17.62	16 42 56.21	59 53.871
15	68 15 32.29	16 49 12.97	59 55.785

The position of α Tauri is

$\mathrm{A} = 66^\circ\ 49'\ 53''.1;\ \mathrm{D} = +16^\circ\ 12'\ 3''.4.$

omputation of the co-ordinates x and y

Greenwich mean Time.	January 23, 11h.	January 23, 12h.	January 23, 13h.	January 23, 14h.	January 23, 15h.
	° ′ ″	′ ″	′ ″	′ ″	° ′ ″
$\alpha - A$	−1 2 49.40	−25 49.8	+11 14.85	+48 24.52	+1 25 39.19
log. $(\alpha - A)$	3.5762722n	3.1902757n	2.8292073	3.4630744	3.7108946
log. sin.−log.	4.6855507	4.6855708	4.6855741	4.6855605	4.6855299
cos. δ	9.9819801	9.9817344	9.9814909	9.9812495	9.9810105
cosec. π	1.7595890	1.7593527	1.7591178	1.7588846	1.7586534
log. x	0.0033920n	9.6169336n	9.2553901	9.8887690	0.1360884
x	−1.007841	−.413936	+.180049	+.774050	+1.368007
$\delta + D$	32° 35′ 32″.31	32° 42′ 7″.5	32° 48′ 36″.62	32° 54′ 59″.61	33° 1′ 16″.37
$\delta - D$	0 11 25.51	0 18 0.7	0 24 29.82	0 30 52.81	0 37 9.57
cos. $\frac{1}{2}(\alpha - A)$	9.9999819	9.9999969	9.9999994	9.9999892	9.9999663
cos. $\frac{1}{2}(\alpha - A)$	9.9999819	9.9999969	9.9999994	9.9999892	9.9999663
log. $(\delta - D)$	2.8360138	3.0337052	3.1672641	3.2678308	3.3482212
log. sin.−log.	4.6855741	4.6855729	4.6855712	4.6855690	4.6855664
cosec. π	1.7595890	1.7593527	1.7591178	1.7588846	1.7586534
log. (1)	9.2811407	9.4786246	9.6119519	9.7122628	9.7923736
log. $\frac{1}{2}(\alpha - A)$	3.27524	2.88925	2.52818	3.16204	3.40986
log. sin.−log.	4.68557	4.68557	4.68557	4.68557	4.68556
sin. $\frac{1}{2}(\alpha - A)$	7.96081	7.57482	7.21375	7.84761	8.09542
sin. $(\delta + D)$	9.73131	9.73261	9.73388	9.73513	9.73636
cosec. π	1.75959	1.75935	1.75912	1.75888	1.75865
log. (2)	7.41252	6.64160	5.92050	7.18923	7.68585
(1)	.191047	.301040	.409216	.515540	.619974
(2)	.002586	.000438	.000083	.001546	.004851
$(1)+(2)=y$	.193633	.301478	.409299	.517086	.624825

The following are the results:

Hour.	x	x'	Diff.	y	y'	Diff.
11	−1.007841			+.193633		
		+.593905			+.107845	
12	− .413936		+80	+.301478		−24
		+.593985			+.107821	
13	+ .180049		+16	+.409299		−34
		+.594001			+.107787	
14	+ .774050		−44	+.517086		−48
		+.593957			+.107739	
15	+1.368007			+.624825		

For the first trial, we may assume T = 13 hours Greenwich mean time, and we shall obtain the approximate times of immersion and emersion. As, however, this example has already been computed on page 259, we will suppose the approximate times to be known, and will assume 12 hours for immersion, and 13.25 hours for emersion. The work will then be as follows:

	Immersion.	Emersion.
T	12h. Gr. m. t.	13.25h. Gr. m. t.
x	−.413936	+.328551
y	+.301478	+.436250
x'	+.593945	+.593997
y'	+.107833	+.107795
$\mu - A$	−14° 59′ 54″.67	+3° 48′ 10″.14
ρ cos. ϕ'	9.8691208	9.8691208
sin. $(\mu - A)$	9.4129543*n*	8.8216638
log. ξ	9.2820751*n*	8.6907846
ξ	−.191459	+.049066
ρ sin. ϕ'	9.8264412	
cos. D	9.9824020	
log. (1)	9.8088432	
(1)	+.643937	+.643937
ρ cos, ϕ'	9.8691208	9.8691208
sin. D	9.4456150	9.4456150
cos. $(\mu - A)$	9.9849468	9.9990427
log. (2)	9.2996826	9.3137785
(2)	+.199380	+.205958
$(1) - (2) = \eta$	+.444557	+.437979
ρ cos. ϕ'	9.8691208	9.8691208
cos. $(\mu - A)$	9.9849468	9.9990427
$d\mu$	9.4191561	9.4191561
log. $d\xi$	9.2732237	9.2873196
$d\xi$	+.187596	+.193785

For immersion, $Q_1 = \qquad N-\psi = 57^\circ\ 44'$.
" emersion, $Q_2 = 180^\circ + N + \psi = 270^\circ\ 45'$.

These results are nearly the same as found on page 261.

(297.) We may obtain a check upon the accuracy of our computations in the same manner as shown for a solar eclipse on page 289. Equation (5), page 267, becomes, in the case of an occultation,

$$(x-\xi)^2+(y-\eta)^2=k^2=.074256,$$

the quantities x, y, ξ, and η being supposed to be computed for the instant of immersion or emersion. If these quantities have been computed for a time, T, which differs from the instant of immersion or emersion by a small interval, t, we shall have

$$[x-\xi+(x'-d\xi)t]^2+[y-\eta+(y'-d\eta)t]^2=k^2.$$

Thus in the preceding example, for immersion, $t=-.019565$.

$x-\xi=-.222477$
$(x'-d\xi)t=-.007950$
Sum $=-.230427$, whose square is .053097.

Also, $y-\eta=-.143079$
$(y'-d\eta)t=-.002384$
Sum $=-.145463$, whose square is .021159.

The sum of these two squares is .074256.

For emersion, $t=-.017511$.

$x-\xi=+.279485$
$(x'-d\xi)t=-.007008$
Sum $=\ \ .272477$, whose square is .074244.

Also, $y-\eta=-.001729$
$(y'-d\eta)t=-.001825$
Sum $=\ \ .003554$, whose square is .000012.

The sum of these two squares is .074256.

(298.) *Ex.* 2. Required the time of occultation of γ Virginis, January 9, 1855, for Washington Observatory.

Apparent conjunction takes place between 23 and 24 hours of Greenwich mean time. The moon's places for 23, 24, and 25 hours, Greenwich time, according to the American Nautical Almanac, are as follows:

Greenwich m. t.	α.	δ.	π.
h.	h. m. s.	° ′ ″	′ ″
23	12 33 21.97	+0 8 30.8	55 31.18
24	12 35 12.33	−0 5 32.3	55 32.80
25	12 37 2.80	−0 19 36.2	55 34.43

The position of γ Virginis is

$$A=12^h.\ 34^m.\ 18^s.43;\quad D=-0^\circ\ 39'\ 12''.1.$$

Computation of the co-ordinates x and y.

Greenwich mean Time.	23h.	24h.	25h.
$\alpha-A$	$-14'\ 6''.9$	$+13'\ 28''.5$	$+41'\ 5''.55$
log. π	3.5225981	3.5228093	3.5230216
log. sin. − log.	4.6855560	4.6855560	4.6855559
sin. π	8.2081541	8.2083653	8.2085775
log. $(\alpha-A)$	2.9278321*n*	2.9076800	3.3919138
log. sin. − log.	4.6855736	4.6855738	4.6855645
cos. δ	9.9999987	9.9999994	9.9999929
cosec. π	1.7918459	1.7916347	1.7914225
log. x	9.4052503*n*	9.3848879	9.8688937
x	−.254244	+.242598	+.739424
$\delta+D$	$-30'\ 41''.3$	$-44'\ 44''.4$	$-58'\ 48''.3$
$\delta-D$	$+47'\ 42''.9$	$+33'\ 39''.8$	$+19'\ 35''.9$
cos. $\frac{1}{2}(\alpha-A)$	9.9999991	9.9999992	9.9999922
cos. $\frac{1}{2}(\alpha-A)$	9.9999991	9.9999992	9.9999922
log. $(\delta-D)$	3.4568062	3.3053084	3.0703704
log. sin. − log.	4.6855609	4.6855679	4.6855725
cosec. π	1.7918459	1.7916347	1.7914225
log. (1)	9.9342112	9.7825094	9.5473498
log. $\frac{1}{2}(\alpha-A)$	2.62680	2.60665	3.09088
log. sin. − log.	4.68557	4.68557	4.68557
sin. $\frac{1}{2}(\alpha-A)$	7.31237	7.29222	7.77645
sin. $(\delta+D)$	7.95069*n*	8.10440*n*	8.23250*n*
cosec. π	1.79185	1.79163	1.79142
log. (2)	4.36728*n*	4.48047*n*	5.57682*n*
(1)	+.859431	+.606051	+.352655
(2)	−.000002	−.000003	−.000038
$(1)+(2)=y$	+.859429	+.606048	+.352617

The following are the results:

Hour.	x	x'	Diff.	y	y'	Diff.
23	−.254244			+.859429		
		+.496842			−.253381	
24	+.242598		−16	+.606048		−50
		+.496826			−.253431	
25	+.739424			+.352617		

For a first approximation we assume T=24 hours Greenwich mean time. The corresponding sidereal time at Washington is 14h. 9m. 35.06s.; whence

$$\mu-A=+23^\circ\ 49'\ 9''.45.$$

Also, ρ sin. $\phi'=9.7955439$, and ρ cos. $\phi'=9.8917226$.

Hence we obtain

$$x = +.24260;\ \xi = +.31474;\ m \sin. M = -.07214.$$
$$y = +.60605;\ \eta = +.63261;\ m \cos. M = -.02656.$$
$$M = 249^\circ\ 47'\ 16'',\ \log. m = 8.885779.$$
$$x' = +.49683;\ d\xi = +.18716;\ n \sin. N = +.30967.$$
$$y' = -.25341;\ d\eta = -.00094;\ n \cos. N = -.25247.$$
$$N = 129^\circ\ 11'\ 24'',\ \log. n = 9.601567.$$
$$\psi = 14^\circ\ 3'\ 12''.$$
$$t_1 = -.56368;\ t_2 = +.75955.$$

Greenwich mean Time. Washington mean Time.
Time of immersion = 23.43632h., or 18h. 17m. 59s.
" emersion = 24.75955h., " 19h. 37m. 23s.

For the second approximation we will assume 23h. 25m. for immersion, and 24h. 45m. for emersion. The results are as follows:

	Immersion.	Emersion.
T	23h. 25m. Gr. m. t.	24h. 45m. Gr. m. t.
x	$-.047224$	$+.615218$
y	$+.753860$	$+.415979$
x'	$+.496843$	$+.496821$
y'	$-.253377$	$-.253443$
$\mu - A$	15° 2′ 43″.2	35° 6′ 0″.3
ξ	$+.202302$	$+.448121$
η	$+.633058$	$+.631747$
$d\xi$	$+.197574$	$+.167383$
$d\eta$	$-.000606$	$-.001341$
M	295° 49′ 58″.2	142° 14′ 41″.2
log. m	9.4428399	9.4360119
N	130° 11′ 7″.3	127° 25′ 29″.9
log. n	9.5929907	9.6178712
ψ	14° 36′ 25″.7	14° 50′ 32″.5
t	$+.012470$	$-.001011$
Q	115° 35′	322° 16′

Hence we have Greenwich mean Time. Washington mean Time.
Time of immersion, 23h. 25m. 44.9s., or 18h. 17m. 33.7s.
" emersion, 24h. 44m. 56.4s., " 19h. 36m. 45.2s.

Check.—For immersion,

$$x - \xi = -.249526$$
$$(x' - d\xi)t = +.003732$$
$$\text{Sum} = -.245794, \text{ whose square is } .060414.$$

Also,

$$y - \eta = +.120802$$
$$(y' - d\eta)t = -.003152$$
$$\text{Sum} = +.117650, \text{ whose square is } .013842.$$

The sum of these two squares is .074256.

For emersion,

$x-\xi=+.167097$

$(x'-d\xi)t=-.000333$

Sum $=+.166764$, whose square is .027810.

Also, $y-\eta=-.215768$

$(y'-d\eta)t=+.000255$

Sum $=-.215513$, whose square is .046446.

The sum of these two squares is .074256.

In the Tables from which the American Nautical Almanac is computed, the value of k is assumed to be 0.272278. In the English Nautical Almanac for 1857 the value of k is assumed to be 0.273114. The value employed in Burckhardt's Tables of the Moon is 0.2725.

(299.) In the American Nautical Almanac, and also in the Berlin Jahrbuch, are furnished elements by which the preceding computations are materially abridged. These elements are the co-ordinates x and y, with their hourly variations. In the American Almanac, p. 375–397, is given a list of all the stars, to the sixth magnitude inclusive, contained in the B. A. Catalogue, which can be occulted by the moon. It also furnishes for each star the Washington mean time (T) of conjunction with the moon; the Washington hour angle of the star at the time T; and the co-ordinates for the same time, with their hourly variations. At the instant of conjunction x reduces to zero, and is therefore omitted from the almanac.

Thus for γ Virginis, January 9, 1855, we find on page 375,

T = Washington mean time of conjunction, 18h. 22.5m.
H = Washington hour angle of star at time T, +1h. 5m. 51s.
Y = the co-ordinate which Bessel represents by y, +0.7298
p' = hourly variation of p, Bessel represents by x', +0.4968
q' = hourly variation of q, Bessel represents by y', −0.2530
log. sin. D = log. sine of star's declination, −8.0570
log. cos. D = log. cosine of star's declination, 0.0000

Having the assistance of these numbers, we are relieved from the necessity of the preliminary computations on page 297, and the approximate times of immersion and emersion are obtained with very little labor, especially if we employ logarithms to only four decimal places, which will generally furnish results correct to the nearest minute.

CHAPTER XII.

LONGITUDE.

SECTION I.

LONGITUDE DETERMINED BY TRANSPORTATION OF CHRONOMETERS.

(300.) THE manufacture of chronometers has attained to such a degree of perfection as to afford the means of determining the difference of longitude of two stations, not too remote from each other, with a precision superior to that of most other methods. The following are the essential steps of this method: The time is accurately determined at one station, Greenwich, for instance, and the chronometer is carefully compared with the transit clock; hence the error of the chronometer on the meridian of Greenwich is known. The chronometer being carried to a second station, for example, Cambridge Observatory, is compared with the transit clock at that place. Thus the error of the chronometer on the meridian of Cambridge is known; but its error on the meridian of Greenwich at the same instant is known, if its *rate* be known, and the longitude is the difference of these two errors. In grand Chronometric expeditions, it is customary to employ a large number of chronometers, from twenty to fifty, or more, as checks upon each other.

(301.) The most serious difficulty in the application of this method consists in determining the rate of the chronometers during the journey, for chronometers generally have a different rate when transported from place to place, either by land or by sea, from that which they maintain in an observatory. When it is proposed to determine the difference of longitude of two stations with the greatest accuracy, the error of the chronometers should be determined at the commencement of the expedition, at the first station; the same thing should be done at the second station; then, as soon as possible, the chronometers should be brought back to the first station, and their error determined anew. The chronometers should thus be transported back and forth a considerable number of times.

Let us designate the eastern station by A, the western by B, and the west longitude of the place B from A we will designate by ω. We will suppose that at the time t, at the place A, the error of one of the chronometers was a; that on its arrival at B, at the time t', the error was b; and again, on its return to A at the time t'', the error was a'. If we regard a day as the unit of time, and represent the mean daily rate of the chronometer during the journey by m, we shall have

$$m=\frac{a'-a}{t''-t};$$

whence we may conclude that

$$\omega=a+m\,(t'-t)-b,$$

or

$$\omega=a'-m(t''-t')-b.$$

Each chronometer will afford an independent determination of the value of ω; and in order to detect any irregularity in the rates of the chronometers, they should be compared daily with each other throughout the entire journey.

The following observations were made to determine the difference of longitude between two stations, A and B:

Station A	t = September 15, 11.55h.	$a=+34$m. 20.1s.
Station B	t' = September 17, 18.85h.	$b=+31$m. 0.6s.
Station A	t'' = September 18, 11.55h.	$a'=+34$m. 4.4s.

Consequently we have $m=-\frac{15.7\text{s.}}{3}=-5.23\text{s.}$

$$\begin{aligned} a&=+34\text{m. } 20.1\text{s.}\\ m(t'-t)=-5.23\times 2.304&=\qquad -12.1\text{s.}\\ -b&=-31\text{m. } 0.6\text{s.}\\ \text{Longitude}=\omega&=\quad 3\text{m. } 7.4\text{s.} \end{aligned}$$

(302.) Since chronometers almost invariably indicate a different rate, according as they are traveling or at rest, if the observer remains for several days at the station B, the error of the chronometers should be determined immediately upon arrival, and again before departing from B; and the interval of rest should not be included in the determination of the value of m. Suppose we have determined the chronometer errors

$$a,\ b,\ b',\ a',$$

corresponding to the times

$$t,\ t',\ t'',\ t''',$$

where a and a' are supposed to have been obtained at the place

A; b was the error on first arriving at B, and b' the error on departing from B.

Then the interval of time embraced in the two journeys is

$$(t'-t)+(t'''-t'');$$

and the change in the error of the chronometer for the same time is

$$(a'-a)-(b'-b).$$

Hence we have $$m=\frac{(a'-a)-(b'-b)}{(t'-t)+(t'''-t'')}.$$

The following example is taken from Struve's chronometric expedition, undertaken in 1843, between Pulkova and Altona:

Pulkova	t = May 19, 21.54h.	$a=$ +6m. 38.10s.
Altona	t' = May 24, 22.66h.	$b=$ −1h. 14m. 39.92s.
Altona	t'' = May 26, 10.72h.	$b'=$ −1h. 14m. 36.77s.
Pulkova	t''' = May 31, 0.00h.	$a'=$ +7m. 9.58s.

Here we have
$$\begin{aligned} a'-a&=31.48\text{s.}\\ b'-b&=\ \,3.15\text{s.}\\ \hline (a'-a)-(b'-b)&=28.33\text{s.}\end{aligned}$$

Also,
$$\begin{aligned} t'-t&=5\text{d. }\ 1.12\text{h.}\\ t'''-t''&=4\text{d. }13.28\text{h.}\\ (t'-t)+(t'''-t'')&=9\text{d. }14.40\text{h.}\end{aligned}$$

Hence $$m=\frac{28.33\text{s.}}{9.6}=+2.951\text{s.}$$

$$\begin{aligned} a&=\qquad +6\text{m. }38.10\text{s.}\\ m(t'-t)=2.951\times5.047&=\qquad\qquad +14.89\text{s.}\\ -b&=+1\text{h. }14\text{m. }39.92\text{s.}\\ \hline \text{Longitude}=\omega&=\ \ 1\text{h. }21\text{m. }32.91\text{s.}\end{aligned}$$

(303.) It is here assumed that the rate of the chronometer was the same during the journey from Pulkova to Altona as during the journey from Altona to Pulkova. In order to eliminate any error which might arise from this supposition, Struve begins the next calculation with Altona, so that any change in the rate of the chronometer will produce the opposite effect from that which would result if the computation commenced with Pulkova. The following combination is the one which immediately succeeded that of the former example:

Altona	t = May 26, 10.72h.	$b=$ −1h. 14m. 36.77s.
Pulkova	t' = May 31, 0.00h.	$a=$ +7m. 9.58s.
Pulkova	t'' = June 3, 5.62h.	$a'=$ +7m. 19.36s.
Altona	t''' = June 7, 20.52h.	$b'=$ −1h. 14m. 0.35s.

Here we have

$$\begin{aligned}
b'-b&=36.42\text{s.}\\
a'-a&=\ \ 9.78\text{s.}\\
(b'-b)-(a'-a)&=\overline{26.64\text{s.}}\\
t'-t&=4\text{d. } 13.28\text{h.}\\
t'''-t''&=4\text{d. } 14.90\text{h.}\\
(t'-t)+(t'''-t'')&=9\text{d. } \ 4.18\text{h.}\\
m&=\frac{26.64\text{s.}}{9.174}=+2.904\text{s.}\\
b&=-1\text{h. } 14\text{m. } 36.77\text{s.}\\
m(t'-t)=2.904\times 4.553&=\qquad\qquad +13.22\text{s.}\\
-a&=\qquad -7\text{m. } \ 9.58\text{s.}\\
\text{Longitude}=\omega&=-\overline{1\text{h. } 21\text{m. } 33.13\text{s.}}
\end{aligned}$$

In the chronometric expedition already referred to, nine voyages were made from Pulkova to Altona, and eight from Altona to Pulkova, in which 81 chronometers were employed. The results of 13 of this number, having shown greater discordances than the rest, were rejected, and the deduced longitude was made to depend upon 68 chronometers.

This result was

1h. 21m. 32.52s.,

with a probable error, according to Struve, of only 0.04s.

(304.) It is indispensable to the accuracy of these results that the time be obtained at each station with the greatest precision. Struve recommends that the time be determined with a transit instrument, by observations of stars near the zenith, inasmuch as a slight deviation of the transit instrument from the plane of the meridian does not affect the time of passage of a zenith star. It is necessary, however, to know the inclination of the axis with the greatest accuracy; and the axis should be reversed upon its supports during each series of observations, so as to eliminate the effect of unequal pivots and of collimation error. In order to eliminate the effect of any error in the right ascensions of the stars employed, the same stars should, if possible, be observed at both stations. For this purpose, a catalogue of all the stars which pass near the zenith, and of a magnitude sufficient to be observed without inconvenience, should be prepared beforehand, and a copy furnished to each observer. If the places of any of the stars are too imperfectly known, they should be carefully observed with the instruments of some large observatory.

(305.) The comparisons of the chronometers should all be made by observing the *coincidence* of beats. If we undertake to compare two clocks which beat seconds of the same kind of time, unless they happen to tick at the same instant, there is a fraction of a second which must be estimated by the ear. This estimation is extremely difficult, and practiced observers will differ among themselves by a quarter of a second, and sometimes even more. When, however, the two clocks happen to tick together, there is no fraction of a second to be estimated; and a practiced ear will detect any deviation from coincidence in beats amounting to 0.01s. Now a sidereal clock gains upon a solar clock one second in about six minutes; and if two such clocks are placed side by side, they must tick together once in every six minutes. In order to compare two such clocks, we notice their movements, and wait until the beats sensibly coincide, when we know that their difference amounts to an entire number of seconds, which is readily discovered. Chronometers generally make two beats in a second; so that between a clock which beats seconds of sidereal time, and a chronometer which ticks half seconds of solar time, there must be a coincidence every three minutes. Chronometers are sometimes made to tick 13 times in 6 seconds. Such a chronometer, regulated to mean time, makes 121 ticks in 56 seconds of sidereal time; that is, the coincidences between such a chronometer and a sidereal seconds-pendulum would occur every 56 seconds. Moreover, the intervals between the ticks of the chronometer is 0.4628s. sidereal time; and 13 of these intervals are equal to 6.016s. sidereal time; 54 are equal to 24.991s.; 67 are equal to 31.007s.; and 121 are equal to 55.999s; that is, in the course of 56 seconds there are five coincidences within the limits ±0.02s. Such a chronometer affords the means of comparing by coincidences with great rapidity; a consideration of no trifling importance where 80 chronometers are to be compared daily. Chronometers are frequently made to beat five times in two seconds, which gives a coincidence at every 36 seconds with a half-second sidereal chronometer.

(306.) It is also indispensable to the accuracy of the results that the personal equation of all the observers employed in obtaining the time should be carefully determined. The mode of doing this has already been explained on page 80. This correction is the most difficult to obtain satisfactorily, especially as personal equation is not always a constant quantity, but is liable to vary with the physical condition of the observer. It is the opinion of Mr. Airy, that when a tolerable number of chronometers is used for a moderate distance, and in good observing weather, the *variation* of personal equation is the error to be most apprehended.

A grand chronometric expedition has been for several years in progress, at the expense of the United States coast survey, for the purpose of determining the difference of longitude between Greenwich and Cambridge, Massachusetts. A large number of chronometers have been transported by means of the Cunard steamers from the Liverpool Observatory to Cambridge, and back again to Liverpool. During the summer of 1849, forty-four different chronometers were employed in several trials, and during the progress of the expedition more than four hundred exchanges of chronometers have been made. For facility of comparing the chronometers, Mr. Bond used a chronometer beating half seconds, and gaining 12 minutes daily on mean solar time, which furnished a coincidence of beats every 90 seconds.

SECTION II.

LONGITUDE DETERMINED BY THE ELECTRIC TELEGRAPH.

(307.) The difference of the local times of two places may be determined by means of any signal which can be seen or heard at both places at the same instant. When the places are not very distant, the explosion of a rocket or the flash of gunpowder may serve this purpose. Six or eight ounces of powder at night makes a good signal at a distance of twenty-five to thirty miles; but for a distance of ten miles, two or three ounces are sufficient, if the observers are provided with telescopes.

(308.) But the electric telegraph affords the means of transmitting signals to a distance of a thousand miles or more with scarcely any appreciable loss of time. The first experiments of this kind any where made were undertaken in the United States; and, with the exception of a rude experiment of Captain Wilkes in 1844, all the experiments in this country have been made in connection with the United States Coast Survey. Suppose there are two observatories at a considerable distance from each other, and that each is provided with a good clock and a transit instrument for determining its error; then, if they are connected by a telegraph wire, they have the means of transmitting signals at pleasure from either observatory to the other, for the purpose of comparing their local times. The signal is given at either station by pressing a key, as in the usual mode of telegraphing; and the observer at the other station hears the click caused by the motion of the armature of his electro-magnet. Four different methods of comparison have been practiced in the experiments by the United States Coast Survey.

(309.) The first method is the most obvious one, and consists in simply striking on the signal key at intervals of ten seconds; the party at one station recording the time when the signals were given, and the other party recording the time when the signals were received. After about twenty signals have been transmitted from the first station to the second, a similar set of signals is returned from the second station to the first. This mode of comparison has but one serious imperfection, and this is, that it requires the fraction of a second to be estimated by the ear. The party giving the signals strikes his key in coincidence with the beats of his clock, so that at this station there is no fraction of a second to be estimated; but at the other station the armature click will not probably be heard in coincidence with the beats of the clock, and the fraction of a second is to be estimated by the ear. Now this fraction can not be estimated with the accuracy which is demanded in this kind of comparison. It is found that observers generally estimate the fraction of a second too small when using the ear alone, unassisted by the eye. This error is greatest at the middle date between two clock beats, and is found to vary from 0.06 to 0.18 of a second with different observers.

(310.) This evil suggested the second method of observation, which relies on the coincidences of a mean solar and sidereal clock or chronometer. The following is the method pursued: After transmitting a few signals by the former method, so as to determine the difference between the local times of the two stations within a small fraction of a second, the party at the first station commences striking on his signal key every second, in coincidence with the beats of his mean solar chronometer, and continues to do so for ten or fifteen minutes without interruption. The party at the second station compares the armature click of his magnet with the beats of his sidereal clock, and watches for a coincidence, and records the time when the coincidence takes place. When he has obtained two or three coincidences, which generally requires from ten to fifteen minutes, he breaks the electric circuit, in order to notify the first party to stop beating. He then commences beating seconds by striking his own signal key in coincidence with the beats of his sidereal clock; and the party at the first station compares the armature clicks of his magnet with the beats of his solar chronometer, and watches for a coincidence. When he has obtained three or four coincidences, which generally requires ten or twelve minutes, he breaks the electric circuit, in order to notify the other party to stop beating. The comparison of times at the two stations is now complete.

(311.) The following observations were made August 1, 1849, for the purpose of determining the difference of longitude between the High School Observatory in Philadelphia and Western Reserve College Observatory at Hudson, Ohio. The time-keeper employed at Philadelphia was a mean solar chronometer, beating half seconds; the time-keeper at Hudson was a sidereal clock.

	Signals given at Philadelphia, mean Time.			Signals received at Hudson, sid. Time.			Signals given at Hudson, sid. Time.			Signals received at Philadel., mean Time.		
	h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.
	14	15	30	17	44	23.5	18	13	0	14	44	1.6
			40			33.5			10			11.8
			50			43.5			20			21.8
		16	0			53.7			30			31.9
			10		45	3.8			40			41.8
		20	20		49	14.2			50			51.7
			30			24.3		14	0		45	1.7
			40			34.4			10			11.4
			50			44.4			20			21.6
		21	0			54.5			30			31.6
			10		50	4.5			40			41.4
Result,	14	20	40	17	49	34.4	18	14	0	14	45	1.7

From these comparisons we may conclude that 14h. 20m. 40s. on the Philadelphia chronometer corresponds to 17h. 49m. 34.4s.

on the Hudson clock; and 18h. 14m. 0s. on the Hudson clock corresponds to 14h. 45m. 1.7s. on the Philadelphia chronometer.

The Philadelphia observer beat seconds for ten minutes, and two coincidences were recorded at Hudson; viz., at

17h. 58m. 0s.,
18h. 4m. 10s.

The Hudson observer beat seconds for eleven minutes, and three coincidences were recorded at Philadelphia; viz., at

14h. 54m. 25s.,
14h. 57m. 28.5s.,
15h. 0m. 39s.

The former comparisons show us that

By the Philadelphia Chronometer.		*By the Hudson Clock.*
14h. 29m. 4s.	correspond to	17h. 58m. 0s.
14h. 35m. 13s.	"	18h. 4m. 10s.
14h. 54m. 25s.	"	18h. 23m. 25s.
14h. 57m. 28.5s.	"	18h. 26m. 29s.
15h. 0m. 39s.	"	18h. 29m. 40s.

At 16h. 0m. the Philadelphia chronometer was 4h. 48m. 25.78s. fast, and losing 2.22s. per day.

At 18h. 6m. the Hudson clock was 8.13s. fast, and losing 1.02s. per day.

In the following table, column first shows the corrected Philadelphia mean times; column second the corresponding Philadelphia sidereal times; column third the corrected Hudson sidereal times; and column fourth shows the differences between the numbers in the two preceding columns, or the difference of longitude between the two places.

Philadel. mean Time.	Philadelphia sid. Time.	Hudson sid. Time.	Diff. of Longitude.
h. m. s.	*h. m. s.*	*h. m. s.*	*m. s.*
9 40 38.08	18 22 57.43	17 57 51.87	25 5.56
9 46 47.09	18 29 7.45	18 4 1.87	5.58
10 5 59.12	18 48 22.63	18 23 16.89	5.74
10 9 2.62	18 51 26.64	18 26 20.89	5.75
10 12 13.13	18 54 37.66	18 29 31.89	5.77

Mean of results by eastern signals, 25m. 5.57s.
" " western signals, 25m. 5.75s.
Mean of both, 25m. 5.66s.

The difference between the results by eastern and western signals is partly due to the time required for the transmission of the signals; but this effect disappears from the mean of both sets of signals.

(312.) A third method of comparing local times is by telegraphing transits of stars. This method was practiced in the summer of 1848, between New York and Cambridge, in the following manner: A list of zenith stars is selected beforehand, and furnished to each observer. When every thing is prepared for observation, the Cambridge astronomer points his telescope upon one of the selected stars as it is passing his meridian, and strikes the key of his register at the instant the star appears to coincide with the first wire of his transit. He makes a record of the time by his own chronometer, and the New York astronomer, hearing the click of his magnet, records the time by his own clock. As the star passes over the second wire of the transit instrument, the Cambridge astronomer again strikes the key of his register, and the time is recorded both at Cambridge and New York. The same operation is repeated for each of the other wires. The Cambridge astronomer now points his telescope upon the next star of the list, which culminates after an interval of five or six minutes, and telegraphs its transit in the same manner. In about twelve minutes from the former observation, the first star passes the meridian of New York, when the New York astronomer points his transit instrument upon the same star, and strikes the key of his register at the instant the star passes each wire of his transit. The times are recorded both at New York and Cambridge. The second star is telegraphed in a similar manner. The same operations are now repeated upon a second pair of stars, and so on as long as may be thought desirable.

The chief objection to this method is, that it involves the estimation of fractions of a second, as in the usual mode of transit observations; that is, it involves the personal equation of the observers.

(313.) The fourth method of comparison obviates this evil in some degree, by printing the signals upon a cylinder or a fillet of paper. There must be a clock at one of the stations for breaking the electric circuit every second, as described in Art. 102; and there must be a register at each of the stations for recording the beats of the clock and any other signals which may be required, as described in Art. 106. When the connections are properly made, there will be heard a click of the magnets at each station simultaneously with the beats of the electric clock, and the registers will all be graduated into second spaces. The method is not limited to two stations, but any number of stations may be compared at the same time. In January, 1849, Cambridge, New York, Philadelphia, and Washington were connected in this manner. The mode of observation is the same as described in the preceding article, except that the observations are all recorded by the operation of machinery. The Cambridge astronomer strikes the key of his register as the star passes successively each wire of his transit instrument, and the dates are printed not only upon his own register, but also upon those at New York, Philadelphia, and Washington. When the same star comes over the meridian of New York, the observer there goes through the same operation, and his observations are printed upon all four of the registers. The Philadelphia observer does the same when the star comes upon his own meridian, and we proceed in the same manner whatever be the number of stations. Thus we have four or more registers all graduated into equal parts by the ticking of the same clock, and upon these we have printed the instants at which the star was seen to pass each wire of the transit telescopes at the several stations. These observations furnish the difference of longitude of the stations, independently of the tabular place of the star employed, and also independently of the absolute error of the clock. The observers now read their levels, and reverse their transit instruments. A second star is now telegraphed successively over each meridian, and so on as long as may be desired.

(314.) The following example is derived from observations made in the summer of 1852, to determine the difference of longitude between Seaton Station, in Washington, District of Columbia, and Roslyn Station, near Petersburgh, Virginia. The observations at Seaton were recorded upon Bond's spring governor, and those at Roslyn upon Saxton's register, and also a Morse register. The diaphragms of the transits consisted of twenty-five wires, arranged in groups of five. The following are the observations of star 6150, British Association Catalogue, July 7, 1852, with the complete reduction for determining the clock error:

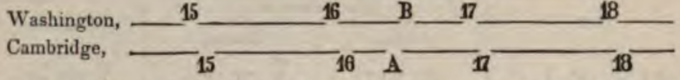
	Seaton Station.	Roslyn Station.
	h. m. s.	h. m. s.
Mean of all the wires,	18 2 9.341	18 3 45.108
Reduction to middle wire,	+.030	.000
Diurnal aberration,	−.018	−.018
Level correction,	−.014	−.076
Azimuth correction,	−.082	−.156
Collimation correction,	−.106	−.192
Personal equation,	−.146	−.000
Sum,	18 2 9.005	18 3 44.666
Star's right ascension,	18 1 48.288	18 1 48.288
Clock error,	20.717	1 56.378

The following table shows the clock errors, derived in a similar manner, from the transits of 15 stars, on the night of July 7th:

Star.	Clock errors at		Difference.
	Seaton.	Roslyn.	
	s.	m. s.	m. s.
6150, B. A. C.	20.717	1 56.378	1 35.661
6268 "	21.214	56.710	35.496
6355 "	20.977	56.539	35.562
6404 "	21.134	56.705	35.571
6599 "	20.912	56.535	35.623
6667 "	20.942	56.663	35.721
6722 "	21.023	56.652	35.629
6784 "	20.964	56.659	35.695
7048 "	21.398	56.909	35.591
7114 "	21.183	56.768	35.585
7204 "	21.017	56.739	35.722
7277 "	21.034	56.713	35.679
7333 "	21.015	56.642	35.627
7398 "	20.938	56.347	35.409
7521 "	20.917	56.597	35.680
Mean of observations July 7, 1852			1 35.617
Mean of all the observations on six nights . .			1 35.603

(315.) This method of observation is so accurate as to furnish a tolerable measurement of the velocity of the electric fluid. If the fluid requires no time for its transmission, then the signals given at either station ought to be similarly printed at all the stations; and the fraction of a second registered upon any one scale should be identically the same as upon every other. But if the fluid requires time for its transmission, these fractions will be different. Suppose the clock to be at Washington; that an arbitrary signal is made at Cambridge; and that the time requisite for the transmission of a signal between the two places is the thirtieth of a second. Then the clock-pause will be registered at Cambridge $\frac{1}{30}$th of a second after it took place and was recorded at Washington, and the arbitrary signal-pause will be recorded at Cambridge as soon as it is made, or $\frac{1}{30}$th of a second before it reaches Washington. We shall thus have the interval between the signal-pause and the preceding clock-pause longer at Washington than at Cambridge, and the excess on the Washington register will measure twice the time consumed in the transmission of the signals between the two stations.

Thus, in the following figure, let the upper line represent a portion of the Washington time scale, corresponding to 15, 16, etc., seconds, and the lower line the same for Cambridge, each division being a little later than the corresponding one for Washington. Then, if an arbitrary signal is made at Cambridge between 16 and 17 seconds, and printed at A, the record on the Washington scale will be at B, and the interval from 16 to B will exceed that from 16 to A by twice the time consumed in the transmission of the signals from Cambridge to Washington.



Numerous observations have been made under the direction of the superintendent of the Coast Survey for the purpose of determining the velocity of the electric fluid, and the general result is about 16,000 miles a second.

SECTION III.

LONGITUDE DETERMINED BY MOON-CULMINATING STARS.

(316.) The moon's motion in right ascension is very rapid, amounting to about one minute in arc for every two minutes of time.

If, then, the right ascension of the moon has been observed at two different stations, we may infer the difference of longitude of the two meridians from the difference of the observed right ascensions compared with the times of observation. If we have a transit instrument adjusted to the meridian, and observe the passage of the moon's limb and some known star, we can deduce the right ascension of the moon's limb from the known right ascension of the star. If we select for comparison a star which is near the moon, the errors of the instrument will have but little influence upon the result, since these errors will be nearly the same for the moon and star. The English and American Nautical Almanacs both furnish the moon's place and those of certain neighboring stars on every day upon which it is possible to observe the moon. These stars are called moon-culminating stars, and are generally four in number for each day, two preceding and two following the moon, and nearly on the same parallel of declination.

(317.) The Nautical Almanac furnishes the right ascension of the moon's bright limb for the lower as well as the upper culmination, L. C. being put to denote the lower culmination, and U. C. the upper culmination. The right ascension of the moon's bright limb is given *for every day*, with a view to the more accurate determination of its variation, when required. It also furnishes the variation in right ascension of the moon's limb in one hour of longitude; that is, the variation during the interval of her transit over two meridians, equidistant from that of Greenwich, and one hour distant from each other. These numbers are deduced from the right ascensions of the *bright limb*, and therefore include the effect produced by the change of the semidiameter.

(318.) These numbers enable us to determine the difference of longitude of any two places where corresponding observations of the moon's limb have been made. The observations furnish the right ascension of the moon's bright limb at its transit over each meridian, which we will represent by A and A′; hence we know the moon's motion in right ascension, A′−A, during the interval of the two transits. But the Almanac furnishes the variation of the moon's right ascension corresponding to one hour, which we will represent by V.

We shall therefore have the proportion

V : A′−A :: 1 hour : the difference of longitude.

Ex. 1. The right ascension of the moon's first limb, September 6, 1840, was observed at Washington to be 19h. 21m. 29.90s., and on the same evening, at Hudson, Ohio, 19h. 22m. 9.72s. Required the difference of longitude of the two places.

Here $A'-A=39.82s.$

That value of V must be taken which corresponds to the middle of the interval between the observations, which is found by interpolation to be 135.55s. Hence we have

135.55s. : 39.82s. :: 1 hour : 17m. 37.56s.,

which is the required difference of longitude.

(319.) Since the moon's motion in right ascension is not uniform, this method of reduction can not be relied upon when the distance between the meridians is considerable. The following method in such cases is to be preferred:

Let ω represent the approximate longitude of the station to be compared with Greenwich, as Washington, for example, and $\omega+x$ the true longitude to be determined. Let A and A′ be the observed right ascensions of the moon's limb at the moments of its passing the meridians of Greenwich and Washington respectively. These will evidently be the sidereal times of her transit at those places. Find, by interpolation from the Nautical Almanac, the moon's right ascension for the assumed longitude ω, and call it A″. Now A′, the sidereal time of transit of the moon's limb at Washington, is her right ascension for the true longitude $\omega+x$, and consequently $A'-A''$ is the *increase* of the moon's right ascension for the small arc of longitude x.

Let $m'=A'-A=$ the observed increase of right ascension of the moon's limb between the two meridians.

$m=A''-A=$ the increase computed for the assumed longitude ω.

Then $m'-m=A'-A''=$ the excess of the observed increase above the computed increase.

And we shall have

$$m:\omega::m'-m:x;$$

that is,
$$x=\frac{\omega}{m}(m'-m).$$

The true longitude $=\omega+x$.

Ex. 2. The increase of right ascension of the moon's bright limb between her transits over the meridians of Greenwich and

Hudson, Ohio, September 6, 1840, was found to be 12m. 17.95s. Required the difference of longitude.

According to the Nautical Almanac, the right ascension of the moon's bright limb for Greenwich transit was,

Date.	R. A. Moon's Limb.	D′.	D″.	D‴.	D⁗.
	h. m. s.	m. s.	s.	s.	s.
Sept. 5, U. C.	18 14 45.36				
		+27 40.48			
L. C.	18 42 25.84		−14.80		
		+27 25.68		−5.77	
" 6, U. C.	19 9 51.52		$c_{,}=-20.57$		$e_{,,}=+1.68$
		$b_0=+27$ 5.11		$d_{,}=-4.09$	
L. C.	19 36 56.63		$c_0=-24.66$		$e_{,}=+1.94$
		+26 40.45		−2.15	
" 7, U. C.	20 3 37.08		−26.81		
		+26 13.64			
L. C.	20 29 50.72				

and the successive orders of differences are found as above.

Hence we have, by Art. 223,

b.	c.	d.	e.
+1625.11s.	−22.615s.	−4.09s.	+1.81s.

We will assume ω to be 5h. 25m. 40s. The process for finding the value of A″−A, by Art. 223, will be as follows:

5h. 25m. 40s. = 19540s. = 4.2909246	log. $t(t-1)=9.3940n$
12h. = 43200s. = 4.6354837	$t-\frac{1}{2}=-.047685=8.6784n$
$t=.452315=9.6554409$	6 comp. = 9.2218
$b=1625.11$s. = 3.2108828	$d=-4.09$s. = 0.6117n
+735.061s. = 2.8663237	−0.008s. = 7.9059n
	log. $t(t-1)=9.3940n$
log. $t=9.65544$	$t+1=1.452315=0.1621$
$t-1=-.547685=9.73853n$	$t-2=-1.547685=0.1897n$
2 comp. = 9.69897	24 comp. = 8.6198
$c=-22.615$s. = 1.35440n	$e=+1.81$s. = 0.2577
+2.801s. = 0.44734	+0.042s. = 8.6233

Hence

A″−A = m = 735.061s. + 2.801s. − .008s. + .042s. = 737.896s.

But m' = 737.95s. = the observed increase of right ascension.

Hence $m'-m$ = +0.054s. = the observed excess.

Therefore

737.896s. : 19540s. :: 0.054s. : x = 1.43s.

The assumed longitude was . . . . 5h. 25m. 40s.
The correction is . . . . +1.43s.
The longitude from this observation is . . 5h. 25m. 41.43s.

(320.) The increase of right ascension of the moon's bright limb should, if possible, be derived from actual observations at Greenwich; or, at all events, the errors of the tables should be corrected by observations at some standard observatory.

The chief disadvantage of this method consists in this circumstance, that an error in the observed increase of right ascension will produce an error between 20 and 30 times as great in the computed longitude. The increase of right ascension of the moon's limb in one hour of longitude varies from 112 seconds to 180 seconds. In the former case, an error of one second in the observed increase of right ascension would cause an error of 32 seconds in the deduced longitude; and in the latter case, it would cause an error of 20 seconds. Hence, to obtain a satisfactory result by this method, requires a series of observations made with the utmost care, and continued through a long period of time.

(321.) It is found that telescopes of different optical power do not exhibit the moon of the same diameter; and the determination of longitude from a single observed moon-culmination is always liable to error from this source. In order to eliminate this error, we should so arrange the series of observations that the error shall sometimes be in excess, and at other times in defect; and this is accomplished by observing successively both limbs of the moon; that is, by observations of the first limb before full moon, and of the second limb after full moon. The observer should also take care that the apparent diameter of the moon is not magnified by imperfect optical adjustment of his telescope; for which purpose he must see that the eye-piece is accurately adjusted to the focus, in order that the moon and the spider lines may both appear sharp and distinct at the same time.

(322.) The method of determining longitude by lunar distances is closely allied to the method of moon-culminating stars; but this method, being little used in fixed observatories, is not treated of in the present volume. The common mode of reducing a lunar distance yields very imperfect results; but in the American Nautical Almanac for 1855, Professor Chauvenet has given a method of making the reductions with entire accuracy, and has furnished tables by which the computations are made with great facility.

SECTION IV.

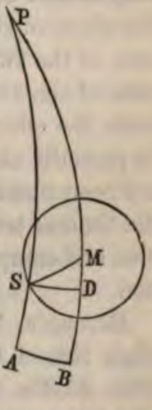
LONGITUDE DETERMINED FROM OCCULTATIONS OF STARS, BY FINDING THE TIME OF TRUE CONJUNCTION.

(323.) On account of the moon's parallax, it often happens that a star which is occulted by the moon to an observer at one station is not occulted at a second station, or the occultation begins at a different instant of time. We can not, therefore, use an occultation as an instantaneous signal for comparing directly the local times at the two stations; but we may deduce from the observed occultation the time of true conjunction of the moon and star; that is, the time of conjunction as seen from the centre of the earth; and this is a phenomenon which happens at the same absolute instant for every observer on the earth's surface. For this purpose, we must determine from the observed instant of immersion or emersion,

1*st*. The apparent difference of right ascension between the moon and star.

2*d*. The true difference of right ascension between the moon and star.

3*d*. The time of true conjunction.



(324.) In the annexed figure, let P represent the pole of the equator, M the centre of the moon, and S the star at the instant of immersion, when its apparent distance from the moon's centre is equal to the moon's semi-diameter. Let SD be a parallel of declination, passing through S, and let AB be the arc of the equator, intercepted between the hour circles PS and PM prolonged. Then MD is the apparent difference of declinations between the star and the moon's centre, which we will represent by δ; and $\frac{\text{SD}}{\cos.\text{AS}}$ is their apparent difference of right ascensions, which we will represent by a. Also, we will represent SM, the moon's semi-diameter, by Δ.

Now the triangle SMD being necessarily very small, we may regard it as a plane triangle, and we shall have

$$\text{SD}^2=\Delta^2-\delta^2=(\Delta+\delta)(\Delta-\delta).$$

Whence $$a=\frac{\sqrt{(\Delta+\delta)(\Delta-\delta)}}{\cos. d},$$

putting d for AS, or, more properly, $\frac{1}{2}$(AS+BM).

If we represent the moon's parallax in right ascension by π, the difference between the true right ascensions of the moon and star will be represented by

$$a\pm\pi.$$

The time required by the moon to describe this arc may be found by the proportion

$$m:3600\text{s.}::a\pm\pi:t,$$

where m is the hourly motion of the moon in right ascension, corresponding to the middle of the interval between the observed time and that of true conjunction of the moon and star.

Hence $$t=\frac{3600}{m}(a\pm\pi).$$

Let T represent the observed instant of immersion or emersion; then $\text{T}\pm t$ will be the instant of the true conjunction.

(325.) If the occultation has been observed under a second meridian, we may in the same way determine the instant of true conjunction at the second place. Now the absolute instant of this phenomenon is the same for both places; hence the difference of the two results thus obtained is the difference of longitude of the two stations. If the two stations are not very remote, the effect of any small error in the tables of the moon will be partially eliminated from the result. If the occultation has not been observed under a second meridian, we must calculate the time of true conjunction for Greenwich according to the tables, and compare this time with that deduced from the observation.

Example. The immersion of η Tauri was observed at the High School Observatory, Philadelphia, July 6, 1839, at 16h. 30m. 25.39s. mean time; and at Hudson, Ohio, at 16h. 2m. 21.67s. mean time. Required the difference of longitude of the two places.

We will assume the longitude of Philadelphia to be 5h. 0m. 42.5s., and that of Hudson to be 5h. 25m. 41.3s.; the corresponding Greenwich times of observation will be 21h. 31m. 7.89s., and 21h. 28m. 2.97s.

For 21h. 31m. 7.89s. Greenwich time, the moon's equatorial parallax, by Adams' Tables, is 59′ 41″.7, which, reduced to the latitude of Philadelphia, is 59′ 36″.8. The moon's parallax in right ascension for this case was computed in *Ex.* 1, page 189, and found to be 44′ 17″.1. The parallax in declination was computed in *Ex.* 1, page 194, and found to be 26′ 10″.1. The moon's true semi-diameter is 16′ 16″.0. The augmentation was computed in *Ex.* 2, page 201, and found to be 10″.15. Hence the augmented semi-diameter is 16′ 26″.2 $=\Delta$. Also, $m=$ 2286″.2.

The moon's true declination . . = 24° 5′ 11″.6 N.
Parallax in declination . . . . . = 26′ 10″.1
Moon's apparent declination . . = 23° 39′ 1″.5
Star's declination . . . . = 23° 36′ 17″.1
δ = difference . . . . = 2′ 44″.4 = 164″.4.

Hence $\Delta+\delta=1150''.6$
$\Delta-\delta=821''.8$
$d=23°\ 37'\ 39''.3$

$$a=\text{apparent difference of R. A.}=\frac{\sqrt{1150.6\times 821.8}}{\cos.\ d}=1061''.9$$

$$\pi=2657''.1$$

$a+\pi$ = the true difference of right ascension . . . . . = 3719″.0

$$t=\frac{3719.0\times 3600}{2286.2}=5856.2\text{s.}=1\text{h. }37\text{m. }36.2\text{s.},$$

which, added to 16h. 30m. 25.39s., gives 18h. 8m. 1.59s. for the Philadelphia time of true conjunction.

So also for 21h. 28m. 2.97s. Greenwich time, the moon's equatorial parallax is 59′ 41″.7, which, reduced to the latitude of Hudson, is 59′ 36″.5. The moon's parallax in right ascension for this case was computed in *Ex.* 3, page 190, and found to be 45′ 56″.5. The parallax in declination was computed in *Ex.* 3, page 195, and found to be 29′ 17″.9. The augmentation of the moon's semi-diameter was computed in *Ex.* 4, page 201, and found to be 8″.9. Hence the augmented semi-diameter is 16′ 24″.9 $=\Delta$. Also, $m=2286''.1$.

The moon's true declination . . . . = 24° 4′ 41″.7
Parallax in declination . . . . = 29′ 17″.9
Moon's apparent declination . . . . = 23° 35′ 23″.8
Star's declination . . . . = 23° 36′ 17″.1
δ = difference . . . . = 53″.3

Hence
$$\Delta+\delta=1038''.2$$
$$\Delta-\delta=931''.6$$
$$d=23^\circ\ 35'\ 50''.4$$

$$a=\text{apparent difference of R. A.}=\frac{\sqrt{1038.2\times 931.6}}{\cos.\ d}=1073''2.$$
$$\pi=2756''.5$$
$$a+\pi=\text{the true difference of right ascension}\ \ldots\ =\overline{3829''.7}$$
$$t=\frac{3829.7\times 3600}{2286.1}=6030.8\text{s.}=1\text{h. }40\text{m. }30.8\text{s.},$$

which, added to 16h. 2m. 21.67s., gives 17h. 42m. 52.47s. for the Hudson time of true conjunction.

Subtracting the Hudson time of true conjunction from the Philadelphia time, we obtain 25m. 9.1s., which is, therefore, the difference of longitude of the two places, as determined by these observations.

SECTION V.

LONGITUDE DETERMINED FROM OCCULTATIONS OF STARS BY USING THE MOON'S MOTION IN ITS APPARENT ORBIT.

(326.) From the supposed longitude of the place we must deduce the Greenwich time of the observation, and for this time find the true place of the moon, and compute its parallax in right ascension and declination, from which we derive the moon's apparent place. Subtracting the place of the moon from that of the occulted star, we obtain the apparent distance of the star from the moon's centre. If this distance is equal to the moon's semi-diameter, augmented for its apparent altitude, the assumed longitude is correct; but if these quantities are not equal, the assumed longitude is erroneous, and the correction of the longitude may be obtained according to the principles of Section III. of Chapter XI.

It is here supposed that the places of the moon given in the Nautical Almanac are perfectly correct. In order that the longitude may be obtained with the greatest accuracy, the corrections of the tables should be deduced from observations at some place whose longitude is well known, and these corrections should be applied to the tabular places before computing the distance between the moon and star.

Example. The immersion of α Tauri was observed at Cambridge, January 23, 1850, at 7h. 14m. 39.05s. mean time; the emersion at 8h. 29m. 50.25s. mean time. Required the longitude of Cambridge from Greenwich.

Assuming the longitude of Cambridge from Greenwich to be 4h. 44m. 30s., the corresponding Greenwich times of immersion and emersion will be 11h. 59m. 9.05s., and 13h. 14m. 20.25s. For these times we find the right ascension and declination of the moon from the Nautical Almanac, and apply the corrections found on page 340. At the time of immersion, the moon's hour angle was 14° 46′ 11″.85 E.; its horizontal parallax, reduced to the latitude of Cambridge, was 59′ 44″.6; and its semi-diameter, augmented for altitude, 16′ 33″.4. At the time of emersion, the hour angle was 3° 18′ 12″.95 W.; its reduced horizontal parallax, 59′ 47″.0; and its augmented semi-diameter, 16′ 34″.6. Hence we obtain the following results:

	For Immersion.		For Emersion.	
	R. A. h. m. s.	Dec. ° ′ ″	R. A. h. m. s.	Dec. ° ′ ″
Moon's true place	4 25 34.12	16 29 58.5 N.	4 28 40.02	16 38 5.3 N.
Correction to do.	−0.52	−2.3	−0.52	−2.3
Moon's parallax	47.70	26 43.5	10.80	26 13.2
Moon's apparent place ...	4 26 21.30	16 3 12.7	4 28 28.70	16 11 49.8
Star's place	4 27 19.54	16 12 3.4	4 27 19.54	16 12 3.4
Difference	58.24	8 50.7	1 9.16	13.6
Reduced to seconds of arc.	839.5	530.2	996.2	12.9

The hourly motion in right ascension is 1464″.9, and in declination 412″.8.

Hence, in the triangle HMM′ (see fig. next page),

$$1464''.9 : 412''.8 :: 1 : \text{tang. } HMM' = 15^\circ\ 44'\ 16''.2,$$

$$\text{cos. } HMM' : 1 :: 1464''.9 : MM' = 1521''.97,$$

which is the hourly motion in orbit.

In the triangle DSM,

$$530''.2 : 839''.5 :: 1 : \text{tang. } DSM = 57^\circ\ 43'\ 33''.2,$$

$$\text{sin. } DSM : 839''.5 :: 1 : SM = 992''.94.$$

Hence $MSC = 73^\circ\ 27'\ 49''.4,$

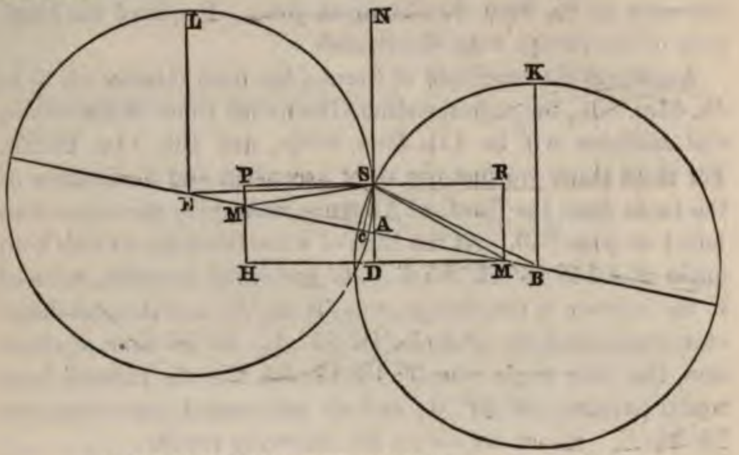
$$1 : 992''.94 :: \text{cos. } MSC : SC = 282''.6,$$

$$SB = 993''.4 : 282''.6 :: 1 : \text{cos. } BSC = 73^\circ\ 28'\ 17''.8.$$

Hence $BSM = 28''.4,$

$$\text{sin. } SBM : 992''.94 :: \text{sin. } BSM : BM = 0''.48.$$

The time of describing BM = 1.14s., which is the correction to the assumed longitude, as deduced from the observed immersion.



$$12''.9 : 996''.2 :: 1 : \text{tang. } DSM' = 89^\circ\ 15'\ 29''.2,$$

$$\text{sin. } DSM' : 996''.2 :: 1 : SM' = 996''.31.$$

Hence $M'SC = 73^\circ\ 31'\ 13''.0,$

$$994''.6 : 282''.6 :: 1 : \text{cos. } ESC = 73^\circ\ 29'\ 31''.7.$$

Hence $ESM' = 1'\ 41''.3,$

$$\text{sin. } SEM' : 996''.31 :: \text{sin. } ESM' : EM' = 1''.72.$$

The time of describing EM′ = 4.07s., which is the correction to the assumed longitude, as deduced from the observed emersion.

Hence the longitude of Cambridge, derived from the observed
Immersion, is . . 4h. 44m. 30s. − 1.14s. = 4h. 44m. 28.86s.
Emersion, is . . . 4h. 44m. 30s. − 4.07s. = 4h. 44m. 25.93s.
The mean of the two results is . . . . . . 4h. 44m. 27.40s.

In a similar manner, we may determine the longitude from a solar eclipse.

SECTION VI.

BESSEL'S METHOD OF COMPUTING THE LONGITUDE OF A PLACE, AND THE ERROR OF THE TABLES, FROM OBSERVATIONS OF A SOLAR ECLIPSE.

(327.) We obtain directly from observation either the sidereal time or the apparent solar time of the different phases of an eclipse; but to deduce the corresponding mean time requires at least an approximate knowledge of the longitude of the place. We may, however, generally assume that the longitude is known with sufficient precision to enable us, without material error, to deduce the mean time from the known sidereal time, or apparent solar time. We shall, therefore, suppose that both the sidereal time and the mean time of the phases of an eclipse are known, and also the latitude of the place of observation.

(328.) The general elements of the eclipse in question, A, D, i, l, x, y, and z, must be computed from hour to hour for the mean time of the meridian of the ephemeris, and these hours must be so selected as to comprehend the entire duration of the eclipse. The formulas for these quantities have been given on page 277.

The values of A, D, and i change but slowly, and we may assume them to be pretty accurately known for the time of observation; for i is extremely small, while A and D depend chiefly upon the place of the sun, which the tables furnish with tolerable precision. Indeed, this assumption is a necessary one, for it is impossible from the observations of an eclipse to detect any error which may exist in these values. The errors, however, existing in the assumed values of x, y, and l may be determined with great accuracy; and we shall therefore substitute for these quantities the expressions $x+\Delta x$, $y+\Delta y$, $l+\Delta l$, where Δx, Δy, and Δl represent the errors of x, y, and l, of which we are in search. Equation (5), page 267, accordingly becomes

$$(x+\Delta x-\xi)^2+(y+\Delta y-\eta)^2=(l+\Delta l-i\zeta)^2.$$

(329.) Let now the values of a, δ, π, a', δ', and π', be taken from the ephemeris for the time T of the first meridian. Let $T+T'$ represent the required time of the first meridian at which a phase of an eclipse was observed. Let x_0 and y_0 denote the values of x and y for the time T, and x' and y' the hourly variations of x and y; then we shall have

$$x=x_0+x'T', \text{ and } y=y_0+y'T'.$$

We may, in the same manner, consider ξ, η, and ζ as also composed of two parts. Since, however, these magnitudes change but slowly, and we generally have an approximate knowledge of the difference of longitude, and consequently the time of the first meridian corresponding to the time of observation, we may assume these quantities as known for this time. The preceding equation therefore becomes

$$(x_{\circ}-\xi+x'\mathrm{T}'+\Delta x)^2+(y_{\circ}-\eta+y'\mathrm{T}'+\Delta y)^2=(l+\Delta l-i\zeta)^2.$$

(330.) If the variations of x and y were proportional to the time, x' and y' would be constant, and the knowledge of the time $\mathrm{T}+\mathrm{T}'$ would not be necessary for computing them. This, however, is not the case; but since the variations of x' and y' are small in comparison with those of x and y, the above equation may be solved by successive approximations, which rapidly converge to the truth.

Let us assume

$$m \sin.\ \mathrm{M}=x_{\circ}-\xi, \qquad n \sin.\ \mathrm{N}=x',$$
$$m \cos.\ \mathrm{M}=y_{\circ}-\eta, \qquad n \cos.\ \mathrm{N}=y',$$
$$l-i\zeta=\mathrm{L},$$

the preceding equation will become

$$(\mathrm{L}+\Delta l)^2=(m \sin.\ \mathrm{M}+n\mathrm{T}' \sin.\ \mathrm{N}+\Delta x)^2$$
$$+(m \cos.\ \mathrm{M}+n\mathrm{T}' \cos.\ \mathrm{N}+\Delta y)^2.$$

Substitute for L its value $\frac{m \sin.\ (\mathrm{M}-\mathrm{N})}{\sin.\ \psi}$, and expand the second member of this equation, remembering that $m^2 \sin.^2 \mathrm{M}+m^2 \cos.^2 \mathrm{M}=m^2$, we obtain

$$\left(\frac{m \sin.\ (\mathrm{M}-\mathrm{N})}{\sin.\ \psi}+\Delta l\right)^2=m^2+2mn\mathrm{T}' \cos.\ (\mathrm{M}-\mathrm{N})+n^2\mathrm{T}'^2$$
$$+2\Delta x(m \sin.\ \mathrm{M}+n\mathrm{T}' \sin.\ \mathrm{N})+(\Delta x)^2,$$
$$+2\Delta y(m \cos.\ \mathrm{M}+n\mathrm{T}' \cos.\ \mathrm{N})+(\Delta y)^2,$$
$$=\{n\mathrm{T}'+m \cos.\ (\mathrm{M}-\mathrm{N})+\Delta x \sin.\ \mathrm{N}+\Delta y \cos.\ \mathrm{N}\}^2,$$
$$+\{m \sin.\ (\mathrm{M}-\mathrm{N})+\Delta x \cos.\ \mathrm{N}-\Delta y \sin.\ \mathrm{N}\}^2.$$

Let us put $\quad n\lambda=\Delta x \sin.\ \mathrm{N}+\Delta y \cos.\ \mathrm{N},$

$$n\lambda'=-\Delta x \cos.\ \mathrm{N}+\Delta y \sin.\ \mathrm{N},$$

and we shall have

$$\left\{\frac{m \sin.\ (\mathrm{M}-\mathrm{N})}{\sin.\ \psi}+\Delta l\right\}^2=\{n\mathrm{T}'+m \cos.\ (\mathrm{M}-\mathrm{N})+n\lambda\}^2$$
$$+\{m \sin.\ (\mathrm{M}-\mathrm{N})-n\lambda'\}^2,$$

or $\quad \{n\mathrm{T}'+m \cos.\ (\mathrm{M}-\mathrm{N})+n\lambda\}^2=\left\{\frac{m}{\sin.\ \psi} \sin.\ (\mathrm{M}-\mathrm{N})+\Delta l\right\}^2$

$$-\{m \sin.\ (\mathrm{M}-\mathrm{N})-n\lambda'\}^2$$
$$=\frac{m^2}{\sin.^2 \psi} \sin.^2 (\mathrm{M}-\mathrm{N})+\frac{2m}{\sin.\ \psi} \sin.\ (\mathrm{M}-\mathrm{N})\Delta l$$
$$-m^2 \sin.^2 (\mathrm{M}-\mathrm{N})+2mn\lambda' \sin.\ (\mathrm{M}-\mathrm{N})$$

$$=\mathrm{L}^2 \cos.^2 \psi+2\mathrm{L}\Delta l+2\mathrm{L}n\lambda' \sin.\ \psi,$$

where we have neglected the small terms Δl^2 and $n^2\lambda'^2$.

Extracting the square root, and neglecting the higher powers of Δl and $n\lambda'$, we have

$$n\mathrm{T}'+m \cos.(\mathrm{M}-\mathrm{N})+n\lambda=\mp\{\mathrm{L}\cos.\psi+\Delta l \sec.\psi+n\lambda' \operatorname{tang}.\psi\},$$

or

$$\mathrm{T}'=-\frac{m}{n}\cos.(\mathrm{M}-\mathrm{N})\mp\frac{\mathrm{L}\cos.\psi}{n}-\lambda\mp\lambda' \operatorname{tang}.\psi\mp\frac{\Delta l}{n}\sec.\psi,$$

or $$\mathrm{T}'=-\frac{m}{n}.\frac{\sin.(\mathrm{M}-\mathrm{N}\pm\psi)}{\sin.\psi}-\lambda\mp\lambda' \operatorname{tang}.\psi\mp\frac{\Delta l}{n}\sec.\psi.$$

(331.) Since now the time of immersion is always earlier than that of emersion, T′, for an immersion, must have a less positive value, or a greater negative value, than for emersion. Hence, if we always take the angle ψ either in the first or fourth quadrant, the upper sign belongs to an immersion, the lower to an emersion. If, however, for an immersion we take ψ in the first or fourth quadrant, but for an emersion in the second or third quadrant, we shall have in either case,

$$\mathrm{T}'=-\frac{m\sin.(\mathrm{M}-\mathrm{N}+\psi)}{n\sin.\psi}-\lambda-\lambda' \operatorname{tang}.\psi-\frac{\Delta l}{n}\sec.\psi,$$

or

$$\mathrm{T}'=-\frac{m}{n}\cos.(\mathrm{M}-\mathrm{N})-\frac{\mathrm{L}\cos.\psi}{n}-\lambda-\lambda' \operatorname{tang}.\psi-\frac{\Delta l}{n}\sec.\psi\,.\ (1)$$

In the case of annular eclipses, at the internal contact the emersion precedes the immersion. We must, therefore, in this case, for the immersion, take ψ in the second or third quadrant, and for the emersion in the first or fourth.

(332.) Equation (1) may be solved by successive approximations. We must, for this purpose, compute the values of x, y, z, A, D, g, l, and i, for several successive hours, so that the values of $x_\circ$ and $y_\circ$, as well as their hourly variations, can be found for any time by interpolation. We then assume a time, T, as accurate as the provisional knowledge of the difference of longitude will permit, and interpolate for this time the quantities $x_\circ$, $y_\circ$, x', and y', and thence find, by formula (1), an approximate value of T′. With the value T+T′, we repeat, if necessary, the preceding computation. Represent by T the value assumed in the last approximation, and the correction obtained by T′; then $\mathrm{T}+\mathrm{T}'=t-\omega$, where t is the time of observation, and ω the east longitude of the station from the first meridian, by which we understand that meridian whose time is employed in the computation of x, y, z, etc.

We therefore have

$$\begin{aligned}\omega &= t-\mathrm{T}+\frac{m}{n}\cos.\,(\mathrm{M}-\mathrm{N})+\frac{\mathrm{L}}{n}\cos.\,\psi+\lambda+\lambda'\,\mathrm{tang.}\,\psi+\frac{\Delta l}{n}\sec.\,\psi \\ &= t-\mathrm{T}+\frac{m\sin.\,(\mathrm{M}-\mathrm{N}+\psi)}{n\sin.\,\psi}+\lambda+\lambda'\,\mathrm{tang.}\,\psi+\frac{\Delta l}{n}\sec.\,\psi \quad . \; . \; (2)\end{aligned}$$

(333.) Since the mean solar hour has been employed as the unit in the values of x' and y', the preceding formula supposes the same unit of time for ω. If we wish to obtain the difference of longitude in seconds of time, we must multiply the formula by s, the number of seconds belonging to an hour of that species of time in which the observation is expressed; $t-\mathrm{T}$ will then be expressed in seconds of the same kind of time in which t is given; or T represents the same kind of time with t.

Equation (2) does not properly furnish the difference of longitude of the place of observation from the first meridian, but rather the relation between this quantity and the errors of the elements employed. If the same eclipse has been observed at different places, we may obtain for each place as many such equations as there are instants of observation. By combining these equations, we may eliminate, as will be seen hereafter, the error of one or more of the elements of computation, and thus render the result, as far as possible, independent of the error of the tables.

(334.) We must now develop the quantities λ and λ', which are determined by the equations

$$\begin{aligned} n\lambda &= \sin.\,\mathrm{N}\Delta x+\cos.\,\mathrm{N}\Delta y, \\ n\lambda' &= \sin.\,\mathrm{N}\Delta y-\cos.\,\mathrm{N}\Delta x.\end{aligned}$$

The quantities x and y, as will be seen from the equations on page 271, depend upon $\alpha-\mathrm{A}$, $\delta-\mathrm{D}$, and π. If we assume these magnitudes to require correction, we shall have

$$\begin{aligned}\Delta x &= a\,\Delta(\alpha-\mathrm{A})+b\,\Delta(\delta-\mathrm{D})+c\,\Delta\pi, \\ \Delta y &= a'\Delta(\alpha-\mathrm{A})+b'\Delta(\delta-\mathrm{D})+c'\Delta\pi,\end{aligned}$$

where a, b, c are the differential coefficients of x in respect to $\alpha-\mathrm{A}$, $\delta-\mathrm{D}$, and π; while a', b', and c' are the same differential coefficients of y. Since $\Delta(\alpha-\mathrm{A})$, $\Delta(\delta-\mathrm{D})$, and $\Delta\pi$ are very small quantities, in the expressions for the differential coefficients we may neglect the terms which contain sin. $(\alpha-\mathrm{A})$ and sin. $(\delta-\mathrm{D})$ as factors, and assume cos. $(\alpha-\mathrm{A})$ and cos. $(\delta-\mathrm{D})$ as equal to unity. We thus obtain, by differentiating the values of x and y on page 271,

$$a=\frac{\cos.\ \delta}{\sin.\ \pi}\cos.\ (\alpha-\mathrm{A})=\frac{\cos.\ \delta}{\sin.\ \pi},$$

$$b=-\frac{\sin.\ \delta\ \sin.\ (\alpha-\mathrm{A})}{\sin.\ \pi}=0,$$

$$c=-\frac{\cos.\ \delta\ \sin.\ (\alpha-\mathrm{A})\ \cos.\ \pi}{\sin.^2\ \pi}=-\frac{x}{\text{tang.}\ \pi},$$

$$a'=\frac{\cos.\ \delta\ \sin.\ \mathrm{D}\ \sin.\ (\alpha-\mathrm{A})}{\sin.\ \pi}=0,$$

$$b'=\frac{\cos.\ (\delta-\mathrm{D})}{\sin.\ \pi}=\frac{1}{\sin.\ \pi},$$

$$c'=-\frac{y}{\text{tang.}\ \pi}.$$

(335.) Since now λ and λ', as also $\Delta(\alpha-\mathrm{A})$, $\Delta(\delta-\mathrm{D})$, and $\Delta\pi$ are expressed in parts of radius, if we wish to obtain the errors of the elements in seconds, these differential coefficients must be divided by 206265. Let us, then, put

$$h=\frac{s}{206265\,.\,n\ \sin.\ \pi},$$

and we shall have

$$\begin{aligned}\lambda &= h\ \sin.\ \mathrm{N}\ \cos.\ \delta\Delta(\alpha-\mathrm{A})+h\ \cos.\ \mathrm{N}\Delta(\delta-\mathrm{D})\\ &\quad -h\ \cos.\ \pi\Delta\pi[x\ \sin.\ \mathrm{N}+y\ \cos.\ \mathrm{N}],\\ \lambda' &= -h\ \cos.\ \mathrm{N}\ \cos.\ \delta\Delta(\alpha-\mathrm{A})+h\ \sin.\ \mathrm{N}\Delta(\delta-\mathrm{D})\\ &\quad +h\ \cos.\ \pi\Delta\pi[x\ \cos.\ \mathrm{N}-y\ \sin.\ \mathrm{N}].\end{aligned}$$

If we multiply the former equation by cos. ψ, and the latter by sin. ψ, then add the two equations together, and divide by h, we shall obtain

$$\begin{aligned}[\lambda+\lambda'\ \text{tang.}\ \psi]\frac{\cos.\ \psi}{h} &= \sin.\ (\mathrm{N}-\psi)\ \cos.\ \delta\Delta(\alpha-\mathrm{A})\\ &\quad +\cos.\ (\mathrm{N}-\psi)\Delta(\delta-\mathrm{D})\\ &\quad -\cos.\ \pi\Delta\pi[x\ \sin.\ (\mathrm{N}-\psi)+y\ \cos.\ (\mathrm{N}-\psi)].\end{aligned}$$

Hence we obtain from equation (2), page 326,

$$\begin{aligned}\omega=t-\mathrm{T}+\frac{m}{n}.s\,\frac{\sin.\,(\mathrm{M}-\mathrm{N}+\psi)}{\sin.\,\psi}&+h\,\frac{\sin.\,(\mathrm{N}-\psi)}{\cos.\,\psi}\cos.\,\delta\Delta(a-\mathrm{A})\\&+h\,\frac{\cos.\,(\mathrm{N}-\psi)}{\cos.\,\psi}\Delta(\delta-\mathrm{D})\\&+\frac{h\,.\,206265\,\sin.\,\pi\Delta l}{\cos.\,\psi}\\-h\cos.\,\pi\Delta\pi&\left[\frac{x\sin.\,(\mathrm{N}-\psi)+y\cos.\,(\mathrm{N}-\psi)}{\cos.\,\psi}\right].\end{aligned}$$

(336.) Every observation of the instant of an eclipse furnishes one equation of the preceding form, and as this contains five unknown quantities, five such equations are sufficient for their determination. The magnitudes Δl and $\Delta\pi$ can not generally be determined, unless observations are made at places widely separated from each other. Nevertheless, the computation of the coefficients will always show what influence any error in the values of π and l may have upon the result. We therefore generally seek to free the difference of longitude only from the errors of a and δ; but the value of Δa can not be determined unless we know the longitude of one of the stations from the first meridian.

(337.) The following is a synopsis of the preceding results:

Compute the values of e, A, D, and g, also the co-ordinates x, y, and z, from the formulas, page 277, with the quantities i and l, all of which quantities are general for all places on the earth.

Compute also the following formulæ:

$$\xi=\rho\cos.\,\phi'\sin.\,(\mu-\mathrm{A}),$$
$$\eta=\rho\sin.\,\phi'\cos.\,\mathrm{D}-\rho\cos.\,\phi'\sin.\,\mathrm{D}\cos.\,(\mu-\mathrm{A}),$$
$$\zeta=\rho\sin.\,\phi'\sin.\,\mathrm{D}+\rho\cos.\,\phi'\cos.\,\mathrm{D}\cos.\,(\mu-\mathrm{A}),$$

where all the symbols have the same signification as on page 278, except μ, which here represents the *observed* sidereal time of contact.

Let T represent the approximate time of the first meridian, corresponding to the phase observed.

Let x_0 represent the value of x for the time T;
y_0 represent the value of y for the time T;
x' represent the hourly variation of x;
y' represent the hourly variation of y.

$$m \sin. \mathrm{M} = x_0 - \xi,$$
$$m \cos. \mathrm{M} = y_0 - \eta,$$
$$n \sin. \mathrm{N} = x',$$
$$n \cos. \mathrm{N} = y',$$
$$l - i\zeta = \mathrm{L},$$
$$\sin. \psi = \frac{m}{\mathrm{L}} \sin. (\mathrm{M} - \mathrm{N}).$$

For the first contact, ψ must be taken in the first or fourth quadrant; for the last contact, in the second or third quadrant.

$$\mathrm{T}' = -\frac{m \sin. (\mathrm{M} - \mathrm{N} + \psi)}{n \sin. \psi} = -\frac{m}{n} \cos. (\mathrm{M} - \mathrm{N}) - \frac{\mathrm{L} \cos. \psi}{n}.$$

Then

$$\omega = t - \mathrm{T} - \mathrm{T}' + h \frac{\sin. (\mathrm{N} - \psi)}{\cos. \psi} \cos. \delta \Delta(\alpha - \mathrm{A})$$
$$+ h \frac{\cos. (\mathrm{N} - \psi)}{\cos. \psi} \Delta(\delta - \mathrm{D},$$

where $$h = \frac{s}{206265 . n \sin. \pi};$$

$s = 3600 =$ the number of seconds in an hour;

$t =$ observed mean time of contact.

(338.) *Example.* On the 28th of July, 1851, occurred an eclipse of the sun, which was observed as follows:

At Königsberg, Prussia.

Beginning	3h. 38m. 10.8s.	Königsberg m. t.
Beginning of total darkness .	4h. 38m. 57.6s.	"
End of total darkness	4h. 41m. 54.2s.	"
End	5h. 38m. 32.9s.	"

At Washington, District of Columbia.

Beginning	7h. 21m. 31.2s. A.M.	Washington m. t.
End	8h. 50m. 38.0s.	"

It is required to determine the error of the tables and the longitude of Washington.

The general co-ordinates for this eclipse have already been given on pages 280 to 285. Our first object is to deduce the error of the tables from the observations at Königsberg.

Computation for Königsberg.

	Beginning.	Beginning of total Darkness.	End of total Darkness.	End.
t	3h. 38m. 10.8s. m. t.	4h. 38m. 57.6s. m. t.	4h. 41m. 54.2s. m. t.	5h. 38m. 32.9s. m. t.
	12h. 0m. 46.44s. s. t.	13h. 1m. 43.23s. s. t.	13h. 4m. 40.31s. s. t.	14h. 1m. 28.31s. s. t.
μ	180° 11′ 36″.62	195° 25′ 48″.39	196° 10′ 4″.64	210° 22′ 4″.72
	$\phi' =$ 54° 31′ 58″.36	log. $\rho = 9.9990344$		
Assume T	2.25h.	3.3h.	3.3h.	4.25h.
$\mu - A$	52° 59′ 59″.28	68° 11′ 41″.69	68° 55′ 57″.94	83° 5′ 42″.89
D	19° 4′ 0″.50	19° 3′ 24″.99	19° 3′ 24″.99	19° 2′ 52″.84
ρ cos. ϕ'	9.7626389	9.7626389	9.7626389	9.7626389
sin. $(\mu - A)$	9.9023474	9.9677599	9.9699558	9.9968388
log. ξ	9.6649863	9.7303988	9.7325947	9.7594777
ξ	+.462366	+.537525	+.540250	+.574748
ρ sin. ϕ'	9.9098980	9.9098980	9.9098980	9.9098980
cos. D	9.9754953	9.9755212	9.9755212	9.9755446
log. (1)	9.8853933	9.8854192	9.8854192	9.8854426
ρ cos. ϕ'	9.7626389	9.7626389	9.7626389	9.7626389
sin. D	9.5141097	9.5138933	9.5138933	9.5136974
cos. $(\mu - A)$	9.7794650	9.5699006	9.5556545	9.0799738
log. (2)	9.0562136	8.8464328	8.8321867	8.3563101
(1)	+.768057	+.768103	+.768103	+.768144
(2)	+.113819	+.070215	+.067950	+.022715
$(1) - (2) = \eta$	+.654238	+.697888	+.700153	+.745429

ρ sin. ϕ'	9.9098980	9.9098980	9.9098980	9.9098980
sin. D	9.5141097	9.5138933	9.5138933	9.5136974
log. (3)	9.4240077	9.4237913	9.4237913	9.4235954
ρ cos. ϕ'	9.7626389	9.7626389	9.7626389	9.7626389
cos. D	9.9754953	9.9755212	9.9755212	9.9755446
cos. $(\mu-A)$	9.7794650	9.5699006	9.5556545	9.0799738
log. (4)	9.5175992	9.3080607	9.2938146	8.8181573
(3)	+.265465	+.265333	+.265333	+.265213
(4)	+.329306	+.203264	+.196705	+.065790
$(3)+(4)=\zeta$	+.594771	+.468597	+.462038	+.331003
log. ζ	9.7743498	9.6707995	9.6646777	9.5198319
log. i	7.6632481	7.6611347n	7.6611347n	7.6632519
log. $i\zeta$	7.4375979	7.3319342n	7.3258124n	7.1830838
l	+.534162	+.012199	+.012199	+.533977
$i\zeta$	+.002739	−.002148	−.002117	+.001524
$l-i\zeta=L$	+.531423	+.014347	+.014316	+.532453
$x_\circ$	−.057369	+.540639	+.540639	+1.081623
ξ	+.462366	+.537525	+.540250	+ .574748
$x_\circ-\xi$	−.519735	+.003114	+.000389	+ .506875
$y_\circ$	+.781216	+.693200	+.693200	+.613242
η	+.654238	+.697888	+.700153	+.745429
$y_\circ-\eta$	+.126978	−.004688	−.006953	−.132187

Computation for Königsberg.

	Beginning.	Beginning of total Darkness.	End of total Darkness.	End.
t	3h. 38m. 10.8s. m. t.	4h. 38m. 57.6s. m. t.	4h. 41m. 54.2s. m. t.	5h. 38m. 32.9s. m. t.
	12h. 0m. 46.44s. s. t.	13h. 1m. 43.23s. s. t.	13h. 4m. 40.31s. s. t.	14h. 1m. 28.31s. s. t.
μ	180° 11′ 36″.62	195° 25′ 48″.39	196° 10′ 4″.64	210° 22′ 4″.72
	$\phi' =$ 54° 31′ 58″.36	log. $\rho = 9.9990344$		
Assume T	2.25h.	3.3h.	3.3h.	4.25h.
$\mu - A$	52° 59′ 59″.28	68° 11′ 41″.69	68° 55′ 57″.94	83° 5′ 42″.89
D	19° 4′ 0″.50	19° 3′ 24″.99	19° 3′ 24″.99	19° 2′ 52″.84
ρ cos. ϕ'	9.7626389	9.7626389	9.7626389	9.7626389
sin. $(\mu - A)$	9.9023474	9.9677599	9.9699558	9.9968388
log. ξ	9.6649863	9.7303988	9.7325947	9.7594777
ξ	+.462366	+.537525	+.540250	+.574748
ρ sin. ϕ'	9.9098980	9.9098980	9.9098980	9.9098980
cos. D	9.9754953	9.9755212	9.9755212	9.9755446
log. (1)	9.8853933	9.8854192	9.8854192	9.8854426
ρ cos. ϕ'	9.7626389	9.7626389	9.7626389	9.7626389
sin. D	9.5141097	9.5138933	9.5138933	9.5136974
cos. $(\mu - A)$	9.7794650	9.5699006	9.5556545	9.0799738
log. (2)	9.0562136	8.8464328	8.8321867	8.3563101
(1)	+.768057	+.768103	+.768103	+.768144
(2)	+.113819	+.070215	+.067950	+.022715
$(1)-(2)=\eta$	+.654238	+.697888	+.700153	+.745429

ρ sin. ϕ'	9.9098980	9.9098980	9.9098980	9.9098980
sin. D	9.5141097	9.5138933	9.5138933	9.5136974
log. (3)	9.4240077	9.4237913	9.4237913	9.4235954
ρ cos. ϕ'	9.7626389	9.7626389	9.7626389	9.7626389
cos. D	9.9754953	9.9755212	9.9755212	9.9755446
cos. $(\mu-\mathrm{A})$	9.7794650	9.5699006	9.5556545	9.0799738
log. (4)	9.5175992	9.3080607	9.2938146	8.8181573
(3)	+.265465	+.265333	+.265333	+.265213
(4)	+.329306	+.203264	+.196705	+.065790
$(3)+(4)=\zeta$	+.594771	+.468597	+.462038	+.331003
log. ζ	9.7743498	9.6707995	9.6646777	9.5198319
log. i	7.6632481	7.6611347n	7.6611347n	7.6632519
log. $i\zeta$	7.4375979	7.3319342n	7.3258124n	7.1830838
l	+.534162	+.012199	+.012199	+.533977
$i\zeta$	+.002739	−.002148	−.002117	+.001524
$l-i\zeta=\mathrm{L}$	+.531423	+.014347	+.014316	+.532453
$x_\circ$	−.057369	+.540639	+.540639	+1.081623
ξ	+.462366	+.537525	+.540250	+ .574748
$x_\circ-\xi$	−.519735	+.003114	+.000389	+ .506875
$y_\circ$	+.781216	+.693200	+.693200	+.613242
η	+.654238	+.697888	+.700153	+.745429
$y_\circ-\eta$	+.126978	−.004688	−.006953	−.132187

	Beginning.	Beginning of total Darkness.	End of total Darkness.	End.
log. $(x_\circ - \xi)$	9.7157820n	7.4933186	6.5899496	9.7049009
log. $(y_\circ - \eta)$	9.1037285	7.6709876n	7.8421722n	9.1211887n
tang. M	0.6120535n	9.8223310n	8.7477774n	0.5837122n
M	283° 43′ 45″.0	146° 24′ 21″.4	176° 47′ 52″.1	104° 36′ 59″.4
sin. M	9.9874108n	9.7429647	8.7470987	9.9857123
m	9.7283712	7.7503539	7.8428509	9.7191886
x'	+.569543	+.569495		+.569395
y'	−.083642	−.084004		−.084328
log. x'	9.7555265	9.7554899		9.7554137
log. y'	8.9224244n	8.9243000n		8.9259718n
tang. N	0.8331021n	0.8311899n		0.8294419n
N	98° 21′ 16″.7	98° 23′ 27″.5	98° 23′ 27″.5	98° 25′ 27″.6
sin. N	9.9953665	9.9953260		9.9952886
n	9.7601600	9.7601639		9.7601251
M−N	185° 22′ 28″.3	48° 0′ 53″.9	78° 24′ 24″.6	6° 11′ 31″.8
sin. (M−N)	8.9715806n	9.8711756	9.9910484	9.0328742
m	9.7283712	7.7503539	7.8428509	9.7191886
comp. L	0.2745596	1.8432389	1.8441783	0.2737187
sin. ψ	8.9745114n	9.4647684	9.6780776	9.0257815
ψ	354° 35′ 20″.3	16° 57′ 10″.7	151° 32′ 30″.8	173° 54′ 30″.7
M−N+ψ	179° 57′ 48″.6	64° 58′ 4″.6	229° 56′ 55″.4	180° 6′ 2″.5

sin. $(M-N+\psi)$	6.8041702	9.9571623	9.8839278*n*	7.2448826*n*
m	9.7283712	7.7503539	7.8428509	9.7191886
comp. n	0.2398400	0.2398361	0.2398361	0.2398749
cosec. ψ	1.0254886*n*	0.5352316	0.3219224	0.9742185
$s=3600$	3.5563025	3.5563025	3.5563025	3.5563025
log. T'	1.3541725*n*	2.0388864	1.8448397*n*	1.7344671*n*
T'	+22.60s.	−109.37s.	+69.96s.	+54.26s.
$t-T-T'$	1h. 22m. 48.20s.	1h. 22m. 46.97s.	1h. 22m. 44.24s.	1h. 22m. 38.64s.
$N-\psi$	103° 45′ 56″.4	81° 26′ 16″.8	306° 50′ 56″.7	284° 30′ 56″.9
$s=3600$	3.5563025	3.5563025	3.5563025	3.5563025
comp. 206265	4.6855749	4.6855749	4.6855749	4.6855749
comp. n	0.2398400	0.2398361	0.2398361	0.2398749
cosec. π	1.7545339	1.7544023	1.7544023	1.7542862
h	0.2362513	0.2361158	0.2361158	0.2360385
sin. $(N-\psi)$	9.9873431	9.9951327	9.9032083*n*	9.9859106*n*
sec. ψ	0.0019396	0.0192949	0.0559292*n*	0.0024591*n*
	0.2255340	0.2505434	0.1952533	0.2244082
coeffic. of cos. $\delta \Delta a$	+1.6809	+1.7805	+1.5677	+1.6765
h	0.2362513	0.2361158	0.2361158	0.2360385
cos. $(N-\psi)$	9.3764884*n*	9.1728350	9.7779408	9.3990626
sec. ψ	0.0019396	0.0192949	0.0559292*n*	0.0024591*n*
	9.6146793*n*	9.4282457	0.0699858*n*	9.6375602*n*
coefficient of $\Delta\delta$	−.4118	+2681	−1.1749	−.4341

Hence we obtain the following equations for Königsberg:

$$\begin{aligned}
\omega &= 1\text{h. }22\text{m. }48.20\text{s.}+1.6809\,\Delta\alpha-\ \ .4118\,\Delta\delta,\\
\omega &= 1\text{h. }22\text{m. }46.97\text{s.}+1.7805\,\Delta\alpha+\ \ .2681\,\Delta\delta,\\
\omega &= 1\text{h. }22\text{m. }44.24\text{s.}+1.5677\,\Delta\alpha-1.1749\,\Delta\delta,\\
\omega &= 1\text{h. }22\text{m. }38.64\text{s.}+1.6765\,\Delta\alpha-\ \ .4341\,\Delta\delta.
\end{aligned}$$

If we assume the longitude of Königsberg equal to 1h. 22m. 0.5s., we shall have

$$\begin{aligned}
0 &= 47.70\text{s.}+1.6809\,\Delta\alpha-\ \ .4118\,\Delta\delta,\\
0 &= 46.47\text{s.}+1.7805\,\Delta\alpha+\ \ .2681\,\Delta\delta,\\
0 &= 43.74\text{s.}+1.5677\,\Delta\alpha-1.1749\,\Delta\delta,\\
0 &= 38.14\text{s.}+1.6765\,\Delta\alpha-\ \ .4341\,\Delta\delta.
\end{aligned}$$

These equations should be solved by the method of least squares, explained in Art. 239. Multiplying each equation by the coefficient of $\Delta\alpha$ in that equation, and taking the sum of all these products, we obtain

$$0 = 295.4297+11.2638\,\Delta\alpha-2.7844\,\Delta\delta.$$

Multiplying each equation by the coefficient of $\Delta\delta$ in that equation, and taking the sum of all these products, we obtain

$$0 = -75.1291-2.7844\,\Delta\alpha+1.8102\,\Delta\delta.$$

Solving these equations in the usual manner, we find

$$\Delta\delta = +1''.87 = \text{error in declination},$$

$$\cos.\ \delta\Delta\alpha = -25''.77,$$

or the error in right ascension $= -27''.26$.

$\Delta\alpha$ in the above equations is used, for the sake of brevity, in place of $\cos.\ \delta\Delta\alpha$. The error in right ascension is multiplied by $\cos.\ \delta$, to reduce it to an arc of a great circle.

Computation for Washington.

	Beginning.	End.
t	7h. 21m. 31.2s. m. t.	8h. 50m. 38.0s. m. t.
	3h. 43m. 49.35s. s. t.	5h. 13m. 10.79s. s. t.
μ	$55^\circ\ 57'\ 20''.2$	$78^\circ\ 17'\ 41''.8$
	$\phi'=38^\circ\ 42'\ 24''.7$	$\log.\ \rho=9.9994302$
Assume T	0.5h.	2.0h.
$\mu-\text{A}$	$-71^\circ\ 10'\ \ 8''.2$	$-48^\circ\ 53'\ 20''.0$
D	$19^\circ\ \ 4'\ 59''.7$	$19^\circ\ \ 4'\ \ 9''.0$
$\rho\ \cos.\ \phi'$	9.8917227	9.8917227
$\sin.\ (\mu-\text{A})$	9.9761089*n*	9.8770463*n*
$\log.\ \xi$	9.8678316*n*	9.7687690*n*
ξ	$-.737618$	$-.587177$

	Beginning.	End.
ρ sin. ϕ'	9.7955437	9.7955437
cos. D	9.9754523	9.9754892
log. (1)	9.7709960	9.7710329
ρ cos. ϕ'	9.8917227	9.8917227
sin. D	9.5144703	9.5141615
cos. $(\mu-A)$	9.5089050	9.8179099
log. (2)	8.9150980	9.2237941
(1)	+.590196	+.590246
(2)	+.082243	+.167415
$(1)-(2)=\eta$	+.507953	+.422831
ρ sin. ϕ'	9.7955437	9.7955437
sin. D	9.5144703	9.5141615
log. (3)	9.3100140	9.3097052
ρ cos. ϕ'	9.8917227	9.8917227
cos. D	9.9754523	9.9754892
cos. $(\mu-A)$	9.5089050	9.8179099
log. (4)	9.3760800	9.6851218
(3)	+.204180	+.204035
(4)	+.237728	+.484308
$(3)+(4)=\zeta$	+.441908	+.688343
log. ζ	9.6453318	9.8378049
log. i	7.6632448	7.6632477
log. $i\zeta$	7.3085766	7.5010526
l	.534242	.534181
$i\zeta$	.002035	.003170
$l-i\zeta=L$	.532207	.531011
$x_\circ$	−1.054056	−0.199758
ξ	−0.737618	−0.587177
$x_\circ-\xi$	−0.316438	+0.387419
$y_\circ$	+0.927049	+0.802116
η	+0.507953	+0.422831
$y_\circ-\eta$	+0.419096	+0.379285
log. $(x_\circ-\xi)$	9.5002887*n*	9.5881809
log. $(y_\circ-\eta)$	9.6223135	9.5789657
tang. M	9.8779752*n*	0.0092152
M	322° 56′ 43″.5	45° 36′ 28″.2
sin. M	9.7800115*n*	9.8540438
m	9.7202772	9.7341371

	Beginning.	End.
x'	$+.569487$	$+.569544$
y'	$-.083015$	$-.083555$
log. x'	9.7554838	9.7555272
log. y'	8.9191566n	8.9219724n
tang. N	0.8363272n	0.8335548n
N	98° 17′ 37″.2	98° 20′ 45″.8
sin. N	9.9954341	9.9953761
n	9.7600497	9.7601511
$M-N$	224° 39′ 6″.3	307° 15′ 42″.4
sin. $(M-N)$	9.8468292n	9.9008463n
m	9.7202772	9.7341371
comp. L	0.2739194	0.2748965
sin. ψ	9.8410258n	9.9098799n
ψ	316° 5′ 41″.3	234° 21′ 4″.7
$M-N+\psi$	180° 44′ 47″.6	181° 36′ 47″.1
sin. $(M-N+\psi)$	8.1149272n	8.4494768n
m	9.7202772	9.7341371
comp. n	0.2399503	0.2398489
cosec. ψ	0.1589742n	0.0901201n
$s=3600$	3.5563025	3.5563025
log. T′	1.7904314	2.0698854
T′	−61.72s.	−117.46s.
$t-T-T'$	−5h. 7m. 27.08s.	−5h. 7m. 24.54s.
$N-\psi$	142° 11′ 55″.9	223° 59′ 41″.1
$s=3600$	3.5563025	3.5563025
comp. 206265	4.6855749	4.6855749
comp. n	0.2399503	0.2398489
cosec. π	1.7547617	1.7545655
h	0.2365894	0.2362918
sin. $(N-\psi)$	9.7874058	9.8417300n
sec. ψ	0.1423731	0.2344702n
	0.1663683	0.3124920
coefficient of cos. $\delta\Delta\alpha$	+1.4668	+2.0535
h	0.2365894	0.2362918
cos. $(N-\psi)$	9.8977056n	9.8569725n
sec. ψ	0.1423731	0.2344702n
	0.2766681n	0.3277345
coefficient of $\Delta\delta$	−1.8909	+2.1268

Hence we obtain the following equations for Washington:

$$\omega=-5\text{h. }7\text{m. }27.08\text{s.}+1.4668\Delta\alpha-1.8909\Delta\delta,$$
$$\omega=-5\text{h. }7\text{m. }24.54\text{s.}+2.0535\Delta\alpha+2.1268\Delta\delta.$$

Employing the values of cos. $\delta\Delta a$ and $\Delta\delta$, found on page 334, we obtain

$$\omega=-5\text{h. }7\text{m. }27.08\text{s.}-37.79\text{s.}-3.54\text{s.}=-5\text{h. }8\text{m. }8.41\text{s.}$$

$$\omega=-5\text{h. }7\text{m. }24.54\text{s.}-52.91\text{s.}+3.98\text{s.}=-5\text{h. }8\text{m. }13.47\text{s.}$$

The mean of the two results is

$$\omega=-5\text{h. }8\text{m. }10.94\text{s.},$$

which is the longitude of Washington from Greenwich, according to the observations of the solar eclipse of July 28, 1851.

SECTION VII.

BESSEL'S METHOD OF COMPUTING THE LONGITUDE OF A PLACE AND THE ERROR OF THE TABLES FROM AN OBSERVED OCCULTATION.

(339.) The formulas of the preceding section are applicable to an occultation of a fixed star, with the modifications indicated in Art. 295. The computation of e, A, D, g, i, and l is dispensed with, as also z and ζ. We must compute the values of x and y from the formulæ

$$x=\frac{\cos.\ \delta\ \sin.\ (a-\text{A})}{\sin.\ \pi},$$

$$y=\frac{\sin.\ (\delta-\text{D})\cos.^2\ \tfrac{1}{2}(a-\text{A})+\sin.\ (\delta+\text{D})\sin.^2\ \tfrac{1}{2}(a-\text{A})}{\sin.\ \pi}.$$

Also the values of ξ and η from the formulæ

$$\xi=\rho\cos.\ \phi'\ \sin.\ (\mu-\text{A}),$$

$$\eta=\rho\sin.\ \phi'\cos.\ \text{D}-\rho\cos.\ \phi'\sin.\ \text{D}\cos.\ (\mu-\text{A}),$$

where μ represents the *observed* sidereal time of immersion or emersion.

Let T represent the approximate time of the first meridian, corresponding to the phase observed.

Let $x_\circ$ represent the value of x for the time T;
$y_\circ$ represent the value of y for the time T;
x' represent the hourly variation of x;
y' represent the hourly variation of y.

$$m\sin.\ \text{M}=x_\circ-\xi,$$

$$m\cos.\ \text{M}=y_\circ-\eta,$$

$$n\sin.\ \text{N}=x',$$

$$n\cos.\ \text{N}=y',$$

$$\sin.\ \psi=\frac{m}{k}\sin.\ (\mathrm{M}-\mathrm{N}),$$

$$\log.\ k=9.4353665.$$

For immersion, ψ must be taken in the first or fourth quadrant; for emersion, in the second or third quadrant.

$$\mathrm{T}'=-\frac{m\sin.\ (\mathrm{M}-\mathrm{N}+\psi)}{n\sin.\ \psi}=-\frac{m}{n}\cos.\ (\mathrm{M}-\mathrm{N})-\frac{k\cos.\ \psi}{n}.$$

Then

$$\omega=t-\mathrm{T}-\mathrm{T}'+h\,\frac{\sin.\ (\mathrm{N}-\psi)}{\cos.\ \psi}\cos.\ \delta\Delta a+h\frac{\cos.\ (\mathrm{N}-\psi)}{\cos.\ \psi}\Delta\delta,$$

where $$h=\frac{s}{206265\,.\,n\sin.\ \pi};$$

$s=3600=$ the number of seconds in an hour;

$t=$ the observed mean time of immersion or emersion.

(340.) *Example.* On the 23d of January, 1850, the occultation of a Tauri was observed as follows:

At Greenwich, England.

Immersion . . . . 13h. 32m. 38.66s. Greenwich m. t.
Emersion . . . . 14h. 1m. 24.52s. "

At Cambridge, Massachusetts.

Immersion . . . . 7h. 14m. 39.05s. Cambridge m. t.
Emersion . . . . 8h. 29m. 50.25s. "

It is required to determine the longitude of Cambridge from Greenwich.

We will first determine the error of the tables according to the Greenwich observations. The co-ordinates x and y have already been given on page 294.

Computation for Greenwich,
$\phi'=51°\ 17'\ 24''.6.$

	Immersion.	Emersion.
	h. m. s.	h. m. s.
t	13 32 38.66 m. t.	14 1 24.52 m. t.
μ	9 44 43.78 s. t.	10 13 34.37 s. t.
A	4 27 19.54	4 27 19.54
$\mu-\mathrm{A}$	5 17 24.24	5 46 14.83
In arc	79° 21′ 3″.6	86° 33′ 42″.45
$x_\circ$	.503232	.787997
$y_\circ$	.467948	.519616
x'	.593999	.593978
y'	.107785	.107762

	Immersion.	Emersion.
ρ	9.9991134	9.9991134
cos. ϕ'	9.7961416	9.7961416
sin. $(\mu-\mathrm{A})$	9.9924552	9.9992176
log. ξ	9.7877102	9.7944726
ξ	.613353	.622978
ρ	9.9991134	
sin. ϕ'	9.8922744	
cos. D	9.9824020	
log. (1)	9.8737898	
ρ cos. ϕ'	9.7952550	9.7952550
sin. D	9.4456150	9.4456150
cos. $(\mu-\mathrm{A})$	9.2666830	8.7779489
log. (2)	8.5075530	8.0188189
(1)	.747807	.747807
(2)	.032178	.010443
$(1)-(2)=\eta$	.715629	.737364
$x_\circ-\xi$	−.110121	+.165019
$y_\circ-\eta$	−.247681	−.217748
log. $(x_\circ-\xi)$	9.0418701*n*	9.2175340
log. $(y_\circ-\eta)$	9.3938927*n*	9.3379542*n*
tang. M	9.6479774	9.8795798*n*
M	203° 58′ 13″.2	142° 50′ 36″.8
sin. M	9.6088078*n*	9.7810323
m	9.4330623	9.4365017
log. x'	9.7737857	9.7737703
log. y'	9.0325583	9.0324656
tang. N	0.7412274	0.7413047
N	79° 42′ 54″.8	79° 43′ 1″.2
sin. N	9.9929653	9.9929678
n	9.7808204	9.7808025
M−N	124° 15′ 18″.4	63° 7′ 35″.6
sin. (M−N)	9.9172630	9.9503684
m	9.4330623	9.4365017
comp. k	0.5646335	0.5646335
sin. ψ	9.9149588	9.9515036
ψ	55° 18′ 7″.2	116° 34′ 33″.5
$\mathrm{M}-\mathrm{N}+\psi$	179° 33′ 25″.6	179° 42′ 9″.1
sin. $(\mathrm{M}-\mathrm{N}+\psi)$	7.8881678	7.7153218
m	9.4330623	9.4365017
comp. n	0.2191796	0.2191975
cosec. ψ	0.0850412	0.0484964
$s=3600$	3.5563025	3.5563025
log. T′	1.1817534	0.9758199
T′	−15.20s.	−9.46s.

	Immersion	Emersion.
$N-\psi$	$24^\circ\ 24'\ 47''.6$	$-36^\circ\ 51'\ 32''.3$
$s=3600$	3.5563025	3.5563025
comp. 206265	4.6855749	4.6855749
comp. n	0.2191796	0.2191975
cosec. π	1.7589910	1.7588792
h	0.2200480	0.2199541
sin. $(N-\psi)$	9.6162807	9.7780408n
sec. ψ	0.2446963	0.3493195n
	0.0810250	0.3473144
coefficient of cos. $\delta\Delta a$	$+1.2051$	$+2.2249$
h	0.2200480	0.2199541
cos. $(N-\psi)$	9.9593220	9.9031521
sec. ψ	0.2446963	0.3493195n
	0.4240663	0.4724257n
coefficient of $\Delta\delta$	$+2.6550$	-2.9677

Hence we have the two equations,

$$0=15.20+1.2051\Delta a+2.6550\Delta\delta,$$
$$0=\ \ 9.46+2.2249\Delta a-2.9677\Delta\delta.$$

From which we obtain

$$\cos.\ \delta\Delta a=-7''.405,$$
$$\Delta\delta=-2''.364.$$

Computation for Cambridge.

	Immersion.	Emersion.
	h. m. s.	h. m. s.
t	7 14 39.05 m. t.	8 29 50.25 m. t.
μ	3 26 28.81 s. t.	4 41 52.36 s. t.
A	4 27 19.54	4 27 19.54
$\mu-A$	-1 0 50.73	14 32.82
In arc	$-15^\circ\ 12'\ 40''.95$	$3^\circ\ 38'\ 12''.3$
Assume T	**12h.**	**13.25h.**
x_0	$-.413936$	$+.328553$
y_0	$+.301478$	$+.436250$
x'	$+.593945$	$+.593997$
y'	$+.107833$	$+.107795$
ρ cos. ϕ'	9.8691208	9.8691208
sin. $(\mu-A)$	9.4189320n	8.8022991
log. ξ	9.2880528n	8.6714199
ξ	$-.194112$	$+.046927$

	Immersion.	Emersion.
ρ sin. ϕ'	9.8264412	
cos. D	9.9824020	
log. (1)	9.8088432	
ρ cos. ϕ'	9.8691208	9.8691208
sin. D	9.4456150	9.4456150
cos. $(\mu-\mathrm{A})$	9.9845113	9.9991246
log. (2)	9.2992471	9.3138604
(1)	+.643937	+.643937
(2)	+.199181	+.205997
$(1)-(2)=\eta$	+.444756	+.437940
$x_\circ-\xi$	−.219824	+.281626
$y_\circ-\eta$	−.143278	−.001690
log. $(x_\circ-\xi)$	9.3420751*n*	9.4496727
log. $(y_\circ-\eta)$	9.1561795*n*	7.2278867*n*
tang. M	0.1858956	2.2217860*n*
M	236° 54′ 15″.5	90° 20′ 37″.8
sin. M	9.9231195*n*	9.9999922
m	9.4189556	9.4496805
log. x'	9.7737463	9.7737842
log. y'	9.0327517	9.0325986
tang. N	0.7409946	0.7411856
N	79° 42′ 35″.3	79° 42′ 51″.3
sin. N	9.9929578	9.9929640
n	9.7807885	9.7808202
M−N	157° 11′ 40″.2	10° 37′ 46″.5
sin. (M−N)	9.5883883	9.2658995
m	9.4189556	9.4496805
comp. k	0.5646335	0.5646335
sin. ψ	9.5719774	9.2802135
ψ	21° 54′ 54″.0	169° 0′ 35″.6
$\mathrm{M}-\mathrm{N}+\psi$	179° 6′ 34″.2	179° 38′ 22″.1
sin. $(\mathrm{M}-\mathrm{N}+\psi)$	8.1914938	7.7988132
m	9.4189556	9.4496805
comp. n	0.2192115	0.2191798
cosec. ψ	0.4280226	0.7197865
$s=3600$	3.5563025	3.5563025
log. T′	1.8139860	1.7437625
T′	−65.16s.	−55.43s.
$\omega=t-\mathrm{T}-\mathrm{T}'$	−4h. 44m. 15.79s.	−4h. 44m. 14.32s.
$\mathrm{N}-\psi$	57° 47′ 41″.3	−89° 17′ 44″.3
$s=3600$	3.5563025	3.5563025
comp. 206265	4.6855749	4.6855749
comp. n	0.2192115	0.2191798
cosec. π	1.7593527	1.7590595

The following recapitulation of the formulæ most frequently used in an observatory is added for convenience of reference.

To compute the corrections to be applied to the observed transit of a star, in order to obtain the correct apparent right ascension. See page 73.

$$\text{R. A.} = \text{T} + dt + a\,.\,\frac{\sin.\ z}{\cos.\ \delta} + b\,.\,\frac{\cos.\ z}{\cos.\ \delta} + \frac{c}{\cos.\ \delta}.$$

By a close circumpolar star (see page 69),

$$a = \tfrac{1}{2}\Delta\ \sec.\ \phi\ \cot.\ \delta.$$

By two stars differing considerably in declination (see page 70),

$$a = \frac{\Delta'\ \cos.\ \delta\ \cos.\ \delta'}{\cos.\ \phi\ \sin.\ (\delta' - \delta)}.$$

R. A. = the apparent right ascension required;

T = the observed time of transit, as shown by the clock;

dt = the correction for the error of the clock, plus when the clock is too slow;

z = the zenith distance of the star;

δ = the declination of star observed;

δ' = the declination of the second star;

a = the deviation of the telescope in azimuth, plus when the eastern pivot deviates to the north of east.

b = the inclination of the axis of the telescope (see page 63), plus when the west end of the axis is too high;

c = the error in collimation (see page 65), plus when the mean of the wires falls on the east side of the optical axis;

Δ = the interval between two successive transits, minus 12 hours;

Δ' = the difference of the observed times, minus the diff[illegible] of right ascensions;

ϕ = the latitude of the place;

$z = \phi - \delta$, if the observations be made [illegible]

$z = \delta - \phi$, if to the north [illegible]

$z = 180° - (\phi + \delta)$, [illegible]

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7099

To find the altitude, azimuth, and parallactic angle of a star, its declination and hour angle being given, as well as the latitude of the place. See pages 108 and 110.

$$\text{tang. } y = \text{cos. P cot. } \delta;$$

$$\text{cos. } z = \frac{\text{sin. } \delta \text{ sin. } (y+\phi)}{\text{cos. } y};$$

$$\text{sin. } (y+\phi) = \frac{\text{cos. } z \text{ cos. } y}{\text{sin. } \delta};$$

$$\text{cot. A} = \frac{\text{cot. P cos. } (y+\phi)}{\text{sin. } y};$$

$$\text{sin. } p = \frac{\text{cos. } \phi \text{ sin. P}}{\text{sin. } z}.$$

A = the azimuth of the star, counted from the north;
z = the zenith distance of the star;
P = the hour angle of the star;
δ = the declination of the star;
ϕ = the latitude of the place;
p = the parallactic angle.

When only p is required,

$$\text{tang. } x = \text{cos. P cot. } \phi;$$

$$\text{tang. } p = \frac{\text{sin. } x \text{ tang. P}}{\text{cos. } (x+\delta)}.$$

To compute the distance between two stars whose right ascensions and declinations are known. See page 111.

$$\text{cot. B} = \text{cos. } (a-a') \text{ cot. } \delta;$$

$$\text{cos. } x = \frac{\text{cos. } (\delta'-\text{B}) \text{ sin. } \delta}{\text{sin. B}}.$$

a = the right ascension of one star;
δ = its declination;
a' = the right ascension of the second star;
δ' = its declination;
x = the angular distance required.

To compute the hour angle at the pole: the latitude of the place, the declination, and zenith distance of the sun or star being given. See page 133.

$$\sin.\ \tfrac{1}{2}\mathrm{P}=\sqrt{\frac{\sin.\left\{\dfrac{z+\phi-\delta}{2}\right\}\times\sin.\left\{\dfrac{z-\phi+\delta}{2}\right\}}{\cos.\ \phi\ \cos.\ \delta}}.$$

P = the hour angle at the pole;
ϕ = the latitude of the place;
δ = the declination of the star;
z = the true zenith distance of the star.

To compute the correction for the reduction to the meridian. See page 142.

$$x=\frac{2\ \sin.^2\ \tfrac{1}{2}\mathrm{P}\ \cos.\ \phi\ \cos.\ \delta}{\sin.\ 1''\ \sin.\ z}-\left(\frac{\sin.^2\ \tfrac{1}{2}\mathrm{P}\ \cos.\ \phi\ \cos.\ \delta}{\sin.\ z}\right)^2\frac{2\ \cot.\ z}{\sin.\ 1''}.$$

P = the hour angle at the pole, as shown by a well-regulated clock;
ϕ = the latitude of the place;
δ = the declination of the star;
z = the meridional zenith distance of the star;
x = the required correction in seconds.

To compute the latitude of a place, from observations of the pole star at any time of the day. See page 152.

$$\phi=\mathrm{H}-d\ \cos.\ \mathrm{P}+\tfrac{1}{2}\ \sin.\ 1''\ (d\ \sin.\ \mathrm{P})^2\ \mathrm{tang.}\ \mathrm{H}$$
$$-\tfrac{1}{3}\ \sin.^2\ 1''\ (d\ \cos.\ \mathrm{P})\ (d\ \sin.\ \mathrm{P})^2.$$

H = the observed altitude of the star, corrected for refraction;
d = the apparent polar distance of the star, expressed in seconds of arc;
P = the hour angle of the star from the meridian;
ϕ = the latitude required.

To find the altitude and hour angle of a star when it is upon the prime vertical, together with the latitude of the place. See pages 113 and 157.

$$\text{cos. P} = \text{cot. } \phi \text{ tang. } \delta;$$

$$\text{sin. A} = \frac{\text{sin. } \delta}{\text{sin. } \phi};$$

$$\text{cos. A} = \text{sin. P cos. } \delta;$$

$$\text{tang. } \phi = \frac{\text{tang. } \delta}{\text{cos. P}};$$

$$\text{sin. } \phi = \frac{\text{sin. } \delta}{\text{sin. A}};$$

$$\text{cos. } \phi = \text{cot. P cot. A.}$$

P = the hour angle of the star at the pole;
A = the altitude of the star;
δ = the declination of the star;
ϕ = the latitude of the place.

To find the longitude and latitude of a star when its right ascension and declination are known, and vice versâ. See pages. 174 and 176.

Make $\text{tang. } a = \text{sin. R. A. cot. Dec.}$

$$\text{tang. L} = \text{sin. } (a+\omega) \text{ tang. R. A. cosec. } a,$$

$$\text{tang. } l = \text{cot. } (a+\omega) \text{ sin. L},$$

$$\text{sin. } l = \text{cos. } (a+\omega) \text{ sin. Dec. sec. } a,$$

$$\text{sin. L} = \text{tang. } (a+\omega) \text{ tang. } l.$$

Make $\text{tang. } a = \text{sin. L cot. } l.$

$$\text{tang. R. A.} = \text{sin. } (a-\omega) \text{ tang. L cosec. } a,$$

$$\text{tang. Dec.} = \text{cot. } (a-\omega) \text{ sin. R. A.}$$

$$\text{sin. Dec.} = \text{cos. } (a-\omega) \text{ sin. } l \text{ sec. } a,$$

$$\text{sin. R. A.} = \text{tang. } (a-\omega) \text{ tang. Dec.}$$

L = the longitude of the star;
l = the latitude of the star;
R. A. = the right ascension of the star;
Dec. = the declination of the star;
ω = the obliquity of the ecliptic.

To compute the longitude, right ascension, and declination of the sun; any one of these quantities, together with the obliquity of the ecliptic, being given. See page 178.

$$\text{tang. R. A.} = \text{tang. Long. cos. } \omega;$$
$$\text{sin. R. A.} = \text{tang. Dec. cot. } \omega;$$
$$\text{tang. Dec.} = \text{sin. R. A. tang. } \omega;$$
$$\text{sin. Dec.} = \text{sin. Long. sin. } \omega;$$
$$\text{tang. Long.} = \frac{\text{tang. R. A.}}{\text{cos. } \omega};$$
$$\text{sin. Long.} = \frac{\text{sin. Dec.}}{\text{sin. } \omega};$$
$$\text{cos. Long.} = \text{cos. R. A. cos. Dec.};$$
$$\text{cos. R. A.} = \frac{\text{cos. Long.}}{\text{cos. Dec.}};$$

Long. = the sun's longitude;
R. A. = the sun's right ascension;
Dec. = the sun's declination;
ω = the obliquity of the ecliptic.

To compute the correction in time, to be applied to the mean of the times of observed equal altitudes of the sun, in order to obtain the time of its meridional passage. See page 128.

$$dP = \frac{d\delta}{15}(\text{tang. } \phi \text{ cosec. P} - \text{tang. } \delta \text{ cot. P}).$$

dP = the increase of the hour angle in time;
$d\delta$ = increase of declination from the meridian to the afternoon observation;
P = hour angle from the meridian, supposing no change in declination;
δ = declination of the sun on the meridian;
ϕ = the latitude of the place.

For the Angle of the Vertical.

$$\text{tang. } \phi' = \text{tang. } \phi \times 0.9933254.$$

For the Radius of the Earth.

$$r = \sqrt{\frac{\text{cos. } \phi}{\text{cos. } \phi' \text{ cos. } (\phi' - \phi)}}.$$

For the horizontal Parallax of any Place.

$$p = r\text{P}.$$

For the Moon's Parallax in Altitude.

$$1.\quad \text{sin. } q = \text{sin. } p \text{ sin. } (z + q).$$

$$2.\quad \text{tang. } q = \frac{\text{sin. } p \text{ sin. } z}{1 - \text{sin. } p \text{ cos. } z}.$$

$$3.\quad q = \frac{\text{sin. } p \text{ sin. } z}{\text{sin. } 1''} + \frac{\text{sin.}^2 p \text{ sin. } 2z}{\text{sin. } 2''} + \frac{\text{sin.}^3 p \text{ sin. } 3z}{\text{sin. } 3''} +, \text{ etc.}$$

For the Moon's Parallax in Right Ascension.

Make $a = \dfrac{\text{sin. } p \text{ cos. } \phi'}{\text{cos. } \delta}$.

$$1.\quad \text{sin. } \Pi = a \text{ sin. } (h + \Pi).$$

$$2.\quad \text{tang. } \Pi = \frac{a \text{ sin. } h}{1 - a \text{ cos. } h}.$$

$$3.\quad \Pi = \frac{a \text{ sin. } h}{\text{sin. } 1''} + \frac{a^2 \text{ sin. } 2h}{\text{sin. } 2''} + \frac{a^3 \text{ sin. } 3h}{\text{sin. } 3''} +, \text{ etc.}$$

For the Moon's Parallax in Declination.

$$1.\quad \text{tang. } \delta' = \left(1 - \frac{\text{sin. } p \cdot \text{sin. } \phi'}{\text{sin. } \delta}\right) \frac{\text{sin. } h'}{\text{sin. } h} \text{ tang. } \delta.$$

Make $\text{cot. } b = \dfrac{\text{cos. } (h + \frac{1}{2}\Pi) \text{ cot. } \phi'}{\text{cos. } \frac{1}{2}\Pi}$,

$$c = \frac{\text{sin. } p \text{ sin. } \phi'}{\text{sin. } b}.$$

$$2.\quad \sin.\ \pi = c\ \sin.\ (b-\delta+\pi).$$

$$3.\quad \text{tang.}\ \pi = \frac{c\ \sin.\ (b-\delta)}{1-c\ \cos.\ (b-\delta)}.$$

$$4.\quad \pi = \frac{c\ \sin.\ (b-\delta)}{\sin.\ 1''} + \frac{c^2\ \sin.\ 2(b-\delta)}{\sin.\ 2''} + \frac{c^3\ \sin.\ 3(b-\delta)}{\sin.\ 3''} +, \text{ etc.}$$

For the hourly Variation of Parallax in Right Ascension.

$$d\Pi = \frac{p\ \cos.\ \phi'\ \cos.\ hdh}{\cos.\ \delta}.$$

For the hourly Variation of Parallax in Declination.

$$d\pi = p\ \cos.\ \phi'\ \sin.\ \delta\ \sin.\ hdh.$$

For the Augmentation of the Moon's Semi-diameter.

$$1.\quad x = \text{A}\,.\,s^2\ \cos.\ z' + \tfrac{1}{2}\text{A}^2 s^3 + \tfrac{1}{2}\text{A}^2 s^3\ \cos.^2\ z' +, \text{ etc.,}$$

where $\text{A} = 0.00001779.$

$$2.\quad x = s\ \sin.\ \pi\ \cot.\ (b-\delta) - \tfrac{1}{2}\ s\ \sin.^2\ \pi.$$

Notation.

ϕ = the geographical latitude of the place;
ϕ' = the geocentric latitude of the place;
r = the radius of the earth, corresponding to the latitude ϕ;
z = the true zenith distance of the moon;
P = the moon's horizontal parallax at the equator;
p = the horizontal parallax at the place of observation;
q = the moon's parallax in altitude;
δ = the true declination of the moon;
δ' = the apparent declination of the moon;
h = the true hour angle at the pole = the sidereal time, *minus* the moon's true right ascension;
h' = the apparent hour angle;
Π = the moon's parallax in right ascension;
π = the parallax in declination;
$d\Pi$ = the variation of parallax in right ascension;
$d\pi$ = the variation of parallax in declination;
s = the true semi-diameter of the moon

TRIGONOMETRICAL FORMULÆ.

	Values of sin. A.	Values of cos. A.	Values of tang. A.
1.	cos. A tang. A.	$\frac{\text{sin. A}}{\text{tang. A}}$.	$\frac{\text{sin. A}}{\text{cos. A}}$.
2.	$\frac{\text{cos. A}}{\text{cot. A}}$.	sin. A cot. A.	$\frac{1}{\text{cot. A}}$.
3.	$\sqrt{1-\text{cos.}^2\text{ A}}$.	$\sqrt{1-\text{sin.}^2\text{ A}}$.	$\sqrt{\frac{1}{\text{cos.}^2\text{ A}}-1}$.
4.	$\frac{1}{\sqrt{1+\text{cot.}^2\text{ A}}}$.	$\frac{1}{\sqrt{1+\text{tang.}^2\text{ A}}}$.	$\frac{\text{sin. A}}{\sqrt{1-\text{sin.}^2\text{ A}}}$.
5.	$\frac{\text{tang. A}}{\sqrt{1+\text{tang.}^2\text{ A}}}$.	$\frac{\text{cot. A}}{\sqrt{1+\text{cot.}^2\text{ A}}}$.	$\frac{\sqrt{1-\text{cos.}^2\text{ A}}}{\text{cos. A}}$.
6.	2 sin. ½A cos. ½A.	$\text{cos.}^2\ \tfrac{1}{2}\text{A}-\text{sin.}^2\ \tfrac{1}{2}\text{A}$.	$\frac{2\text{ tang. }\frac{1}{2}\text{A}}{1-\text{tang.}^2\ \frac{1}{2}\text{A}}$.
7.	$\sqrt{\frac{1-\text{cos. 2A}}{2}}$.	$1-2\text{ sin.}^2\ \tfrac{1}{2}\text{A}$.	$\frac{2\text{ cot. }\frac{1}{2}\text{A}}{\text{cot.}^2\ \frac{1}{2}\text{A}-1}$.
8.	$\frac{2\text{ tang. }\frac{1}{2}\text{A}}{1+\text{tang.}^2\ \frac{1}{2}\text{A}}$.	$2\text{ cos.}^2\ \tfrac{1}{2}\text{A}-1$.	$\frac{2}{\text{cot. }\frac{1}{2}\text{A}-\text{tang. }\frac{1}{2}\text{A}}$
9.	$\frac{2}{\text{tang. }\frac{1}{2}\text{A}+\text{cot. }\frac{1}{2}\text{A}}$.	$\sqrt{\frac{1+\text{cos. 2A}}{2}}$.	cot. A − 2 cot. 2A.
10.	$\frac{1}{\text{cosecant A}}$.	$\frac{1-\text{tang.}^2\ \frac{1}{2}\text{A}}{1+\text{tang.}^2\ \frac{1}{2}\text{A}}$.	$\frac{1-\text{cos. 2A}}{\text{sin. 2A}}$.
11.	$2\text{ sin.}^2(45^\circ+\tfrac{1}{2}\text{A})-1$.	$\frac{\text{cot. }\frac{1}{2}\text{A}-\text{tang. }\frac{1}{2}\text{A}}{\text{cot. }\frac{1}{2}\text{A}+\text{tang. }\frac{1}{2}\text{A}}$.	$\frac{\text{sin. 2A}}{1+\text{cos. 2A}}$.
12.	$1-2\text{ sin.}^2(45^\circ-\tfrac{1}{2}\text{A})$.	$\frac{1}{1+\text{tang. A tang. }\frac{1}{2}\text{A}}$.	$\sqrt{\frac{1-\text{cos. 2A}}{1+\text{cos. 2A}}}$.
13.		$\frac{1}{\text{secant A}}$.	

Relative to two Arcs, A *and* B.

$$\sin.\ (A+B)=\sin.\ A\ \cos.\ B+\cos.\ A\ \sin.\ B$$

$$\sin.\ (A-B)=\sin.\ A\ \cos.\ B-\cos.\ A\ \sin.\ B,$$

$$\cos.\ (A+B)=\cos.\ A\ \cos.\ B-\sin.\ A\ \sin.\ B,$$

$$\cos.\ (A-B)=\cos.\ A\ \cos.\ B+\sin.\ A\ \sin.\ B,$$

$$\text{tang.}\ (A+B)=\frac{\text{tang.}\ A+\text{tang.}\ B}{1-\text{tang.}\ A\,.\,\text{tang.}\ B},$$

$$\text{tang.}\ (A-B)=\frac{\text{tang.}\ A-\text{tang.}\ B}{1+\text{tang.}\ A\,.\,\text{tang.}\ B},$$

$$\frac{\sin.\ (A+B)}{\sin.\ (A-B)}=\frac{\text{tang.}\ A+\text{tang.}\ B}{\text{tang.}\ A-\text{tang.}\ B}=\frac{\cot.\ B+\cot.\ A}{\cot.\ B-\cot.\ A},$$

$$\frac{\cos.\ (A+B)}{\cos.\ (A-B)}=\frac{\cot.\ B-\text{tang.}\ A}{\cot.\ B+\text{tang.}\ A}=\frac{\cot.\ A-\text{tang.}\ B}{\cot.\ A+\text{tang.}\ B},$$

$$\sin.\ A\ \cos.\ B=\tfrac{1}{2}\sin.\ (A+B)+\tfrac{1}{2}\sin.\ (A-B),$$

$$\cos.\ A\ \sin.\ B=\tfrac{1}{2}\sin.\ (A+B)-\tfrac{1}{2}\sin.\ (A-B),$$

$$\sin.\ A\ \sin.\ B=\tfrac{1}{2}\cos.\ (A-B)-\tfrac{1}{2}\cos.\ (A+B).$$

$$\cos.\ A\ \cos.\ B=\tfrac{1}{2}\cos.\ (A+B)+\tfrac{1}{2}\cos.\ (A-B),$$

$$\sin.\ A+\sin.\ B=2\sin.\ \tfrac{1}{2}(A+B)\,.\,\cos.\ \tfrac{1}{2}(A-B),$$

$$\cos.\ A+\cos.\ B=2\cos.\ \tfrac{1}{2}(A+B)\,.\,\cos.\ \tfrac{1}{2}(A-B),$$

$$\sin.\ A-\sin.\ B=2\sin.\ \tfrac{1}{2}(A-B)\,.\,\cos.\ \tfrac{1}{2}(A+B),$$

$$\cos.\ B-\cos.\ A=2\sin.\ \tfrac{1}{2}(A-B)\,.\,\sin.\ \tfrac{1}{2}(A+B),$$

$$\text{tang.}\ A+\text{tang.}\ B=\frac{\sin.\ (A+B)}{\cos.\ A\,.\,\cos.\ B},$$

$$\cot.\ A+\cot.\ B=\frac{\sin.\ (A+B)}{\sin.\ A\,.\,\sin.\ B},$$

$$\text{tang.}\ A-\text{tang.}\ B=\frac{\sin.\ (A-B)}{\cos.\ A\,.\,\cos.\ B},$$

$$\cot.\ B-\cot.\ A=\frac{\sin.\ (A-B)}{\sin.\ A\,.\,\sin.\ B},$$

$$\frac{\sin.\ A+\sin.\ B}{\sin.\ A-\sin.\ B}=\frac{\text{tang.}\ \tfrac{1}{2}(A+B)}{\text{tang.}\ \tfrac{1}{2}(A-B)},$$

$$\frac{\cos.\ B+\cos.\ A}{\cos.\ B-\cos.\ A}=\frac{\cot.\ \tfrac{1}{2}(A+B)}{\text{tang.}\ \tfrac{1}{2}(A-B)},$$

Multiple Arcs.

$$\sin. 2A = 2 \sin. A \cos. A,$$
$$\sin. 3A = 2 \sin. 2A . \cos. A - \sin. A,$$
$$\sin. 4A = 2 \sin. 3A . \cos. A - \sin. 2A,$$
$$\cos. 2A = 2 \cos. A \cos. A - 1,$$
$$= 1 - 2 \sin.^2 A,$$
$$\cos. 3A = 2 \cos. 2A \cos. A - \cos. A,$$
$$\cos. 4A = 2 \cos. 3A \cos. A - \cos. 2A,$$
$$\text{tang. } 2A = \frac{2 \text{ tang. } A}{1 - \text{tang.}^2 A},$$
$$\text{tang. } 3A = \frac{\text{tang. } 2A + \text{tang. } A}{1 - \text{tang. } 2A \text{ tang. } A},$$
$$\text{tang. } 4A = \frac{\text{tang. } 3A + \text{tang. } A}{1 - \text{tang. } 3A \text{ tang. } A}.$$

Trigonometrical Series.

$$\sin. A = A - \frac{A^3}{2.3} + \frac{A^5}{2.3.4.5} - \frac{A^7}{2.3.4.5.6.7} +, \text{ etc.},$$
$$\cos. A = 1 - \frac{A^2}{2} + \frac{A^4}{2.3.4} - \frac{A^6}{2.3.4.5.6} +, \text{ etc.},$$
$$\text{tang. } A = A + \frac{A^3}{3} + \frac{2A^5}{3.5} + \frac{17A^7}{3^2.5.7} +, \text{ etc.},$$
$$\cot. A = \frac{1}{A} - \frac{A}{3} - \frac{A^3}{3^2.5} - \frac{2A^5}{3^3.5.7} -, \text{ etc.}$$

Differentials of Trigonometrical Lines.

$$d \sin. x = + \cos. x \, dx,$$
$$d \cos. x = - \sin. x \, dx,$$
$$d \text{ tang. } x = + \frac{dx}{\cos.^2 x},$$
$$d \cot. x = - \frac{dx}{\sin.^2 x}.$$

TABLES.

POSITIONS OF THE PRINCIPAL FOREIGN OBSERVATORIES.

North Latitudes and West Longitudes are indicated by the sign +; South Latitudes and East Longitudes by the sign —.

Place.	Latitude.	Longitude from Washington in Time.	Longitude from Greenwich in Time.	Longitude from Greenwich in Arc.
	° ′ ″	h. m. s.	h. m. s.	° ′ ″
Abo	+60 26 56.8	— 6 37 20.0	— 1 29 8.8	— 22 17 11.4
Altona	+53 32 45.3	— 5 47 57.4	— 0 39 46.2	— 9 56 32.2
Armagh	+54 21 12.7	— 4 41 35.7	+ 0 26 35.5	+ 6 38 52.5
Athens	+37 58 20.0	— 6 43 6.4	— 1 34 55.2	— 23 43 47.8
Berlin	+52 30 16.7	— 6 1 46.1	— 0 53 34.9	— 13 23 43.9
Bilk	+51 12 25.0	— 5 35 16.1	— 0 27 4.9	— 6 46 13.9
Bonn	+50 43 45.0	— 5 36 35.7	— 0 28 24.5	— 7 6 6.9
Breslau	+51 6 56.0	— 6 16 21.2	— 1 8 10.0	— 17 2 30.0
Brussels	+50 51 10.7	— 5 25 38.8	— 0 17 27.6	— 4 21 54.0
Cambridge, England	+52 12 51.8	— 5 8 34.7	— 0 0 23.5	— 0 5 53.1
Cape of Good Hope	—33 56 3.0	— 6 22 7.2	— 1 13 56.0	— 18 28 59.7
Christiania	+59 54 43.7	— 5 51 6.0	— 0 42 54.8	— 10 43 41.4
Copenhagen	+55 40 53.0	— 5 58 30.5	— 0 50 19.3	— 12 34 49.5
Cracow	+50 3 50.0	— 6 28 2.4	— 1 19 51.2	— 19 57 48.6
Dorpat	+58 22 47.1	— 6 55 5.8	— 1 46 54.6	— 26 43 38.4
Dublin	+53 23 13.0	— 4 42 49.2	+ 0 25 22.0	+ 6 20 30.0
Durham	+54 46 6.4	— 5 1 53.2	+ 0 6 18.0	+ 1 34 30.0
Edinburgh	+55 57 23.2	— 4 55 28.2	+ 0 12 43.0	+ 3 10 45.0
Florence	+43 46 40.8	— 5 53 12.9	— 0 45 1.7	— 11 15 24.9
Geneva	+46 11 58.8	— 5 32 48.9	— 0 24 37.7	— 6 9 25.2
Göttingen	+51 31 47.9	— 5 47 57.3	— 0 39 46.1	— 9 56 31.5
Gotha	+50 56 5.2	— 5 51 6.9	— 0 42 55.7	— 10 43 54.9
Greenwich	+51 28 38.2	— 5 8 11.2	0 0 0	0 0 0
Hamburgh	+53 33 5.0	— 5 48 4.8	— 0 39 53.6	— 9 58 23.4
Kasan	+55 47 23.1	— 8 24 43.1	— 3 16 31.9	— 49 7 58.9
Königsberg	+54 42 50.4	— 6 30 11.6	— 1 22 0.4	— 20 30 5.4
Kremsmünster	+48 3 23.8	— 6 4 44.6	— 0 56 33.4	— 14 8 21.3
Leipsic	+51 20 20.4	— 5 57 39.7	— 0 49 28.5	— 12 22 7.5
Leyden	+52 9 28.2	— 5 26 8.6	— 0 17 57.4	— 4 29 21.4
Liverpool	+53 24 47.8	— 4 56 11.1	+ 0 12 0.1	+ 3 0 1.7
Madras	+13 4 9.2	—10 29 8.2	— 5 20 57.0	— 80 14 15.0
Mannheim	+49 29 12.9	— 5 42 2.7	— 0 33 51.5	— 8 27 52.5
Markree	+54 10 31.7	— 4 34 22.8	+ 0 33 48.4	+ 8 27 6.0
Marseilles	+43 17 49.0	— 5 29 40.2	— 0 21 29.0	— 5 22 14.8
Milan	+45 28 0.7	— 5 44 57.8	— 0 36 46.6	— 9 11 39.6
Modena	+44 38 52.8	— 5 51 55.2	— 0 43 44.0	— 10 55 59.5
Moscow	+55 45 19.8	— 7 38 28.5	— 2 30 17.3	— 37 34 19.3
Munich	+48 8 45.0	— 5 54 37.6	— 0 46 26.4	— 11 36 36.6
Naples	+40 51 46.6	— 6 5 12.1	— 0 57 0.9	— 14 15 13.9
Olmutz	+49 35 40.0	— 6 17 11.3	— 1 9 0.1	— 17 15 1.5
Oxford	+51 45 36.0	— 5 3 8.6	+ 0 5 2.6	+ 1 15 39.0
Padua	+45 24 2.5	— 5 55 40.2	— 0 47 29.0	— 11 52 15.4
Palermo	+38 6 44.0	— 6 1 36.7	— 0 53 25.5	— 13 21 21.9
Paramatta	—33 48 49.8	+ 8 47 42.6	—10 4 6.2	—151 1 33.7
Paris	+48 50 13.2	— 5 17 32.7	— 0 9 21.5	— 2 20 21.9
Petersburgh	+59 56 29.7	— 7 9 24.7	— 2 1 13.5	— 30 18 22.2
Prague	+50 5 18.5	— 6 5 53.2	— 0 57 42.0	— 14 25 29.4
Pulkowa	+59 46 18.7	— 7 9 29.9	— 2 1 18.7	— 30 19 40.1
Rome	+41 53 54.0	— 5 58 5.9	— 0 49 54.7	— 12 [illegible] 5
San Fernando	+36 27 45.0	— 4 43 22.1	+ 0 24 49.1	+ [illegible]
Santiago	—33 26 25.9	— 0 25 37.6	+ 4 42 33.6	+ [illegible]
Senftenberg	+50 5 10.1	— 6 14 1.1	— 1 5 49.9	— [illegible]
Vienna	+48 12 35.5	— 6 13 43.7	— 1 5 32.5	— [illegible]
Wilna	+54 40 59.1	— 6 49 23.0	— 1 41 11.8	— [illegible]

POSITIONS OF THE PRINCIPAL FOREIGN OBSERVATORIES.

North Latitudes and West Longitudes are indicated by the sign +; South Latitudes and East Longitudes by the sign —.

Place.	Latitude.	Longitude from Washington in Time.	Longitude from Greenwich in Time.	Longitude from Greenwich in Arc.
	° ′ ″	h. m. s.	h. m. s.	° ′ ″
Abo	+60 26 56.8	— 6 37 20.0	— 1 29 8.8	— 22 17 11.4
Altona	+53 32 45.3	— 5 47 57.4	— 0 39 46.2	— 9 56 32.2
Armagh	+54 21 12.7	— 4 41 35.7	+ 0 26 35.5	+ 6 38 52.5
Athens	+37 58 20.0	— 6 43 6.4	— 1 34 55.2	— 23 43 47.8
Berlin	+52 30 16.7	— 6 1 46.1	— 0 53 34.9	— 13 23 43.9
Bilk	+51 12 25.0	— 5 35 16.1	— 0 27 4.9	— 6 46 13.9
Bonn	+50 43 45.0	— 5 36 35.7	— 0 28 24.5	— 7 6 6.9
Breslau	+51 6 56.0	— 6 16 21.2	— 1 8 10.0	— 17 2 30.0
Brussels	+50 51 10.7	— 5 25 38.8	— 0 17 27.6	— 4 21 54.0
Cambridge, England	+52 12 51.8	— 5 8 34.7	— 0 0 23.5	— 0 5 53.1
Cape of Good Hope	—33 56 3.0	— 6 22 7.2	— 1 13 56.0	— 18 28 59.7
Christiania	+59 54 43.7	— 5 51 6.0	— 0 42 54.8	— 10 43 41.4
Copenhagen	+55 40 53.0	— 5 58 30.5	— 0 50 19.3	— 12 34 49.5
Cracow	+50 3 50.0	— 6 28 2.4	— 1 19 51.2	— 19 57 48.6
Dorpat	+58 22 47.1	— 6 55 5.8	— 1 46 54.6	— 26 43 38.4
Dublin	+53 23 13.0	— 4 42 49.2	+ 0 25 22.0	+ 6 20 30.0
Durham	+54 46 6.4	— 5 1 53.2	+ 0 6 18.0	+ 1 34 30.0
Edinburgh	+55 57 23.2	— 4 55 28.2	+ 0 12 43.0	+ 3 10 45.0
Florence	+43 46 40.8	— 5 53 12.9	— 0 45 1.7	— 11 15 24.9
Geneva	+46 11 58.8	— 5 32 48.9	— 0 24 37.7	— 6 9 25.2
Göttingen	+51 31 47.9	— 5 47 57.3	— 0 39 46.1	— 9 56 31.5
Gotha	+50 56 5.2	— 5 51 6.9	— 0 42 55.7	— 10 43 54.9
Greenwich	+51 28 38.2	— 5 8 11.2	0 0 0	0 0 0
Hamburgh	+53 33 5.0	— 5 48 4.8	— 0 39 53.6	— 9 58 23.4
Kasan	+55 47 23.1	— 8 24 43.1	— 3 16 31.9	— 49 7 58.9
Königsberg	+54 42 50.4	— 6 30 11.6	— 1 22 0.4	— 20 30 5.4
Kremsmünster	+48 3 23.8	— 6 4 44.6	— 0 56 33.4	— 14 8 21.3
Leipsic	+51 20 20.4	— 5 57 39.7	— 0 49 28.5	— 12 22 7.5
Leyden	+52 9 28.2	— 5 26 8.6	— 0 17 57.4	— 4 29 21.4
Liverpool	+53 24 47.8	— 4 56 11.1	+ 0 12 0.1	+ 3 0 1.7
Madras	+13 4 9.2	—10 29 8.2	— 5 20 57.0	— 80 14 15.0
Mannheim	+49 29 12.9	— 5 42 2.7	— 0 33 51.5	— 8 27 52.5
Markree	+54 10 31.7	— 4 34 22.8	+ 0 33 48.4	+ 8 27 6.0
Marseilles	+43 17 49.0	— 5 29 40.2	— 0 21 29.0	— 5 22 14.8
Milan	+45 28 0.7	— 5 44 57.8	— 0 36 46.6	— 9 11 39.6
Modena	+44 38 52.8	— 5 51 55.2	— 0 43 44.0	— 10 55 59.5
Moscow	+55 45 19.8	— 7 38 28.5	— 2 30 17.3	— 37 34 19.3
Munich	+48 8 45.0	— 5 54 37.6	— 0 46 26.4	— 11 36 36.6
Naples	+40 51 46.6	— 6 5 12.1	— 0 57 0.9	— 14 15 13.9
Olmutz	+49 35 40.0	— 6 17 11.3	— 1 9 0.1	— 17 15 1.5
Oxford	+51 45 36.0	— 5 3 8.6	+ 0 5 2.6	+ 1 15 39.0
Padua	+45 24 2.5	— 5 55 40.2	— 0 47 29.0	— 11 52 15.4
Palermo	+38 6 44.0	— 6 1 36.7	— 0 53 25.5	— 13 21 21.9
Paramatta	—33 48 49.8	+ 8 47 42.6	—10 4 6.2	—151 1 33.7
Paris	+48 50 13.2	— 5 17 32.7	— 0 9 21.5	— 2 20 21.9
Petersburgh	+59 56 29.7	— 7 9 24.7	— 2 1 13.5	— 30 18 22.2
Prague	+50 5 18.5	— 6 5 53.2	— 0 57 42.0	— 14 25 29.4
Pulkowa	+59 46 18.7	— 7 9 29.9	— 2 1 18.7	— 30 19 40.1
Rome	+41 53 54.0	— 5 58 5.9	— 0 49 54.7	— 12 28 40.5
San Fernando	+36 27 45.0	— 4 43 22.1	+ 0 24 49.1	+ 6 12 17.1
Santiago	—33 26 25.9	— 0 25 37.6	+ 4 42 33.6	+ 70 38 24.0
Senftenberg	+50 5 10.1	— 6 14 1.1	— 1 5 49.9	— 16 27 28.9
Vienna	+48 12 35.5	— 6 13 43.7	— 1 5 32.5	— 16 23 7.9
Wilna	+54 40 59.1	— 6 49 23.0	— 1 41 11.8	— 25 17 56.5

POSITIONS OF THE PRINCIPAL FOREIGN OBSERVATORIES.

North Latitudes and West Longitudes are indicated by the sign +; South Latitudes and East Longitudes by the sign —.

Place.	Latitude.	Longitude from Washington in Time.	Longitude from Greenwich in Time.	Longitude from Greenwich in Arc.
	° ′ ″	h. m. s.	h. m. s.	° ′ ″
Abo	+60 26 56.8	— 6 37 20.0	— 1 29 8.8	— 22 17 11.4
Altona	+53 32 45.3	— 5 47 57.4	— 0 39 46.2	— 9 56 32.2
Armagh	+54 21 12.7	— 4 41 35.7	+ 0 26 35.5	+ 6 38 52.5
Athens	+37 58 20.0	— 6 43 6.4	— 1 34 55.2	— 23 43 47.8
Berlin	+52 30 16.7	— 6 1 46.1	— 0 53 34.9	— 13 23 43.9
Bilk	+51 12 25.0	— 5 35 16.1	— 0 27 4.9	— 6 46 13.9
Bonn	+50 43 45.0	— 5 36 35.7	— 0 28 24.5	— 7 6 6.9
Breslau	+51 6 56.0	— 6 16 21.2	— 1 8 10.0	— 17 2 30.0
Brussels	+50 51 10.7	— 5 25 38.8	— 0 17 27.6	— 4 21 54.0
Cambridge, England	+52 12 51.8	— 5 8 34.7	— 0 0 23.5	— 0 5 53.1
Cape of Good Hope	—33 56 3.0	— 6 22 7.2	— 1 13 56.0	— 18 28 59.7
Christiania	+59 54 43.7	— 5 51 6.0	— 0 42 54.8	— 10 43 41.4
Copenhagen	+55 40 53.0	— 5 58 30.5	— 0 50 19.3	— 12 34 49.5
Cracow	+50 3 50.0	— 6 28 2.4	— 1 19 51.2	— 19 57 48.6
Dorpat	+58 22 47.1	— 6 55 5.8	— 1 46 54.6	— 26 43 38.4
Dublin	+53 23 13.0	— 4 42 49.2	+ 0 25 22.0	+ 6 20 30.0
Durham	+54 46 6.4	— 5 1 53.2	+ 0 6 18.0	+ 1 34 30.0
Edinburgh	+55 57 23.2	— 4 55 28.2	+ 0 12 43.0	+ 3 10 45.0
Florence	+43 46 40.8	— 5 53 12.9	— 0 45 1.7	— 11 15 24.9
Geneva	+46 11 58.8	— 5 32 48.9	— 0 24 37.7	— 6 9 25.2
Göttingen	+51 31 47.9	— 5 47 57.3	— 0 39 46.1	— 9 56 31.5
Gotha	+50 56 5.2	— 5 51 6.9	— 0 42 55.7	— 10 43 54.9
Greenwich	+51 28 38.2	— 5 8 11.2	0 0 0	0 0 0
Hamburgh	+53 33 5.0	— 5 48 4.8	— 0 39 53.6	— 9 58 23.4
Kasan	+55 47 23.1	— 8 24 43.1	— 3 16 31.9	— 49 7 58.9
Königsberg	+54 42 50.4	— 6 30 11.6	— 1 22 0.4	— 20 30 5.4
Kremsmünster	+48 3 23.8	— 6 4 44.6	— 0 56 33.4	— 14 8 21.3
Leipsic	+51 20 20.4	— 5 57 39.7	— 0 49 28.5	— 12 22 7.5
Leyden	+52 9 28.2	— 5 26 8.6	— 0 17 57.4	— 4 29 21.4
Liverpool	+53 24 47.8	— 4 56 11.1	+ 0 12 0.1	+ 3 0 1.7
Madras	+13 4 9.2	—10 29 8.2	— 5 20 57.0	— 80 14 15.0
Mannheim	+49 29 12.9	— 5 42 2.7	— 0 33 51.5	— 8 27 52.5
Markree	+54 10 31.7	— 4 34 22.8	+ 0 33 48.4	+ 8 27 6.0
Marseilles	+43 17 49.0	— 5 29 40.2	— 0 21 29.0	— 5 22 14.8
Milan	+45 28 0.7	— 5 44 57.8	— 0 36 46.6	— 9 11 39.6
Modena	+44 38 52.8	— 5 51 55.2	— 0 43 44.0	— 10 55 59.5
Moscow	+55 45 19.8	— 7 38 28.5	— 2 30 17.3	— 37 34 19.3
Munich	+48 8 45.0	— 5 54 37.6	— 0 46 26.4	— 11 36 36.6
Naples	+40 51 46.6	— 6 5 12.1	— 0 57 0.9	— 14 15 13.9
Olmutz	+49 35 40.0	— 6 17 11.3	— 1 9 0.1	— 17 15 1.5
Oxford	+51 45 36.0	— 5 3 8.6	+ 0 5 2.6	+ 1 15 39.0
Padua	+45 24 2.5	— 5 55 40.2	— 0 47 29.0	— 11 52 15.4
Palermo	+38 6 44.0	— 6 1 36.7	— 0 53 25.5	— 13 21 21.9
Paramatta	—33 48 49.8	+ 8 47 42.6	—10 4 6.2	—151 1 33.7
Paris	+48 50 13.2	— 5 17 32.7	— 0 9 21.5	— 2 20 21.9
Petersburgh	+59 56 29.7	— 7 9 24.7	— 2 1 13.5	— 30 18 22.2
Prague	+50 5 18.5	— 6 5 53.2	— 0 57 42.0	— 14 25 29.4
Pulkowa	+59 46 18.7	— 7 9 29.9	— 2 1 18.7	— 30 19 40.1
Rome	+41 53 54.0	— 5 58 5.9	— 0 49 54.7	— 12 28 40.5
San Fernando	+36 27 45.0	— 4 43 22.1	+ 0 24 49.1	+ 6 12 17.1
Santiago	—33 26 25.9	— 0 25 37.6	+ 4 42 33.6	+ 70 38 24.0
Senftenberg	+50 5 10.1	— 6 14 1.1	— 1 5 49.9	— 16 27 28.9
Vienna	+48 12 35.5	— 6 13 43.7	— 1 5 32.5	— 16 23 7.9
Wilna	+54 40 59.1	— 6 49 23.0	— 1 41 11.8	— 25 17 56.5

LATITUDES AND LONGITUDES OF PLACES IN THE UNITED STATES.

West Longitudes are considered as positive, East Longitudes as negative.

Place.	Latitude.			Longitude from Washington in Time.			Longitude from Greenwich in Time.			Longitude from Greenwich in Arc.		
	°	′	″	h.	m.	s.	h.	m.	s.	°	′	″
Northfield, Minn., College	44	27	41	+1	4	24.84	+6	12	36.93	+ 93	9	14
Brunswick, Me., College	43	54	29.0	−0	28	22.5	+4	39	49.6	+ 69	57	23.9
Hanover, N. H., Observ.	43	42	15	−0	19	4.13	+4	49	7.96	+ 72	16	59.4
Rochester, N. Y., Observ.	43	8	17	+0	3	12	+5	11	24	+ 77	51	0
Madison, Ind., Observ.	43	4	33	+0	49	24.1	+5	57	36.2	+ 89	24	3
Clinton, N. Y., Observ.	43	3	17.0	−0	6	34.65	+5	1	37.44	+ 75	24	21.6
Buffalo, N. Y., Observ.	42	54	9.5	+0	7	21.68	+5	15	33.77	+ 78	53	26.5
Troy, N. Y., Observ.	42	43	52	−0	13	27.5	+4	54	44.59	+ 73	41	8.8
Williamstown, Mass., Observ.	42	42	49	−0	15	18.6	+4	52	53.5	+ 73	13	22.5
Albany, N. Y., Observ.	42	39	49.5	−0	13	12.87	+4	54	59.22	+ 73	44	48.3
Cambridge, Mass., Observ.	42	22	48.3	−0	23	41.11	+4	44	30.98	+ 71	7	44.7
Amherst, Mass., Observ.	42	22	15.6	−0	18	4.8	+4	50	7.3	+ 72	31	49.5
Boston, Mass., State House	42	21	27.6	−0	23	57	+4	44	15.09	+ 71	3	46.3
Ann Arbor, Mich., Observ.	42	16	48.0	+0	26	43.10	+5	34	55.19	+ 83	43	47.8
Alfred, N. Y., Observ.	42	15	19.8	+0	2	55.00	+5	11	7.09	+ 77	46	46.3
Chicago, Ill., Observ.	41	50	1.0	+0	42	14.69	+5	50	26.78	+ 87	36	41.7
Providence, R. I., Observ.	41	49	46.4	−0	22	34.51	+4	45	37.58	+ 71	24	23.7
Poughkeepsie, N. Y., Observ.	41	41	18	−0	12	38.5	+4	55	33.6	+ 73	53	24.0
Middletown, Conn., College	41	33	8	−0	17	35	+4	50	37.09	+ 72	39	16.3
West Point, N. Y., Observ.	41	23	31	−0	12	22.71	+4	55	49.38	+ 73	57	20.7
New Haven, Conn., Observ.	41	18	36.5	−0	16	29.90	+4	51	42.19	+ 72	55	32.8
Hudson, O., Observ.	41	14	42.6	+0	17	32.06	+5	25	44.15	+ 81	26	2.2
Ogden, Utah, Observ.	41	13	8.6	+2	19	47.52	+7	27	59.61	+111	59	54.1
Hastings, N. Y., Observ.	40	59	25	−0	12	42.4	+4	55	29.7	+ 73	52	25.5
New York, Columbia College	40	45	23.1	−0	12	18.40	+4	55	53.69	+ 73	58	25.3
New York, Ruth. Observ.	40	43	48.5	−0	12	15.47	+4	55	56.62	+ 73	59	9.3
New York, City Hall	40	42	43.2	−0	12	11.0	+4	56	1.09	+ 74	0	16.3
Bethlehem, Pa., Observ.	40	36	23.9	−0	6	40.19	+5	1	31.90	+ 75	52	58.5
Allegheny, Pa., Observ.	40	27	41.6	+0	11	50.84	+5	20	2.93	+ 80	0	43.9
Princeton, N. J., Observ.	40	20	58	−0	9	34.6	+4	58	37.5	+ 74	39	22.5
Haverford, Pa., Observ.	40	0	36.5	−0	6	59.34	+5	1	12.75	+ 75	18	11.2
Philadelphia, Pa., Observ.	39	57	7.5	−0	7	33.64	+5	0	38.45	+ 75	9	36.7
Baltimore, Md., Wash. Mon.	39	17	47.8	−0	1	44.6	+5	6	27.49	+ 76	36	52.3
Glasgow, Mo., Observ.	39	16	16.8	+1	3	6.8	+6	11	18.9	+ 92	49	43.5
Cincinnati, O., New Observ.	39	8	35.5	+0	29	29.33	+5	37	41.42	+ 84	25	21.3
Cincinnati, O., Old Observ.	39	6	26.5	+0	29	46.85	+5	37	58.94	+ 84	29	44.1
Annapolis, Md., Observ.	38	58	53.5	−0	2	15.60	+5	5	56.49	+ 76	29	7.3
Columbia, Mo., Observ.	38	56		+1	1	6	+6	9	18.09	+ 92	19	31.3
Georgetown, D. C., Observ.	38	54	26.2	+0	0	6.20	+5	8	18.29	+ 77	4	34.3
Washington, D. C., Observ.	38	53	38.8	0	0	0	+5	8	12.09	+ 77	3	1.3
St. Louis, Mo., Observ.	38	38	3.64	+0	52	37.02	+6	0	49.11	+ 90	12	1.6
Charlottesville, Va., Univers.	38	2	3	+0	5	53.8	+5	14	5.9	+ 78	31	29
San Francisco, Cal.	37	48	23.6	+3	1	27.4	+8	9	39.49	+122	24	52.3
Monterey, Cal.	36	37	59.9	+2	59	26.3	+8	7	38.39	+121	54	35.8
Nashville, Tenn., University	36	9	33	+0	39	5.0	+5	47	17.09	+ 86	49	16.3
Chapel Hill, N. C.	35	54	21	+0	8	59	+5	17	11.09	+ 79	17	46.3
Santa Fé, New Mexico	35	41	6	+1	55	54	+7	3	6.09	+105	46	31.3
Oxford., Miss., Observ.	34	22	12.64	+0	49	55.03	+5	58	7.12	+ 89	31	46.8
Charleston, S. C.	32	47	5.3	+0	11	32.8	+5	19	44.89	+ 79	56	13.3
San Diego, Cal.	32	41	58.0	+2	40	42.3	+7	48	54.39	+117	13	35.8
Savannah, Ga., Exchange	32	4	53.4	+0	16	9.9	+5	24	21.99	+ 81	5	29.8
Mobile, Ala.	30	41	26.2	+0	43	54.7	+5	52	6.79	+ 88	1	41.8
New Orleans, La., City Hall	29	57	30	+0	51	47.91	+6	0	0	+ 90	0	0
Galveston, Tex., Court-house	29	18	14.5	+1	10	55.0	+6	19	7.09	+ 94	46	46.3

TABLE III.

To convert Hours, Minutes, and Seconds into Decimals of a Day.

Hours.	Decimal.	Min.	Decimal.	Sec.	Decimal.
1	.0416+	1	.000694+	1	.0000116
2	.0833+	2	.001388+	2	.0000231
3	.1250+	3	.002083+	3	.0000347
4	.1666+	4	.002777+	4	.0000463
5	.2083+	5	.003472+	5	.0000579
6	.2500+	6	.004166+	6	.0000694
7	.2916+	7	.004861+	7	.0000810
8	.3333+	8	.005555+	8	.0000925
9	.3750+	9	.006250+	9	.0001042
10	.4166+	10	.006944+	10	.0001157
11	.4583+	11	.007638+	11	.0001273
12	.5000+	12	.008333+	12	.0001389
13	.5416+	13	.009027+	13	.0001505
14	.5833+	14	.009722+	14	.0001620
15	.6250+	15	.010416+	15	.0001736
16	.6666+	16	.011111+	16	.0001852
17	.7083+	17	.011805+	17	.0001968
18	.7500+	18	.012500+	18	.0002083
19	.7916+	19	.013194+	19	.0002199
20	.8333+	20	.013888+	20	.0002315
21	.8750+	21	.014583+	21	.0002431
22	.9166+	22	.015277+	22	.0002546
23	.9583+	23	.015972+	23	.0002662
24	1.0000+	24	.016666+	24	.0002778
		25	.017361+	25	.0002894
		26	.018055+	26	.0003009
		27	.018750+	27	.0003125
		28	.019444+	28	.0003241
		29	.020138+	29	.0003356
		30	.020833+	30	.0003472
		31	.021527+	31	.0003588
		32	.022222+	32	.0003704
		33	.022916+	33	.0003819
		34	.023611+	34	.0003935
		35	.024305+	35	.0004051
		36	.025000+	36	.0004167
		37	.025694+	37	.0004282
		38	.026388+	38	.0004398
		39	.027083+	39	.0004514
		40	.027777+	40	.0004630
		41	.028472+	41	.0004745
		42	.029166+	42	.0004861
		43	.029861+	43	.0004977
		44	.030555+	44	.0005093
		45	.031250+	45	.0005208
		46	.031944+	46	.0005324
		47	.032638+	47	.0005440
		48	.033333+	48	.0005556
		49	.034027+	49	.0005671
		50	.034722+	50	.0005787
		51	.035416+	51	.0005903
		52	.036111+	52	.0006019
		53	.036805+	53	.0006134
		54	.037500+	54	.0006250
		55	.038194+	55	.0006366
		56	.038888+	56	.0006481
		57	.039583+	57	.0006597
		58	.040277+	58	.0006713
		59	.040972+	59	.0006829
		60	.041666+	60	.0006944

The sign +, appended to numbers in this table, signifies that the last figure repeats to infinity.

To convert Decimals of a Day into Hours, Minutes, and Seconds.

Dec.	H.	M.	S.	Dec.	H.	M.	S.
.01	0	14	24	.61	14	38	24
.02	0	28	48	.62	14	52	48
.03	0	43	12	.63	15	7	12
.04	0	57	36	.64	15	21	36
.05	1	12	0	.65	15	36	0
.06	1	26	24	.66	15	50	24
.07	1	40	48	.67	16	4	48
.08	1	55	12	.68	16	19	12
.09	2	9	36	.69	16	33	36
.10	2	24	0	.70	16	48	0
.11	2	38	24	.71	17	2	24
.12	2	52	48	.72	17	16	48
.13	3	7	12	.73	17	31	12
.14	3	21	36	.74	17	45	36
.15	3	36	0	.75	18	0	0
.16	3	50	24	.76	18	14	24
.17	4	4	48	.77	18	28	48
.18	4	19	12	.78	18	43	12
.19	4	33	36	.79	18	57	36
.20	4	48	0	.80	19	12	0
.21	5	2	24	.81	19	26	24
.22	5	16	48	.82	19	40	48
.23	5	31	12	.83	19	55	12
.24	5	45	36	.84	20	9	36
.25	6	0	0	.85	20	24	0
.26	6	14	24	.86	20	38	24
.27	6	28	48	.87	20	52	48
.28	6	43	12	.88	21	7	12
.29	6	57	36	.89	21	21	36
.30	7	12	0	.90	21	36	0
.31	7	26	24	.91	21	50	24
.32	7	40	48	.92	22	4	48
.33	7	55	12	.93	22	19	12
.34	8	9	36	.94	22	33	36
.35	8	24	0	.95	22	48	0
.36	8	38	24	.96	23	2	24
.37	8	52	48	.97	23	16	48
.38	9	7	12	.98	23	31	12
.39	9	21	36	.99	23	45	36
.40	9	36	0	.001	0	1	26.4
.41	9	50	24	.002		2	52.8
.42	10	4	48	.003		4	19.2
.43	10	19	12	.004		5	45.6
.44	10	33	36	.005		7	12.0
.45	10	48	0	.006		8	38.4
.46	11	2	24	.007		10	4.8
.47	11	16	48	.008		11	31.2
.48	11	31	12	.009		12	57.6
.49	11	45	36	.010		14	24.0
.50	12	0	0				
.51	12	14	24	.0001	0	0	8.64
.52	12	28	48	.0002		0	17.28
.53	12	43	12	.0003		0	25.92
.54	12	57	36	.0004		0	34.56
.55	13	12	0	.0005		0	43.20
.56	13	26	24	.0006		0	51.84
.57	13	40	48	.0007		1	0.48
.58	13	55	12	.0008		1	9.12
.59	14	9	36	.0009		1	17.76
.60	14	24	0	.0010		1	26.40

TABLE IV.

To convert intervals of Mean Solar Time into equivalent intervals of Sidereal Time.

HOURS.		MINUTES.		SECONDS.		FRACTIONS.	
Mean T.	Sidereal Time.	Mean T.	Sidereal Time.	Mean T.	Sidereal Time.	Mean T.	Sidereal Time.
h.	h. m. s.	m.	m. s.	s.	s.	s.	s.
1	1 0 9.856	1	1 0.164	1	1.003	0.02	0.020
2	2 0 19.713	2	2 0.329	2	2.005	0.04	0.040
3	3 0 29.569	3	3 0.493	3	3.008	0.06	0.060
4	4 0 39.426	4	4 0.657	4	4.011	0.08	0.080
5	5 0 49.282	5	5 0.821	5	5.014	0.10	0.100
6	6 0 59.139	6	6 0.986	6	6.016	0.12	0.120
7	7 1 8.995	7	7 1.150	7	7.019	0.14	0.140
8	8 1 18.852	8	8 1.314	8	8.022	0.16	0.160
9	9 1 28.708	9	9 1.478	9	9.025	0.18	0.180
10	10 1 38.565	10	10 1.643	10	10.027	0.20	0.201
11	11 1 48.421	11	11 1.807	11	11.030	0.22	0.221
12	12 1 58.278	12	12 1.971	12	12.033	0.24	0,241
13	13 2 8.134	13	13 2.136	13	13.036	0.26	0.261
14	14 2 17.991	14	14 2.300	14	14.038	0.28	0.281
15	15 2 27.847	15	15 2.464	15	15.041	0.30	0.301
16	16 2 37.704	16	16 2.628	16	16.044	0.32	0.321
17	17 2 47.560	17	17 2.793	17	17.047	0.34	0.341
18	18 2 57.417	18	18 2.957	18	18.049	0.36	0.361
19	19 3 7.273	19	19 3.121	19	19.052	0.38	0.381
20	20 3 17.129	20	20 3.285	20	20.055	0.40	0.401
21	21 3 26.986	21	21 3.450	21	21.057	0.42	0.421
22	22 3 36.842	22	22 3.614	22	22.060	0.44	0.441
23	23 3 46.699	23	23 3.778	23	23.063	0.46	0.461
24	24 3 56.555	24	24 3.943	24	24.066	0.48	0.481
		25	25 4.107	25	25.068	0.50	0.501
		26	26 4.271	26	26.071	0.52	0.521
		27	27 4.435	27	27.074	0.54	0.541
		28	28 4.600	28	28.077	0.56	0.562
		29	29 4.764	29	29.079	0.58	0.582
		30	30 4.928	30	30.082	0.60	0.602
		31	31 5.093	31	31.085	0.62	0.622
		32	32 5.257	32	32.088	0.64	0.642
		33	33 5.421	33	33.090	0.66	0.662
		34	34 5.585	34	34.093	0.68	0.682
		35	35 5.750	35	35.096	0.70	0.702
		36	36 5.914	36	36.099	0.72	0.722
		37	37 6.078	37	37.101	0.74	0.742
		38	38 6.242	38	38.104	0.76	0.762
		39	39 6.407	39	39.107	0.78	0.782
		40	40 6.571	40	40.110	0.80	0.802
		41	41 6.735	41	41.112	0,82	0.822
		42	42 6.900	42	42.115	0.84	0.842
		43	43 7.064	43	43.118	0.86	0.862
		44	44 7.228	44	44.120	0.88	0.882
		45	45 7.392	45	45.123	0.90	0.902
		46	46 7.557	46	46.126	0.92	0.923
		47	47 7.721	47	47.129	0.94	0.943
		48	48 7.885	48	48.131	0.96	0.963
		49	49 8.049	49	49.134	0.98	0.983
		50	50 8.214	50	50.137	1.00	1.003
		51	51 8.378	51	51.140		
		52	52 8.542	52	52.142		
		53	53 8.707	53	53.145		
		54	54 8.871	54	54.148		
		55	55 9.035	55	55.151		
		56	56 9.199	56	56.153		
		57	57 9.364	57	57.156		
		58	58 9.528	58	58.159		
		59	59 9.692	59	59.162		

This table is useful for the conversion of mean solar into sidereal time.
Sidereal time required = sidereal time at the preceding mean noon
+ the equivalent to the given mean time.
See Example, p. 123.

To convert intervals of Sidereal Time into equivalent intervals of Mean Solar Time.

HOURS.		MINUTES.		SECONDS.		FRACTIONS.	
Sider. T.	Mean Time.	Sider. T.	Mean Time.	Sider T.	Mean Time.	Sider. T.	Mean Time.
h.	h. m. s.	m.	m. s.	s.	s.	s.	s.
1	0 59 50.170	1	0 59.836	1	0.997	0.02	0.020
2	1 59 40.341	2	1 59.672	2	1.995	0.04	0.040
3	2 59 30.511	3	2 59.509	3	2.992	0.06	0.060
4	3 59 20.682	4	3 59.345	4	3.989	0.08	0.080
5	4 59 10.852	5	4 59.181	5	4.986	0.10	0.100
6	5 59 1.023	6	5 59.017	6	5.984	0.12	0.120
7	6 58 51.193	7	6 58.853	7	6.981	0.14	0.140
8	7 58 41.364	8	7 58.689	8	7.978	0.16	0.160
9	8 58 31.534	9	8 58.526	9	8.975	0.18	0.180
10	9 58 21.704	10	9 58.362	10	9.973	0.20	0.199
11	10 58 11.875	11	10 58.198	11	10.970	0.22	0.219
12	11 58 2.045	12	11 58.034	12	11.967	0.24	0.239
13	12 57 52.216	13	12 57.870	13	12.965	0.26	0.259
14	13 57 42.386	14	13 57.706	14	13.962	0.28	0.279
15	14 57 32.557	15	14 57.543	15	14.959	0.30	0.299
16	15 57 22.727	16	15 57.379	16	15.956	0.32	0.319
17	16 57 12.897	17	16 57.215	17	16.954	0.34	0.339
18	17 57 3.068	18	17 57.051	18	17.951	0.36	0.359
19	18 56 53.238	19	18 56.887	19	18.948	0.38	0.379
20	19 56 43.409	20	19 56.723	20	19.945	0.40	0.399
21	20 56 33.579	21	20 56.560	21	20.943	0.42	0.419
22	21 56 23.750	22	21 56.396	22	21.940	0.44	0.439
23	22 56 13.920	23	22 56.232	23	22.937	0.46	0.459
24	23 56 4.091	24	23 56.068	24	23.934	0.48	0.479
		25	24 55.904	25	24.932	0.50	0.499
		26	25 55.741	26	25.929	0.52	0.519
		27	26 55.577	27	26.926	0.54	0.539
		28	27 55.413	28	27.924	0.56	0.558
		29	28 55.249	29	28.921	0.58	0.578
		30	29 55.085	30	29.918	0.60	0.598
		31	30 54.921	31	30.915	0.62	0.618
		32	31 54.758	32	31.913	0.64	0.638
		33	32 54.594	33	32.910	0.66	0.658
		34	33 54.430	34	33.907	0.68	0.678
		35	34 54.266	35	34.904	0.70	0.698
		36	35 54.102	36	35.902	0.72	0.718
		37	36 53.938	37	36.899	0.74	0.738
		38	37 53.775	38	37.896	0.76	0.758
		39	38 53.611	39	38.894	0.78	0.778
		40	39 53.447	40	39.891	0.80	0.798
		41	40 53.283	41	40.888	0.82	0.818
		42	41 53.119	42	41.885	0.84	0.838
		43	42 52.955	43	42.883	0.86	0.858
		44	43 52.792	44	43.880	0.88	0.878
		45	44 52.628	45	44.877	0.90	0.898
		46	45 52.464	46	45.874	0.92	0.917
		47	46 52.300	47	46.872	0.94	0.937
		48	47 52.136	48	47.869	0.96	0.957
		49	48 51.973	49	48.866	0.98	0.977
		50	49 51.809	50	49.863	1.00	0.997
		51	50 51.645	51	50.861		
		52	51 51.481	52	51.858		
		53	52 51.317	53	52.855		
		54	53 51.153	54	53.853		
		55	54 50.990	55	54.850		
		56	55 50.826	56	55.847		
		57	56 50.662	57	56.844		
		58	57 50.498	58	57.842		
		59	58 50.334	59	58.839		

This table is useful for the conversion of sidereal into mean solar time.
Mean solar time required = mean time at the preceding sidereal noon
+the equivalent to the given sidereal time.
See Example, page 125.

To convert Degrees into Sidereal Time.

Arc.	Time.		Arc.	Time.		Arc.	Time.		Arc.	Time.		Arc.	Time.		Arc.	Time.		Arc.	Time.		Arc.	Time.
°	h.	m.	°	h.	m.	°	h.	m.	°	h.	m.	°	h.	m.	°	h.	m.	′	m.	s.	″	s.
1	0	4	61	4	4	121	8	4	181	12	4	241	16	4	301	20	4	1	0	4	1	0.067
2	0	8	62	4	8	122	8	8	182	12	8	242	16	8	302	20	8	2	0	8	2	0.133
3	0	12	63	4	12	123	8	12	183	12	12	243	16	12	303	20	12	3	0	12	3	0.200
4	0	16	64	4	16	124	8	16	184	12	16	244	16	16	304	20	16	4	0	16	4	0.267
5	0	20	65	4	20	125	8	20	185	12	20	245	16	20	305	20	20	5	0	20	5	0.333
6	0	24	66	4	24	126	8	24	186	12	24	246	16	24	306	20	24	6	0	24	6	0.400
7	0	28	67	4	28	127	8	28	187	12	28	247	16	28	307	20	28	7	0	28	7	0.467
8	0	32	68	4	32	128	8	32	188	12	32	248	16	32	308	20	32	8	0	32	8	0.533
9	0	36	69	4	36	129	8	36	189	12	36	249	16	36	309	20	36	9	0	36	9	0.600
10	0	40	70	4	40	130	8	40	190	12	40	250	16	40	310	20	40	10	0	40	10	0.667
11	0	44	71	4	44	131	8	44	191	12	44	251	16	44	311	20	44	11	0	44	11	0.733
12	0	48	72	4	48	132	8	48	192	12	48	252	16	48	312	20	48	12	0	48	12	0.800
13	0	52	73	4	52	133	8	52	193	12	52	253	16	52	313	20	52	13	0	52	13	0.867
14	0	56	74	4	56	134	8	56	194	12	56	254	16	56	314	20	56	14	0	56	14	0.933
15	1	0	75	5	0	135	9	0	195	13	0	255	17	0	315	21	0	15	1	0	15	1.000
16	1	4	76	5	4	136	9	4	196	13	4	256	17	4	316	21	4	16	1	4	16	1.067
17	1	8	77	5	8	137	9	8	197	13	8	257	17	8	317	21	8	17	1	8	17	1.133
18	1	12	78	5	12	138	9	12	198	13	12	258	17	12	318	21	12	18	1	12	18	1.200
19	1	16	79	5	16	139	9	16	199	13	16	259	17	16	319	21	16	19	1	16	19	1.267
20	1	20	80	5	20	140	9	20	200	13	20	260	17	20	320	21	20	20	1	20	20	1.333
21	1	24	81	5	24	141	9	24	201	13	24	261	17	24	321	21	24	21	1	24	21	1.400
22	1	28	82	5	28	142	9	28	202	13	28	262	17	28	322	21	28	22	1	28	22	1.467
23	1	32	83	5	32	143	9	32	203	13	32	263	17	32	323	21	32	23	1	32	23	1.533
24	1	36	84	5	36	144	9	36	204	13	36	264	17	36	324	21	36	24	1	36	24	1.600
25	1	40	85	5	40	145	9	40	205	13	40	265	17	40	325	21	40	25	1	40	25	1.667
26	1	44	86	5	44	146	9	44	206	13	44	266	17	44	326	21	44	26	1	44	26	1.733
27	1	48	87	5	48	147	9	48	207	13	48	267	17	48	327	21	48	27	1	48	27	1.800
28	1	52	88	5	52	148	9	52	208	13	52	268	17	52	328	21	52	28	1	52	28	1.867
29	1	56	89	5	56	149	9	56	209	13	56	269	17	56	329	21	56	29	1	56	29	1.933
30	2	0	90	6	0	150	10	0	210	14	0	270	18	0	330	22	0	30	2	0	30	2.000
31	2	4	91	6	4	151	10	4	211	14	4	271	18	4	331	22	4	31	2	4	31	2.067
32	2	8	92	6	8	152	10	8	212	14	8	272	18	8	332	22	8	32	2	8	32	2.133
33	2	12	93	6	12	153	10	12	213	14	12	273	18	12	333	22	12	33	2	12	33	2.200
34	2	16	94	6	16	154	10	16	214	14	16	274	18	16	334	22	16	34	2	16	34	2.267
35	2	20	95	6	20	155	10	20	215	14	20	275	18	20	335	22	20	35	2	20	35	2.333
36	2	24	96	6	24	156	10	24	216	14	24	276	18	24	336	22	24	36	2	24	36	2.400
37	2	28	97	6	28	157	10	28	217	14	28	277	18	28	337	22	28	37	2	28	37	2.467
38	2	32	98	6	32	158	10	32	218	14	32	278	18	32	338	22	32	38	2	32	38	2.533
39	2	36	99	6	36	159	10	36	219	14	36	279	18	36	339	22	36	39	2	36	39	2.600
40	2	40	100	6	40	160	10	40	220	14	40	280	18	40	340	22	40	40	2	40	40	2.667
41	2	44	101	6	44	161	10	44	221	14	44	281	18	44	341	22	44	41	2	44	41	2.733
42	2	48	102	6	48	162	10	48	222	14	48	282	18	48	342	22	48	42	2	48	42	2.800
43	2	52	103	6	52	163	10	52	223	14	52	283	18	52	343	22	52	43	2	52	43	2.867
44	2	56	104	6	56	164	10	56	224	14	56	284	18	56	344	22	56	44	2	56	44	2.933
45	3	0	105	7	0	165	11	0	225	15	0	285	19	0	345	23	0	45	3	0	45	3.000
46	3	4	106	7	4	166	11	4	226	15	4	286	19	4	346	23	4	46	3	4	46	3.067
47	3	8	107	7	8	167	11	8	227	15	8	287	19	8	347	23	8	47	3	8	47	3.133
48	3	12	108	7	12	168	11	12	228	15	12	288	19	12	348	23	12	48	3	12	48	3.200
49	3	16	109	7	16	169	11	16	229	15	16	289	19	16	349	23	16	49	3	16	49	3.267
50	3	20	110	7	20	170	11	20	230	15	20	290	19	20	350	23	20	50	3	20	50	3.333
51	3	24	111	7	24	171	11	24	231	15	24	291	19	24	351	23	24	51	3	24	51	3.400
52	3	28	112	7	28	172	11	28	232	15	28	292	19	28	352	23	28	52	3	28	52	3.467
53	3	32	113	7	32	173	11	32	233	15	32	293	19	32	353	23	32	53	3	32	53	3.533
54	3	36	114	7	36	174	11	36	234	15	36	294	19	36	354	23	36	54	3	36	54	3.600
55	3	40	115	7	40	175	11	40	235	15	40	295	19	40	355	23	40	55	3	40	55	3.667
56	3	44	116	7	44	176	11	44	236	15	44	296	19	44	356	23	44	56	3	44	56	3.733
	3	48	117	7	48	177	11	48	237	15	48	297	19	48	357	23	48	57	3	48	57	3.800
		52	118	7	52	178	11	52	238	15	52	298	19	52	358	23	52	58	3	52	58	3.867
		56	119	7	56	179	11	56	239	15	56	299	19	56	359	23	56	59	3	56	59	3.933
		0	120	8	0	180	12	0	240	16	0	300	20	0	360	24	0	60	4	0	60	4.000

To convert Sidereal Time into Degrees.

Time.	Arc.	Time.	Arc.	Time.	Arc.	Time.	Arc.	Time.	Arc.
h.	°	m.	° '	s.	' "	s.	"	s.	"
1	15	1	0 15	1	0 15	0.01	0.15	0.60	9.00
2	30	2	0 30	2	0 30	0.02	0.30	0.61	9.15
3	45	3	0 45	3	0 45	0.03	0.45	0.62	9.30
4	60	4	1 0	4	1 0	0.04	0.60	0.63	9.45
5	75	5	1 15	5	1 15	0.05	0.75	0.64	9.60
6	90	6	1 30	6	1 30	0.06	0.90	0.65	9.75
7	105	7	1 45	7	1 45	0.07	1.05	0.66	9.90
8	120	8	2 0	8	2 0	0.08	1.20	0.67	10.05
9	135	9	2 15	9	2 15	0.09	1.35	0.68	10.20
10	150	10	2 30	10	2 30	0.10	1.50	0.69	10.35
11	165	11	2 45	11	2 45	0.11	1.65	0.70	10.50
12	180	12	3 0	12	3 0	0.12	1.80	0.71	10.65
13	195	13	3 15	13	3 15	0.13	1.95	0.72	10.80
14	210	14	3 30	14	3 30	0.14	2.10	0.73	10.95
15	225	15	3 45	15	3 45	0.15	2.25	0.74	11.10
16	240	16	4 0	16	4 0	0.16	2.40	0.75	11.25
17	255	17	4 15	17	4 15	0.17	2.55	0.76	11.40
18	270	18	4 30	18	4 30	0.18	2.70	0.77	11.55
19	285	19	4 45	19	4 45	0.19	2.85	0.78	11.70
20	300	20	5 0	20	5 0	0.20	3.00	0.79	11.85
21	315	21	5 15	21	5 15	0.21	3.15	0.80	12.00
22	330	22	5 30	22	5 30	0.22	3.30	0.81	12.15
23	345	23	5 45	23	5 45	0.23	3.45	0.82	12.30
24	360	24	6 0	24	6 0	0.24	3.60	0.83	12.45
		25	6 15	25	6 15	0.25	3.75	0.84	12.60
		26	6 30	26	6 30	0.26	3.90	0.85	12.75
		27	6 45	27	6 45	0.27	4.05	0.86	12.90
		28	7 0	28	7 0	0.28	4.20	0.87	13.05
		29	7 15	29	7 15	0.29	4.35	0.88	13.20
		30	7 30	30	7 30	0.30	4.50	0.89	13.35
		31	7 45	31	7 45	0.31	4.65	0.90	13.50
		32	8 0	32	8 0	0.32	4.80	0.91	13.65
		33	8 15	33	8 15	0.33	4.95	0.92	13.80
		34	8 30	34	8 30	0.34	5.10	0.93	13.95
		35	8 45	35	8 45	0.35	5.25	0.94	14.10
		36	9 0	36	9 0	0.36	5.40	0.95	14.25
		37	9 15	37	9 15	0.37	5.55	0.96	14.40
		38	9 30	38	9 30	0.38	5.70	0.97	14.55
		39	9 45	39	9 45	0.39	5.85	0.98	14.70
		40	10 0	40	10 0	0.40	6.00	0.99	14.85
		41	10 15	41	10 15	0.41	6.15	1.00	15.00
		42	10 30	42	10 30	0.42	6.30		
		43	10 45	43	10 45	0.43	6.45		
		44	11 0	44	11 0	0.44	6.60		
		45	11 15	45	11 15	0.45	6.75		
		46	11 30	46	11 30	0.46	6.90		
		47	11 45	47	11 45	0.47	7.05		
		48	12 0	48	12 0	0.48	7.20	Thous. of seconds of Time.	Arc.
		49	12 15	49	12 15	0.49	7.35		
		50	12 30	50	12 30	0.50	7.50		"
		51	12 45	51	12 45	0.51	7.65	0.001	0.015
		52	13 0	52	13 0	0.52	7.80	.002	0.030
		53	13 15	53	13 15	0.53	7.95	.003	0.045
		54	13 30	54	13 30	0.54	8.10	.004	0.060
		55	13 45	55	13 45	0.55	8.25	.005	0.075
		56	14 0	56	14 0	0.56	8.40	.006	0.090
		57	14 15	57	14 15	0.57	8.55	.007	0.105
		58	14 30	58	14 30	0.58	8.70	.008	0.120
		59	14 45	59	14 45	0.59	8.85	.009	0.135
		60	15 0	60	15 0	0.60	9.00	.010	0.150

TABLE VIII.—BESSEL'S REFRACTIONS.

App. Alt.	Mean Refract	Differ. for 10′.	Log. A.	Diff.	M.	N.	App. Alt.	Mean Refract.	Diff.	Log. A.	M.	N
° ′	′ ″	″					° ′	′ ″	″			
0 0	34 54.1	124.9			1.1059	1.7344	10 0	5 16.2	5.0	1.74623	1.0041	1.04
10	32 49.2	116.9	0.75803		1.0952	1.6767	10	5 11.2	4.8	1.74670	1.0040	1.04
20	30 52.3	108.8	1.03248		1.0860	1.6252	20	5 6.4	4.7	1.74714	1.0039	1.03
30	29 3.5	100.8	1.18228	9909	1.0780	1.5789	30	5 1.7	4.5	1.74757	1.0038	1.03
40	27 22.7	92.9	1.28137	7163	1.0710	1.5373	40	4 57.2	4.4	1.74799	1.0037	1.03
50	25 49.8	85.2	1.35300	5464	1.0648	1.4995	50	4 52.8	4.3	1.74839	1.0036	1.03
1 0	24 24.6	77.9	1.40764	4322	1.0593	1.4653	11 0	4 48.5	4.2	1.74876	1.0035	1.03
10	23 6.7	71.1	1.45086	3516	1.0546	1.4341	10	4 44.3	4.1	1.74912	1.0034	1.03
20	21 55.6	64.7	1.48602	2928	1.0505	1.4057	20	4 40.2	3.9	1.74947	1.0033	1.03
30	20 50.9	59.0	1.51530	2480	1.0465	1.3797	30	4 36.3	3.9	1.74981	1.0032	1.03
40	19 51.9	53.9	1.54010	2132	1.0429	1.3560	40	4 32.4	3.7	1.75013	1.0031	1.03
50	18 58.0	49.4	1.56142	1853	1.0397	1.3342	50	4 28.7	3.7	1.75043	1.0030	1.03
2 0	18 8.6	45.6	1.57995	1623	1.0368	1.3141	12 0	4 25.0	3.6	1.75072	1.0030	1.02
10	17 23.0	42.3	1.59618	1423	1.0342	1.2955	10	4 21.4	3.4	1.75101	1.0029	1.02
20	16 40.7	39.8	1.61041	1237	1.0318	1.2783	20	4 18.0	3.4	1.75129	1.0028	1.02
30	16 0.9	37.5	1.62278	1075	1.0298	1.2624	30	4 14.6	3.3	1.75155	1.0027	1.02
40	15 23.4	35.6	1.63353	933	1.0278	1.2477	40	4 11.3	3.3	1.75180	1.0027	1.02
50	14 47.8	33.2	1.64286	828	1.0261	1.2341	50	4 8.0	3.1	1.75205	1.0026	1.02
3 0	14 14.6	30.9	1.65114	755	1.0244	1.2215	13 0	4 4.9	3.1	1.75229	1.0026	1.02
10	13 43.7	28.7	1.65869	691	1.0230	1.2098	10	4 1.8	3.0	1.75252	1.0026	1.02
20	13 15.0	26.7	1.66560	644	1.0216	1.1989	20	3 58.8	2.9	1.75274	1.0025	1.02
30	12 48.3	24.6	1.67204	609	1.0204	1.1888	30	3 55.9	2.9	1.75295	1.0024	1.02
40	12 23.7	23.0	1.67813	570	1.0192	1.1794	40	3 53.0	2.8	1.75316	1.0024	1.02
50	12 0.7	21.8	1.68383	525	1.0182	1.1706	50	3 50.2	2.8	1.75336	1.0023	1.02
4 0	11 38.9	20.6	1.68908	476	1.0172	1.1624	14 0	3 47.4	2.7	1.75355		1.02
10	11 18.3	19.7	1.69384	432	1.0163	1.1549	10	3 44.7	2.6	1.75373		1.02
20	10 58.6	19.0	1.69816	372	1.0155	1.1478	20	3 42.1	2.6	1.75391		1.02
30	10 39.6	18.4	1.70188	317	1.0147	1.1408	30	3 39.5	2.5	1.75408		1.02
40	10 21.2	17.9	1.70505	267	1.0140	1.1342	40	3 37.0	2.5	1.75425		1.02
50	10 3.3	16.8	1.70772	248	1.0133	1.1283	50	3 34.5	2.4	1.75441		1.02
5 0	9 46.5	15.6	1.71020	259	1.0127	1.1229	15 0	3 32.1	13.5	1.75457		1.01
10	9 30.9	14.9	1.71279	243	1.0121	1.1178	16	3 18.6	12.0	1.75543		1.01
20	9 16.0	14.1	1.71522	227	1.0115	1.1130	17	3 6.6	10.8	1.75615		1.01
30	9 1.9	13.5	1.71749	212	1.0110	1.1082	18	2 55.8	9.7	1.75675		1.01
40	8 48.4	12.8	1.71961	199	1.0105	1.1036	19	2 46.1	8.8	1.75726		1.01
50	8 35.6	12.3	1.72160	186	1.0100	1.0992	20	2 37.3	8.0	1.75771		1.01
6 0	8 23.3	11.7	1.72346	173	1.0096	1.0951	21	2 29.3	7.4	1.75809		1.01
10	8 11.6	11.3	1.72519	162	1.0092	1.0914	22	2 21.9	6.7	1.75842		1.00
20	8 0.3	10.8	1.72681	151	1.0088	1.0879	23	2 15.2	6.3	1.75871		1.00
30	7 49.5	10.3	1.72832	142	1.0084	1.0846	24	2 8.9	5.7	1.75897		1.00
40	7 39.2	10.0	1.72974	131	1.0081	1.0815	25	2 3.2	5.4	1.75919		1.00
50	7 29.2	9.5	1.73105	124	1.0078	1.0784	26	1 57.8	5.0	1.75939		1.00
7 0	7 19.7	9.2	1.73229	118	1.0075	1.0754	27	1 52.8	4.6	1.75957		1.00
10	7 10.5	8.8	1.73347	112	1.0073	1.0725	28	1 48.2	4.4	1.75973		1.00
20	7 1.7	8.4	1.73459	105	1.0070	1.0697	29	1 43.8	4.1	1.75988		1.00
30	6 53.3	8.2	1.73564	99	1.0067	1.0671	30	1 39.7	3.9	1.76001		1.00
40	6 45.1	7.9	1.73663	94	1.0065	1.0646	31	1 35.8	3.7	1.76012		1.00
50	6 37.2	7.6	1.73757	88	1.0062	1.0622	32	1 32.1	3.4	1.76023		1.00
8 0	6 29.6	7.3	1.73845	83	1.0060	1.0600	33	1 28.7	3.3	1.76033		1.00
10	6 22.3	7.1	1.73928	79	1.0058	1.0579	34	1 25.4	3.1	1.76042		1.00
20	6 15.2	6.8	1.74007	76	1.0056	1.0559	35	1 22.3	3.0	1.76050		1.00
30	6 8.4	6.6	1.74083	72	1.0054	1.0540	36	1 19.3	2.8	1.76058		1.00
40	6 1.8	6.4	1.74155	68	1.0052	1.0523	37	1 16.5	2.7	1.76065		1.00
50	5 55.4	6.1	1.74223	65	1.0050	1.0508	38	1 13.8	2.6	1.76071		1.00
9 0	5 49.3	6.0	1.74288	64	1.0049	1.0493	39	1 11.2	2.5	1.76077		1.00
10	5 43.3	5.7	1.74352	60	1.0047	1.0479	40	1 8.7	2.4	1.76082		1.00
20	5 37.6	5.6	1.74412	56	1.0046	1.0466	41	1 6.3	2.3	1.76087		1.00
30	5 32.0	5.5	1.74468	53	1.0045	1.0454	42	1 4.0	2.2	1.76092		1.00
40	5 26.5	5.2	1.74521	52	1.0043	1.0442	43	1 1.8	2.1	1.76096		1.00
50	5 21.3	5.1	1.74573	50	1.0042	1.0431	44	0 59.7	2.0	1.76100		1.00

App. Alt.	Mean Ref.	Diff.	Log. A.
°	″	″	
45	57.7	2.0	1.76104
46	55.7	1.9	1.76107
47	53.8	1.9	1.76111
48	51.9	1.7	1.76114
49	50.2	1.8	1.76117
50	48.4	1.7	1.76119
51	46.7	1.6	1.76122
52	45.1	1.6	1.76124
53	43.5	1.6	1.76126
54	41.9	1.5	1.76128
55	40.4	1.5	1.76130
56	38.9	1.4	1.76132
57	37.5	1.4	1.76134
58	36.1	1.4	1.76136
59	34.7	1.4	1.76138
60	33.3	1.3	1.76139
61	32.0	1.3	1.76140
62	30.7	1.3	1.76142
63	29.4	1.2	1.76143
64	28.2	1.3	1.76144
65	26.9	1.2	1.76145
66	25.7	1.2	1.76146
67	24.5	1.2	1.76147
68	23.3	1.1	1.76148
69	22.2	1.2	1.76148
70	21.0	1.1	1.76149
71	19.9	1.1	1.76150
72	18.8	1.1	1.76150
73	17.7	1.1	1.76151
74	16.6	1.1	1.76151
75	15.5	5.3	1.76152
80	10.2	5.1	1.76154
85	5.1	5.1	1.76156
90	0.0		1.76156

Factor B, depending on the Barometer.		
Eng. In.	B.	Log. B.
27.9	0.943	—0.02564
28.0	0.946	—0.02409
28.1	0.949	—0.02254
28.2	0.953	—0.02099
28.3	0.956	—0.01946
28.4	0.960	—0.01793
28.5	0.963	—0.01640
28.6	0.966	—0.01488
28.7	0.970	—0.01336
28.8	0.973	—0.01185
28.9	0.976	—0.01035
29.0	0.980	—0.00885
29.1	0.983	—0.00735
29.2	0.987	—0.00586
29.3	0.990	—0.00438
29.4	0.993	—0.00290
29.5	0.997	—0.00142
29.6	1.000	+0.00005
29.7	1.003	+0.00151
29.8	1.007	+0.00297
29.9	1.010	+0.00443
30.0	1.014	+0.00588
30.1	1.017	+0.00732
30.2	1.020	+0.00876
30.3	1.024	+0.01020
30.4	1.027	+0.01163
30.5	1.031	+0.01306
30.6	1.034	+0.01448
30.7	1.037	+0.01589
30.8	1.041	+0.01731
30.9	1.044	+0.01871
31.0	1.047	+0.02012

Factor *t*, depending on the attached Thermometer. Fahrenheit.					
Fahr. Deg.	*t*.	Log. *t*.	Ft. Deg.	*t*.	Log. *t*.
°			°		
—20	1.005	+0.00203	40	0.999	—0.00031
—15	1.004	+0.00183	45	0.999	—0.00050
—10	1.004	+0.00164	50	0.998	—0.00070
— 5	1.003	+0.00144	55	0.998	—0.00089
0	1.003	+0.00125	60	0.997	—0.00109
+ 5	1.003	+0.00105	65	0.997	—0.00128
+10	1.002	+0.00086	70	0.997	—0.00148
+15	1.002	+0.00066	75	0.996	—0.00167
+20	1.001	+0.00047	80	0.996	—0.00186
+25	1.001	+0.00027	85	0.995	—0.00205
+30	1.000	+0.00008	90	0.995	—0.00225
+35	1.000	—0.00011	95	0.994	—0.00244

Log. Refraction=log. cotang. App. Alt.+log. A +M(log. B+log. *t*)+N log. T.

True Refraction=Mean Refraction×B×*t*×T.

Factor T, depending on the external Thermometer. Fahrenheit.					
Fahr. Deg.	T.	Log. T.	Fahr. Deg.	T.	Log. T.
°			°		
—20	1.156	+0.06279	38	1.022	+0.00924
19	1.153	+0.06181	39	1.019	+0.00837
18	1.150	+0.06083	40	1.017	+0.00750
17	1.148	+0.05985	41	1.015	+0.00664
16	1.145	+0.05887	42	1.013	+0.00578
15	1.143	+0.05790	43	1.011	+0.00492
14	1.140	+0.05693	44	1.009	+0.00406
13	1.138	+0.05596	45	1.007	+0.00320
12	1.135	+0.05500	46	1.005	+0.00234
11	1.132	+0.05403	47	1.003	+0.00149
10	1.130	+0.05307	48	1.001	+0.00064
9	1.128	+0.05211	49	1.000	—0.00021
8	1.125	+0.05115	50	0.998	—0.00106
7	1.123	+0.05020	51	0.996	—0.00191
6	1.120	+0.04924	52	0.994	—0.00275
5	1.118	+0.04829	53	0.992	—0.00360
4	1.115	+0.04734	54	0.990	—0.00444
3	1.113	+0.04640	55	0.988	—0.00528
2	1.110	+0.04545	56	0.986	—0.00612
— 1	1.108	+0.04451	57	0.984	—0.00698
0	1.106	+0.04357	58	0.982	—0.00780
+ 1	1.103	+0.04263	59	0.980	—0.00863
2	1.101	+0.04169	60	0.978	—0.00946
3	1.098	+0.04076	61	0.977	—0.01029
4	1.096	+0.03982	62	0.975	—0.01112
5	1.094	+0.03889	63	0.973	—0.01195
6	1.091	+0.03796	64	0.971	—0.01278
7	1.089	+0.03704	65	0.969	—0.01360
8	1.087	+0.03611	66	0.967	—0.01443
9	1.084	+0.03519	67	0.965	—0.01525
10	1.082	+0.03427	68	0.964	—0.01607
11	1.080	+0.03335	69	0.962	—0.01689
12	1.078	+0.03243	70	0.960	—0.01770
13	1.075	+0.03152	71	0.958	—0.01852
14	1.073	+0.03060	72	0.956	—0.01933
15	1.071	+0.02969	73	0.955	—0.02015
16	1.069	+0.02878	74	0.953	—0.02096
17	1.066	+0.02787	75	0.951	—0.02177
18	1.064	+0.02697	76	0.949	—0.02257
19	1.062	+0.02606	77	0.948	—0.02338
20	1.060	+0.02514	78	0.946	—0.02419
21	1.057	+0.02426	79	0.944	—0.02499
22	1.055	+0.02336	80	0.942	—0.02579
23	1.053	+0.02247	81	0.941	—0.02659
24	1.051	+0.02157	82	0.939	—0.02738
25	1.049	+0.02068	83	0.937	—0.02819
26	1.047	+0.01979	84	0.935	—0.02898
27	1.044	+0.01890	85	0.934	—0.02978
28	1.042	+0.01801	86	0.932	—0.03057
29	1.040	+0.01713	87	0.930	—0.03136
30	1.038	+0.01624	88	0.929	—0.03216
31	1.036	+0.01536	89	0.927	—0.03294
32	1.034	+0.01448	90	0.925	—0.03373
33	1.032	+0.01360	91	0.924	—0.03452
34	1.030	+0.01273	92	0.922	—0.03530
35	1.028	+0.01185	93	0.920	—0.03609
36	1.026	+0.01098	94	0.919	—0.03687
+37	1.024	+0.01011	95	0.917	—0.03765

N.P.D.	Azimuth.	Diff.	Level.	Diff.	Collim.	N.P.D.	Azimuth.	Diff.	Level.	Diff.	Collim.
°						°					
140	+1.555		+0.030		+1.556	80	+0.491		+0.889		+1.015
139	+1.523	.032	+0.056	.026	+1.524	79	+0.477	.014	+0.900	.011	+1.019
138	+1.492	.031	+0.081	.025	+1.494	78	+0.462	.015	+0.912	.012	+1.022
137	+1.463	.029	+0.105	.024	+1.466	77	+0.448	.014	+0.923	.011	+1.026
136	+1.434	.029	+0.128	.023	+1.440	76	+0.434	.014	+0.935	.012	+1.031
135	+1.406	.028	+0.150	.022	+1.414	75	+0.419	.015	+0.947	.012	+1.035
134	+1.379	.027	+0.172	.022	+1.390	74	+0.405	.014	+0.958	.011	+1.040
133	+1.354	.025	+0.193	.021	+1.367	73	+0.390	.015	+0.970	.012	+1.046
132	+1.329	.025	+0.213	.020	+1.346	72	+0.375	.015	+0.982	.012	+1.051
131	+1.304	.025	+0.232	.019	+1.325	71	+0.360	.015	+0.994	.012	+1.058
130	+1.281	.023	+0.251	.019	+1.305	70	+0.345	.015	+1.007	.013	+1.064
129	+1.258	.023	+0.270	.019	+1.287	69	+0.329	.016	+1.019	.012	+1.071
128	+1.236	.022	+0.288	.018	+1.269	68	+0.313	.016	+1.032	.013	+1.079
127	+1.214	.022	+0.305	.017	+1.252	67	+0.298	.015	+1.045	.013	+1.086
126	+1.193	.021	+0.322	.017	+1.236	66	+0.281	.017	+1.058	.013	+1.095
125	+1.173	.020	+0.339	.017	+1.221	65	+0.265	.016	+1.071	.013	+1.103
124	+1.153	.020	+0.355	.016	+1.206	64	+0.248	.017	+1.085	.014	+1.113
123	+1.133	.020	+0.371	.016	+1.192	63	+0.231	.017	+1.098	.013	+1.122
122	+1.114	.019	+0.386	.015	+1.179	62	+0.214	.017	+1.112	.014	+1.133
121	+1.096	.018	+0.401	.015	+1.167	61	+0.196	.018	+1.126	.014	+1.143
120	+1.077	.019	+0.416	.015	+1.155	60	+0.179	.017	+1.141	.015	+1.155
119	+1.059	.018	+0.430	.014	+1.143	59	+0.160	.019	+1.156	.015	+1.167
118	+1.042	.017	+0.444	.014	+1.133	58	+0.142	.018	+1.171	.015	+1.179
117	+1.024	.018	+0.458	.014	+1.122	57	+0.122	.020	+1.186	.015	+1.192
116	+1.007	.017	+0.472	.014	+1.113	56	+0.103	.019	+1.202	.016	+1.206
115	+0.991	.016	+0.486	.014	+1.103	55	+0.083	.020	+1.218	.016	+1.221
114	+0.974	.017	+0.499	.013	+1.095	54	+0.062	.021	+1.234	.016	+1.236
113	+0.958	.016	+0.512	.013	+1.086	53	+0.041	.021	+1.252	.018	+1.252
112	+0.942	.016	+0.525	.013	+1.079	52	+0.020	.021	+1.269	.017	+1.269
111	+0.927	.015	+0.537	.012	+1.071	51	—0.002	.022	+1.287	.018	+1.287
110	+0.911	.016	+0.550	.013	+1.064	50	—0.025	.023	+1.305	.018	+1.305
109	+0.896	.015	+0.562	.012	+1.058	49	—0.049	.024	+1.324	.019	+1.325
108	+0.881	.015	+0.574	.012	+1.051	48	—0.073	.024	+1.344	.020	+1.346
107	+0.866	.015	+0.586	.012	+1.046	47	—0.098	.025	+1.364	.020	+1.367
106	+0.851	.015	+0.599	.013	+1.040	46	—0.124	.026	+1.385	.021	+1.390
105	+0.836	.015	+0.610	.011	+1.035	45	—0.150	.026	+1.406	.021	+1.414
104	+0.822	.014	+0.622	.012	+1.031	44	—0.178	.028	+1.428	.022	+1.440
103	+0.808	.014	+0.633	.011	+1.026	43	—0.207	.029	+1.452	.024	+1.466
102	+0.793	.015	+0.645	.012	+1.022	42	—0.237	.030	+1.476	.024	+1.494
101	+0.779	.014	+0.656	.011	+1.019	41	—0.267	.030	+1.501	.025	+1.524
100	+0.765	.014	+0.668	.012	+1.015	40	—0.300	.033	+1.527	.026	+1.556
99	+0.751	.014	+0.679	.011	+1.012	39	—0.333	.033	+1.554	.027	+1.589
98	+0.737	.014	+0.690	.011	+1.010	38	—0.368	.035	+1.582	.028	+1.624
97	+0.723	.014	+0.701	.011	+1.008	37	—0.405	.037	+1.612	.030	+1.662
96	+0.710	.013	+0.712	.011	+1.006	36	—0.443	.038	+1.643	.031	+1.701
95	+0.696	.014	+0.723	.011	+1.004	35	—0.484	.041	+1.675	.032	+1.743
94	+0.682	.014	+0.734	.011	+1.002	34	—0.526	.042	+1.709	.034	+1.788
93	+0.669	.013	+0.745	.011	+1.001	33	—0.571	.045	+1.745	.036	+1.836
92	+0.655	.014	+0.756	.011	+1.001	32	—0.618	.047	+1.783	.038	+1.887
91	+0.641	.014	+0.767	.011	+1.000	31	—0.667	.049	+1.823	.040	+1.942
90	+0.628	.013	+0.778	.011	+1.000	30	—0.720	.053	+1.866	.043	+2.000
89	+0.614	.014	+0.789	.011	+1.000	29	—0.776	.056	+1.911	.045	+2.063
88	+0.601	.013	+0.800	.011	+1.001	28	—0.836	.060	+1.959	.048	+2.130
87	+0.587	.014	+0.811	.011	+1.001	27	—0.900	.064	+2.011	.052	+2.203
86	+0.573	.014	+0.822	.011	+1.002	26	—0.968	.068	+2.066	.055	+2.281
85	+0.560	.013	+0.833	.011	+1.004	25	—1.041	.073	+2.125	.059	+2.366
84	+0.546	.014	+0.844	.011	+1.006	24	—1.120	.079	+2.189	.064	+2.459
83	+0.532	.014	+0.855	.011	+1.008	23	—1.206	.086	+2.257	.068	+2.559
82	+0.519	.013	+0.867	.012	+1.010	22	—1.298	.092	+2.332	.075	+2.669
81	+0.505	.014	+0.878	.011	+1.012	21	—1.400	.102	+2.414	.082	+2.790

N.P.D.	Azimuth	Diff.	Level.	Diff.	Collimation.	Diff.
°						
+21	− 1.400	.111	+ 2.414	.089	+ 2.790	.134
20	− 1.511	.122	+ 2.503	.099	+ 2.924	.148
19	− 1.633	.134	+ 2.602	.109	+ 3.072	.164
18	− 1.767	.151	+ 2.711	.121	+ 3.236	.184
17	− 1.918	.168	+ 2.832	.136	+ 3.420	.208
16	− 2.086	.191	+ 2.968	.154	+ 3.628	.236
15	− 2.277	.217	+ 3.122	.175	+ 3.864	.270
14	− 2.494	.249	+ 3.297	.201	+ 4.134	.311
13	− 2.743	.291	+ 3.498	.234	+ 4.445	.365
12	− 3.034	.342	+ 3.732	.276	+ 4.810	.431
11	− 3.376	.410	+ 4.008	.332	+ 5.241	.518
10	− 3.786	.500	+ 4.340	.403	+ 5.759	.633
9	− 4.286	.624	+ 4.743	.503	+ 6.392	.793
8	− 4.910	.801	+ 5.246	.646	+ 7.185	
7	− 5.711		+ 5.892	.860	+ 8.206	
6	− 6.777		+ 6.752		+ 9.567	
5	− 8.268		+ 7.955		+11.474	
4	−10.502		+ 9.757		+14.336	
3	−14.223		+12.759		+19.107	
2	−21.660		+18.759		+28.654	
+ 1	−43.961		+36.750		+57.299	
0						
− 1	+45.217		−35.193		−57.299	
− 2	+22.916		−17.202		−28.654	
− 3	+15.479		−11.202		−19.107	
− 4	+11.758		− 8.201		−14.336	
− 5	+ 9.524		− 6.398		−11.474	
− 6	+ 8.033		− 5.196	.860	− 9.567	
− 7	+ 6.967	.801	− 4.336	.647	− 8.206	
− 8	+ 6.166	.624	− 3.689	.503	− 7.185	.793
− 9	+ 5.542	.501	− 3.186	.403	− 6.392	.633
−10	+ 5.041	.409	− 2.783	.331	− 5.759	.518
−11	+ 4.632	.342	− 2.452	.276	− 5.241	.431
−12	+ 4.290	.290	− 2.176	.234	− 4.810	.365
−13	+ 4.000	.251	− 1.942	.202	− 4.445	.311
−14	+ 3.749	.217	-- 1.740	.175	− 4.134	.270
−15	+ 3.532	.190	− 1.565	.154	− 3.864	.236
−16	+ 3.342	.168	− 1.411	.136	− 3.628	.208
−17	+ 3.174	.151	− 1.275	.121	− 3.420	.184
−18	+ 3.023	.135	− 1.154	.109	− 3.236	.164
−19	+ 2.888	.122	− 1.045	.098	− 3.072	.148
−20	+ 2.766	.111	− 0.947	.090	− 2.924	.134
−21	+ 2.655	.101	− 0.857	.081	− 2.790	.121
−22	+ 2.554	.093	− 0.776	.075	− 2.669	.110
−23	+ 2.461	.085	− 0.701	.069	− 2.559	.100
−24	+ 2.376	.079	− 0.632	.064	− 2.459	.093
−25	+ 2.297	.073	− 0.568	.059	− 2.366	.085
−26	+ 2.224	.069	− 0.509	.055	− 2.281	.078
−27	+ 2.155	.063	− 0.454	.051	− 2.203	.073
−28	+ 2.092	.060	− 0.403	.049	− 2.130	.067
−29	+ 2.032	.056	− 0.354	.045	− 2.063	.063
−30	+ 1.976	.053	− 0.309	.042	− 2.000	.058
−31	+ 1.923	.050	− 0.267	.040	− 1.942	.055
−32	+ 1.873	.047	− 0.227	.038	− 1.887	.051
−33	+ 1.826	.044	− 0.189	.036	− 1.836	.048
−34	+ 1.782	.043	− 0.153	.035	− 1.788	.045
−35	+ 1.739	.040	− 0.118	.032	− 1.743	.042
−36	+ 1.699	.038	− 0.086	.031	− 1.701	.039
−37	+ 1.661	.037	− 0.055	.030	− 1.662	.038
--38	+ 1.624		− 0.025		− 1.624	

Polaris. Upper Culmination.

N. P. Dist.	Azimuth.	Level.	Collimation.
° ′ ″			
1 31 0	−28.768	+24.493	+37.782
30 50	− .822	+ .536	+ .851
30 40	− .876	+ .580	+ .921
30 30	− .930	+ .624	+ .991
30 20	− .984	+ .668	+38.061
30 10	−29.039	+ .712	+ .131
30 0	− .094	+ .756	+ .202
29 50	− .150	+ .801	+ .272
29 40	− .205	+ .845	+ .344
29 30	− .261	+ .890	+ .415

Polaris. Lower Culmination.

1 31 0	+30.023	−22.936	−37.782
30 50	+ .077	− .980	− .851
30 40	+ .132	−23.023	− .921
30 30	+ .186	− .067	− .991
30 20	+ .241	− .111	−38.061
30 10	+ .295	− .155	− .131
30 0	+ .350	− .200	− .202
29 50	+ .405	− .244	− .272
29 40	+ .461	− .289	− .344
29 30	+ .516	− .334	− .415

λ Ursæ Minoris. Upper Culmination.

1 9 0	−38.144	+32.057	+49.826
8 50	− .238	+ .132	+ .946
8 40	− .332	+ .209	+50.068
8 30	− .426	+ .285	+ .189
8 20	− .522	+ .362	+ .312
8 10	− .618	+ .439	+ .435

λ Ursæ Minoris. Lower Culmination.

1 9 0	+39.399	−30.500	−49.826
8 50	+ .493	− .576	− .946
8 40	+ .587	− .652	−50.068
8 30	+ .682	− .729	− .189
8 20	+ .778	− .806	− .312
8 10	+ .874	− .883	− .435

51 Cephei. Upper Culmination.

2 45 0	−15.576	+13.850	+20.843
44 50	− .591	+ .864	+ .864
44 40	− .608	+ .877	+ .885
44 30	− .625	+ .890	+ .906

51 Cephei. Lower Culmination.

2 45 0	+16.831	−12.293	−20.843
44 50	+ .848	− .307	− .864
44 40	+ .864	− .320	− .885
44 30	+ .880	− .333	− .906

δ Ursæ Minoris. Upper Culmination.

3 24 20	−12.451	+11.329	+16.834
24 10	− .461	+ .338	+ .848
24 0	− .472	+ .347	+ .862

δ Ursæ Minoris. Lower Culmination.

3 24 20	+13.707	− 9.773	−16.834
24 10	+ .718	− .781	− .848
24 0	+ .729	− .790	− .862

Sec.	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.	9 m.	10 m.	11 m.
	″	″	″	″	″	″	″	″	″	″	″	″
0	0.0	2.0	7.8	17.7	31.4	49.1	70.7	96.2	125.7	159.0	196.3	237.5
1	0.0	2.0	8.0	17.9	31.7	49.4	71.1	96.7	126.2	159.6	197.0	238.3
2	0.0	2.1	8.1	18.1	31.9	49.7	71.5	97.1	126.7	160.2	197.6	239.0
3	0.0	2.2	8.2	18.3	32.2	50.1	71.9	97.6	127.2	160.8	198.3	239.7
4	0.0	2.2	8.4	18.5	32.5	50.4	72.3	98.0	127.8	161.4	198.9	240.4
5	0.0	2.3	8.5	18.7	32.7	50.7	72.7	98.5	128.3	162.0	199.6	241.1
6	0.0	2.4	8.7	18.9	33.0	51.1	73.1	99.0	128.8	162.6	200.3	241.9
7	0.0	2.4	8.8	19.1	33.3	51.4	73.5	99.4	129.3	163.2	200.9	242.6
8	0.0	2.5	8.9	19.3	33.5	51.7	73.9	99.9	129.9	163.8	201.6	243.3
9	0.0	2.6	9.1	19.5	33.8	52.1	74.3	100.4	130.4	164.4	202.2	244.1
10	0.1	2.7	9.2	19.7	34.1	52.4	74.7	100.8	130.9	165.0	202.9	244.8
11	0.1	2.7	9.4	19.9	34.4	52.7	75.1	101.3	131.5	165.6	203.6	245.5
12	0.1	2.8	9.5	20.1	34.6	53.1	75.5	101.8	132.0	166.2	204.2	246.3
13	0.1	2.9	9.6	20.3	34.9	53.4	75.9	102.3	132.6	166.8	204.9	247.0
14	0.1	3.0	9.8	20.5	35.2	53.8	76.3	102.7	133.1	167.4	205.6	247.7
15	0.1	3.1	9.9	20.7	35.5	54.1	76.7	103.2	133.6	168.0	206.3	248.5
16	0.1	3.1	10.1	20.9	35.7	54.5	77.1	103.7	134.2	168.6	206.9	249.2
17	0.2	3.2	10.2	21.2	36.0	54.8	77.5	104.2	134.7	169.2	207.6	249.9
18	0.2	3.3	10.4	21.4	36.3	55.1	77.9	104.6	135.3	169.8	208.3	250.7
19	0.2	3.4	10.5	21.6	36.6	55.5	78.3	105.1	135.8	170.4	208.9	251.4
20	0.2	3.5	10.7	21.8	36.9	55.8	78.8	105.6	136.3	171.0	209.6	252.2
21	0.2	3.6	10.8	22.0	37.2	56.2	79.2	106.1	136.9	171.6	210.3	253.0
22	0.3	3.7	11.0	22.3	37.4	56.5	79.6	106.6	137.4	172.2	211.0	253.6
23	0.3	3.8	11.2	22.5	37.7	56.9	80.0	107.0	138.0	172.9	211.7	254.4
24	0.3	3.8	11.3	22.7	38.0	57.3	80.4	107.5	138.5	173.5	212.3	255.1
25	0.3	3.9	11.5	22.9	38.3	57.6	80.8	108.0	139.1	174.1	213.0	255.9
26	0.4	4.0	11.6	23.1	38.6	58.0	81.3	108.5	139.6	174.7	213.7	256.6
27	0.4	4.1	11.8	23.4	38.9	58.3	81.7	109.0	140.2	175.3	214.4	257.4
28	0.4	4.2	11.9	23.6	39.2	58.7	82.1	109.5	140.7	175.9	215.1	258.1
29	0.5	4.3	12.1	23.8	39.5	59.0	82.5	110.0	141.3	176.6	215.8	258.9
30	0.5	4.4	12.3	24.0	39.8	59.4	83.0	110.4	141.8	177.2	216.4	259.6
31	0.5	4.5	12.4	24.3	40.1	59.8	83.4	110.9	142.4	177.8	217.1	260.4
32	0.6	4.6	12.6	24.5	40.3	60.1	83.8	111.4	143.0	178.4	217.8	261.1
33	0.6	4.7	12.8	24.7	40.6	60.5	84.2	111.9	143.5	179.0	218.5	261.9
34	0.6	4.8	12.9	25.0	40.9	60.8	84.7	112.4	144.1	179.7	219.2	262.6
35	0.7	4.9	13.1	25.2	41.2	61.2	85.1	112.9	144.6	180.3	219.9	263.4
36	0.7	5.0	13.3	25.4	41.5	61.6	85.5	113.4	145.2	180.9	220.6	264.1
37	0.7	5.1	13.4	25.7	41.8	61.9	86.0	113.9	145.8	181.6	221.3	264.9
38	0.8	5.2	13.6	25.9	42.1	62.3	86.4	114.4	146.3	182.2	222.0	265.7
39	0.8	5.3	13.8	26.2	42.5	62.7	86.8	114.9	146.9	182.8	222.7	266.4
40	0.9	5.4	14.0	26.4	42.8	63.0	87.3	115.4	147.5	183.5	223.4	267.2
41	0.9	5.6	14.1	26.6	43.1	63.4	87.7	115.9	148.0	184.1	224.1	268.0
42	1.0	5.7	14.3	26.9	43.4	63.8	88.1	116.4	148.6	184.7	224.8	268.7
43	1.0	5.8	14.5	27.1	43.7	64.2	88.6	116.9	149.2	185.4	225.5	269.5
44	1.1	5.9	14.7	27.4	44.0	64.5	89.0	117.4	149.7	186.0	226.2	270.3
45	1.1	6.0	14.8	27.6	44.3	64.9	89.5	117.9	150.3	186.6	226.9	271.0
46	1.2	6.1	15.0	27.9	44.6	65.3	89.9	118.4	150.9	187.3	227.6	271.8
47	1.2	6.2	15.2	28.1	44.9	65.7	90.3	118.9	151.5	187.9	228.3	272.6
48	1.3	6.4	15.4	28.3	45.2	66.0	90.8	119.5	152.0	188.5	229.0	273.3
49	1.3	6.5	15.6	28.6	45.5	66.4	91.2	120.0	152.6	189.2	229.7	274.1
50	1.4	6.6	15.8	28.8	45.9	66.8	91.7	120.5	153.2	189.8	230.4	274.9
51	1.4	6.7	15.9	29.1	46.2	67.2	92.1	121.0	153.8	190.5	231.1	275.6
52	1.5	6.8	16.1	29.4	46.5	67.6	92.6	121.5	154.4	191.1	231.8	276.4
53	1.5	7.0	16.3	29.6	46.8	68.0	93.0	122.0	154.9	191.8	232.5	277.2
54	1.6	7.1	16.5	29.9	47.1	68.3	93.5	122.5	155.5	192.4	233.2	278.0
55	1.6	7.2	16.7	30.1	47.5	68.7	93.9	123.1	156.1	193.1	234.0	278.8
56	1.7	7.3	16.9	30.4	47.8	69.1	94.4	123.6	156.7	193.7	234.7	279.5
57	1.8	7.5	17.1	30.6	48.1	69.5	94.8	124.1	157.3	194.4	235.4	280.3
58	1.8	7.6	17.3	30.9	48.4	69.9	95.3	124.6	157.8	195.0	236.1	281.1
59	1.9	7.7	17.5	31.2	48.8	70.3	95.7	125.1	158.4	195.7	236.8	281.9

Sec.	12 m.	13 m.	14 m.	15 m.	16 m.	17 m.	18 m.	19 m.	20 m.	21 m.
	″	″	″	″	″	″	″	″	″	″
0	282.7	331.7	384.7	441.6	502.5	567.2	635.9	708.4	784.9	865.3
1	283.5	332.6	385.6	442.6	503.5	568.3	637.0	709.4	786.2	866.6
2	284.3	333.4	386.6	443.6	504.6	569.4	638.2	710.9	787.5	868.0
3	285.0	334.3	387.5	444.6	505.6	570.5	639.4	712.1	788.8	869.4
4	285.8	335.2	388.4	445.6	506.7	571.6	640.6	713.4	790.1	870.8
5	286.6	336.0	389.3	446.5	507.7	572.8	641.7	714.6	791.4	872.1
6	287.4	336.9	390.2	447.5	508.8	573.9	642.9	715.9	792.7	873.5
7	288.2	337.7	391.2	448.5	509.8	575.0	644.1	717.1	794.0	874.9
8	289.0	338.6	392.1	449.5	510.9	576.1	645.3	718.4	795.4	876.3
9	289.8	339.4	393.0	450.5	511.9	577.2	646.5	719.6	796.7	877.6
10	290.6	340.3	393.9	451.5	513.0	578.4	647.7	720.9	798.0	879.0
11	291.4	341.2	394.9	452.5	514.0	579.5	648.9	722.1	799.3	880.4
12	292.2	342.0	395.8	453.5	515.1	580.6	650.0	723.4	800.7	881.8
13	293.0	342.9	396.7	454.5	516.1	581.7	651.2	724.6	802.0	883.2
14	293.8	343.7	397.6	455.5	517.2	582.9	652.4	725.9	803.3	884.6
15	294.6	344.6	398.6	456.5	518.3	584.0	653.6	727.2	804.6	886.0
16	295.4	345.5	399.5	457.5	519.3	585.1	654.8	728.4	806.0	887.4
17	296.2	346.4	400.5	458.5	520.4	586.2	656.0	729.7	807.3	888.8
18	297.0	347.2	401.4	459.5	521.5	587.4	657.2	730.9	808.6	890.2
19	297.8	348.1	402.3	460.5	522.5	588.5	658.4	732.2	809.9	891.6
20	298.6	349.0	403.3	461.5	523.6	589.6	659.6	733.5	811.3	893.0
21	299.4	349.8	404.2	462.5	524.6	590.8	660.8	734.7	812.6	894.4
22	300.2	350.7	405.1	463.5	525.7	591.9	662.0	736.0	813.9	895.8
23	301.0	351.6	406.1	464.5	526.8	593.0	663.2	737.3	815.2	897.2
24	301.8	352.5	407.0	465.5	527.9	594.2	664.4	738.5	816.6	898.6
25	302.6	353.3	408.0	466.5	528.9	595.3	665.6	739.8	817.9	900.0
26	303.5	354.2	408.9	467.5	530.0	596.5	666.8	741.1	819.2	901.4
27	304.3	355.1	409.9	468.5	531.1	597.6	668.0	742.3	820.5	902.8
28	305.1	356.0	410.8	469.5	532.2	598.7	669.2	743.6	821.9	904.2
29	305.9	356.9	411.7	470.5	533.2	599.9	670.4	744.9	823.2	905.6
30	306.7	357.7	412.7	471.5	534.3	601.0	671.6	746.2	824.6	907.0
31	307.5	358.6	413.6	472.6	535.4	602.2	672.8	747.4	825.9	908.4
32	308.4	359.5	414.6	473.6	536.5	603.3	674.1	748.7	827.3	909.8
33	309.2	360.4	415.5	474.6	537.6	604.5	675.3	750.0	828.6	911.2
34	310.0	361.3	416.5	475.6	538.7	605.6	676.5	751.3	829.9	912.6
35	310.8	362.2	417.4	476.6	539.7	606.8	677.7	752.6	831.2	914.0
36	311.6	363.1	418.4	477.6	540.8	607.9	678.9	753.8	832.6	915.5
37	312.5	364.0	419.4	478.7	541.9	609.1	680.1	755.1	833.9	916.9
38	313.3	364.8	420.3	479.7	543.0	610.2	681.3	756.4	835.3	918.3
39	314.1	365.7	421.3	480.7	544.1	611.4	682.6	757.7	836.6	919.7
40	315.0	366.6	422.2	481.7	545.2	612.5	683.8	759.0	838.0	921.1
41	315.8	367.5	423.2	482.8	546.3	613.7	685.0	760.2	839.3	922.5
42	316.6	368.4	424.2	483.8	547.4	614.8	686.2	761.5	840.7	923.9
43	317.4	369.3	425.1	484.8	548.4	616.0	687.4	762.8	842.0	925.3
44	318.3	370.2	426.1	485.8	549.5	617.2	688.7	764.1	843.4	926.8
45	319.1	371.1	427.0	486.9	550.6	618.3	689.9	765.4	844.7	928.2
46	319.9	372.0	428.0	487.9	551.7	619.5	691.1	766.7	846.1	929.6
47	320.8	372.9	429.0	488.9	552.8	620.6	692.4	768.0	847.5	931.0
48	321.6	373.8	429.9	490.0	553.9	621.8	693.6	769.3	848.9	932.4
49	322.4	374.7	430.9	491.0	555.0	623.0	694.8	770.6	850.2	933.8
50	323.3	375.6	431.9	492.0	556.1	624.1	696.0	771.9	851.6	935.2
51	324.1	376.5	432.8	493.1	557.2	625.3	697.3	773.1	852.9	936.6
52	325.0	377.4	433.8	494.1	558.3	626.5	698.5	774.5	854.3	938.1
53	325.8	378.3	434.8	495.2	559.4	627.6	699.7	775.8	855.7	939.5
54	326.7	379.3	435.8	496.2	560.5	628.8	701.0	777.1	857.1	940.9
55	327.5	380.2	436.7	497.2	561.6	630.0	702.2	778.4	858.4	942.3
56	328.4	381.1	437.7	498.3	562.7	631.2	703.5	779.7	859.8	943.8
57	329.2	382.0	438.7	499.3	563.9	632.3	704.7	781.0	861.1	945.2
58	330.0	382.9	439.7	500.3	565.0	633.5	705.9	782.3	862.5	946.6
59	330.9	383.8	440.6	501.4	566.1	634.7	707.1	783.6	863.9	948.1

Sec.	22 m.	23 m.	24 m.	25 m.	26 m.	27 m.	28 m.	29 m.	30 m.	31 m.
	″	″	″	″	″	″	″	″	″	″
0	949.6	1037.8	1129.9	1225.9	1325.9	1429.7	1537.5	1649.0	1764.6	1884.0
1	951.0	1039.3	1131.4	1227.5	1327.6	1431.4	1539.3	1650.9	1766.6	1886.0
2	952.4	1040.8	1133.0	1229.2	1329.3	1433.2	1541.1	1652.8	1768.5	1888.0
3	953.8	1042.3	1134.6	1230.8	1331.0	1434.9	1542.9	1654.7	1770.5	1890.0
4	955.3	1043.8	1136.2	1232.5	1332.7	1436.7	1544.8	1656.6	1772.4	1892.1
5	956.7	1045.3	1137.8	1234.1	1334.4	1438.5	1546.6	1658.5	1774.4	1894.1
6	958.2	1046.8	1139.3	1235.7	1336.1	1440.3	1548.4	1660.4	1776.3	1896.1
7	959.6	1048.3	1140.9	1237.3	1337.8	1442.1	1550.2	1662.3	1778.3	1898.1
8	961.1	1049.8	1142.5	1239.0	1339.5	1443.9	1552.1	1664.2	1780.3	1900.2
9	962.5	1051.3	1144.0	1240.6	1341.2	1445.6	1553.9	1666.1	1782.3	1902.2
10	963.9	1052.8	1145.6	1242.3	1342.9	1447.4	1555.8	1668.0	1784.2	1904.3
11	965.4	1054.3	1147.2	1243.9	1344.6	1449.2	1557.6	1669.9	1786.2	1906.3
12	966.9	1055.9	1148.8	1245.6	1346.3	1451.0	1559.5	1671.9	1788.2	1908.4
13	968.3	1057.4	1150.4	1247.2	1348.0	1452.8	1561.3	1673.8	1790.1	1910.4
14	969.8	1058.9	1152.0	1248.9	1349.7	1454.5	1563.2	1675.7	1792.1	1912.4
15	971.2	1060.4	1153.6	1250.5	1351.4	1456.3	1565.0	1677.6	1794.1	1914.4
16	972.7	1062.0	1155.2	1252.2	1353.2	1458.1	1566.9	1679.5	1796.1	1916.5
17	974.1	1063.5	1156.8	1253.8	1354.9	1459.9	1568.7	1681.4	1798.1	1918.5
18	975.5	1065.0	1158.3	1255.5	1356.6	1461.6	1570.5	1683.3	1800.0	1920.6
19	977.0	1066.5	1159.9	1257.1	1358.3	1463.4	1572.4	1685.2	1802.0	1922.6
20	978.5	1068.1	1161.5	1258.8	1360.1	1465.2	1574.3	1687.2	1804.0	1924.7
21	979.9	1069.6	1163.1	1260.4	1361.8	1466.9	1576.1	1689.1	1805.9	1926.7
22	981.4	1071.1	1164.7	1262.1	1363.5	1468.7	1578.0	1691.0	1807.9	1928.8
23	982.9	1072.6	1166.3	1263.7	1365.2	1470.5	1579.8	1692.9	1809.9	1930.8
24	984.4	1074.2	1167.9	1265.4	1367.0	1472.3	1581.7	1694.8	1811.9	1932.9
25	985.8	1075.7	1169.5	1267.0	1368.7	1474.0	1583.5	1696.7	1813.9	1935.0
26	987.3	1077.2	1171.1	1268.7	1370.4	1475.9	1585.3	1698.6	1815.8	1937.0
27	988.8	1078.7	1172.7	1270.3	1372.1	1477.7	1587.2	1700.5	1817.8	1939.0
28	990.3	1080.3	1174.3	1272.1	1373.9	1479.5	1589.1	1702.5	1819.8	1941.1
29	991.8	1081.8	1175.9	1273.7	1375.6	1481.3	1590.9	1704.4	1821.8	1943.1
30	993.2	1083.3	1177.5	1275.4	1377.4	1483.1	1592.7	1706.3	1823.8	1945.2
31	994.7	1084.8	1179.1	1277.1	1379.0	1484.9	1594.6	1708.2	1825.8	1947.2
32	996.2	1086.4	1180.7	1278.8	1380.8	1486.7	1596.5	1710.2	1827.8	1949.3
33	997.6	1087.9	1182.3	1280.4	1382.5	1488.5	1598.3	1712.1	1829.8	1951.3
34	999.1	1089.5	1183.9	1282.1	1384.2	1490.3	1600.2	1714.0	1831.8	1953.4
35	1000.6	1091.0	1185.5	1283.8	1385.9	1492.1	1602.1	1715.9	1833.8	1955.5
36	1002.1	1092.6	1187.1	1285.5	1387.7	1493.9	1604.0	1717.9	1835.8	1957.6
37	1003.5	1094.1	1188.7	1287.1	1389.4	1495.7	1605.9	1719.8	1837.8	1959.6
38	1005.0	1095.7	1190.3	1288.8	1391.2	1497.5	1607.7	1721.7	1839.8	1961.7
39	1006.5	1097.2	1191.9	1290.5	1392.9	1499.3	1609.6	1723.6	1841.8	1963.7
40	1008.0	1098.8	1193.5	1292.2	1394.7	1501.1	1611.5	1725.6	1843.8	1965.8
41	1009.4	1100.3	1195.1	1293.8	1396.4	1502.9	1613.3	1727.5	1845.8	1967.8
42	1010.9	1101.9	1196.7	1295.5	1398.2	1504.7	1615.2	1729.5	1847.8	1969.9
43	1012.4	1103.4	1198.3	1297.2	1399.9	1506.5	1617.1	1731.5	1849.8	1972.0
44	1013.9	1105.0	1199.9	1298.9	1401.7	1508.4	1619.0	1733.4	1851.8	1974.1
45	1015.4	1106.5	1201.5	1300.5	1403.4	1510.2	1620.8	1735.3	1853.8	1976.1
46	1016.9	1108.1	1203.1	1302.2	1405.2	1512.0	1622.7	1737.2	1855.8	1978.2
47	1018.4	1109.6	1204.7	1303.9	1406.9	1513.8	1624.6	1739.2	1857.8	1980.3
48	1019.9	1111.2	1206.4	1305.6	1408.7	1515.6	1626.5	1741.2	1859.8	1982.4
49	1021.4	1112.7	1208.0	1307.3	1410.4	1517.4	1628.3	1743.1	1861.8	1984.4
50	1022.8	1114.3	1209.6	1309.0	1412.2	1519.2	1630.2	1745.1	1863.8	1986.5
51	1024.3	1115.8	1211.2	1310.7	1413.9	1521.0	1632.1	1747.0	1865.8	1988.6
52	1025.8	1117.4	1212.9	1312.4	1415.7	1522.9	1634.0	1749.0	1867.8	1990.7
53	1027.3	1118.9	1214.5	1314.1	1417.4	1524.7	1635.9	1750.9	1869.8	1992.7
54	1028.8	1120.5	1216.1	1315.7	1419.2	1526.5	1637.7	1752.9	1871.8	1994.8
55	1030.3	1122.0	1217.7	1317.4	1420.9	1528.3	1639.6	1754.8	1873.8	1996.9
56	1031.8	1123.6	1219.4	1319.1	1422.7	1530.2	1641.5	1756.8	1875.9	1999.0
57	1033.3	1125.1	1221.0	1320.8	1424.4	1532.0	1643.3	1758.7	1877.9	2001.0
58	1034.8	1126.7	1222.6	1322.5	1426.2	1533.8	1645.2	1760.7	1879.9	2003.1
59	1036.3	1128.3	1224.2	1324.2	1427.9	1535.6	1647.1	1762.6	1882.0	2005.3

Sec.	32 m.	33 m.	34 m.	35 m.
	″	″	″	″
0	2007.4	2134.6	2265.6	2400.6
1	2009.4	2136.8	2267.8	2402.9
2	2011.5	2138.9	2270.0	2405.2
3	2013.6	2141.1	2272.2	2407.5
4	2015.7	2143.2	2274.5	2409.8
5	2017.8	2145.3	2276.7	2412.0
6	2019.9	2147.5	2278.9	2414.3
7	2022.0	2149.7	2281.2	2416.6
8	2024.1	2151.8	2283.4	2418.9
9	2026.2	2153.9	2285.6	2421.2
10	2028.3	2156.1	2287.8	2423.5
11	2030.5	2158.3	2290.0	2425.8
12	2032.5	2160.5	2292.3	2428.1
13	2034.6	2162.6	2294.5	2430.4
14	2036.7	2164.8	2296.8	2432.7
15	2038.8	2166.9	2299.0	2435.0
16	2040.9	2169.1	2301.3	2437.3
17	2043.0	2171.2	2303.6	2439.6
18	2045.1	2173.4	2305.8	2441.9
19	2047.2	2175.6	2308.0	2444.2
20	2049.3	2177.8	2310.2	2446.5
21	2051.4	2179.9	2312.4	2448.8
22	2053.5	2182.1	2314.7	2451.1
23	2055.7	2184.3	2316.9	2453.4
24	2057.8	2186.4	2319.2	2455.7
25	2059.9	2188.6	2321.5	2458.0
26	2062.0	2190.8	2323.7	2460.3
27	2064.1	2193.0	2325.9	2462.6
28	2066.2	2195.2	2328.2	2464.9
29	2068.3	2197.3	2330.4	2467.2
30	2070.4	2199.5	2332.7	2469.5
31	2072.6	2201.7	2334.9	2471.8
32	2074.7	2203.9	2337.2	2474.2
33	2076.8	2206.1	2339.4	2476.5
34	2078.9	2208.3	2341.7	2478.8
35	2081.0	2210.5	2343.9	2481.1
36	2083.2	2212.7	2346.2	2483.5
37	2085.3	2214.9	2348.5	2485.8
38	2087.4	2217.1	2350.7	2488.1
39	2089.6	2219.3	2353.0	2490.4
40	2091.7	2221.5	2355.2	2492.8
41	2093.8	2223.7	2357.5	2495.1
42	2095.9	2225.9	2359.7	2497.4
43	2098.0	2228.1	2361.9	2499.7
44	2100.2	2230.3	2364.2	2502.1
45	2102.3	2232.5	2366.4	2504.4
46	2104.5	2234.7	2368.7	2506.7
47	2106.6	2236.9	2371.0	2509.0
48	2108.8	2239.1	2373.3	2511.4
49	2110.9	2241.3	2375.5	2513.7
50	2113.1	2243.5	2377.8	2516.1
51	2115.2	2245.7	2380.1	2518.4
52	2117.4	2247.9	2382.4	2520.8
53	2119.6	2250.1	2384.6	2523.1
54	2121.7	2252.3	2386.9	2525.4
55	2123.8	2254.5	2389.2	2527.7
56	2126.0	2256.7	2391.5	2530.1
57	2128.1	2258.9	2393.7	2532.4
58	2130.3	2261.1	2396.0	2534.8
59	2132.4	2263.4	2398.3	2537.1

Part Second.									For Rate.	
m.	s.	″	m.	s.	″	m.	s.	″	r.	Log. r. factor.
									s.	
5	0	0.01	16	0	0.61	26	0	4.26	+30	9.9996985
6	0	0.01		10	0.64		10	4.37	29	7085
	20	0.01		20	0.67		20	4.48	28	7186
	30	0.02		30	0.69		30	4.60	27	7286
	40	0.02		40	0.72		40	4.72	26	7387
	50	0.02		50	0.75		50	4.83	25	7487
7	0	0.02	17	0	0.78	27	0	4.96	24	7588
	10	0.02		10	0.81		10	5.08	23	7688
	20	0.03		20	0.84		20	5.20	22	7789
	30	0.03		30	0.88		30	5.33	21	7889
	40	0.03		40	0.91		40	5.46	20	7990
	50	0.04		50	0.95		50	5.60	19	8090
8	0	0.04	18	0	0.98	28	0	5.73	18	8191
	10	0.04		10	1.02		10	5.87	17	8291
	20	0.05		20	1.06		20	6.01	16	8392
	30	0.05		30	1.09		30	6.15	15	8492
	40	0.05		40	1.13		40	6.30	14	8593
	50	0.06		50	1.18		50	6.44	13	8693
9	0	0.06	19	0	1.22	29	0	6.59	12	8794
	10	0.07		10	1.26		10	6.75	11	8894
	20	0.07		20	1.30		20	6.90	10	8995
	30	0.08		30	1.35		30	7.06	9	9095
	40	0.08		40	1.40		40	7.22	8	9196
	50	0.09		50	1.44		50	7.38	7	9296
10	0	0.09	20	0	1.49	30	0	7.55	6	9397
	10	0.10		10	1.54		10	7.72	5	9497
	20	0.11		20	1.60		20	7.89	4	9598
	30	0.11		30	1.65		30	8.06	3	9698
	40	0.12		40	1.70		40	8.24	2	9799
	50	0.13		50	1.76		50	8.42	+ 1	9.9999899
11	0	0.14	21	0	1.82	31	0	8.61	0	0.0000000
	10	0.15		10	1.87		10	8.79	— 1	0.0000101
	20	0.15		20	1.93		20	8.98	2	0201
	30	0.16		30	1.99		30	9.17	3	0302
	40	0.17		40	2.06		40	9.37	4	0402
	50	0.18		50	2.12		50	9.57	5	0503
12	0	0.19	22	0	2.19	32	0	9.77	6	0603
	10	0.20		10	2.25		10	9.97	7	0704
	20	0.22		20	2.32		20	10.18	8	0804
	30	0.23		30	2.39		30	10.39	9	0905
	40	0.24		40	2.46		40	10.61	10	1005
	50	0.25		50	2.54		50	10.82	11	1106
13	0	0.27	23	0	2.61	33	0	11.04	12	1206
	10	0.28		10	2.69		10	11.27	13	1307
	20	0.30		20	2.77		20	11.50	14	1407
	30	0.31		30	2.85		30	11.73	15	1508
	40	0.33		40	2.93		40	11.96	16	1608
	50	0.34		50	3.01		50	12.20	17	1709
14	0	0.36	24	0	3.10	34	0	12.44	18	1809
	10	0.38		10	3.18		10	12.69	19	1910
	20	0.39		20	3.27		20	12.94	20	2010
	30	0.41		30	3.36		30	13.19	21	2111
	40	0.43		40	3.45		40	13.45	22	2211
	50	0.45		50	3.55		50	13.71	23	2312
15	0	0.47	25	0	3.64	35	0	13.97	24	2412
	10	0.49		10	3.74		10	14.24	25	2513
	20	0.52		20	3.84		20	14.51	26	2613
	30	0.54		30	3.94		30	14.78	27	2714
	40	0.56		40	4.05		40	15.06	28	2814
	50	0.59		50	4.15		50	15.35	29	2915
16	0	0.61	26	0	4.26	36	0	15.63	—30	0.0003015

	Logarithm of A.										
Min.	**2 h.**	**3 h.**	**4 h.**	**5 h.**	**6 h.**	**7 h.**	**8 h.**	**9 h.**	**10 h.**	**11 h.**	**12**
0	9.4109	9.4172	9.4260	9.4374	9.4515	9.4685	9.4884	9.5115	9.5379	9.5680	9.60
2	.4111	.4174	.4263	.4378	.4521	.4691	.4892	.5123	.5389	.5691	.60
4	.4113	.4177	.4266	.4383	.4526	.4697	.4899	.5132	.5398	.5701	.60
6	.4114	.4179	.4270	.4387	.4531	.4704	.4906	.5140	.5408	.5712	.60
8	.4116	.4182	.4273	.4391	.4536	.4710	.4913	.5148	.5417	.5723	.60
10	9.4118	9.4184	9.4277	9.4396	9.4542	9.4716	9.4921	9.5157	9.5427	9.5734	9.60
12	.4120	.4187	.4280	.4400	.4547	.4723	.4928	.5165	.5436	.5745	.60
14	.4121	.4190	.4284	.4405	.4552	.4729	.4935	.5174	.5446	.5756	.61
16	.4123	.4193	.4288	.4409	.4558	.4735	.4943	.5182	.5456	.5767	.61
18	.4125	.4195	.4291	.4414	.4563	.4742	.4950	.5191	.5466	.5778	.61
20	9.4127	9.4198	9.4295	9.4418	9.4569	9.4748	9.4958	9.5199	9.5475	9.5789	9.61
22	.4129	.4201	.4299	.4423	.4574	.4755	.4965	.5208	.5485	.5800	.61
24	.4131	.4204	.4302	.4427	.4580	.4761	.4973	.5217	.5495	.5811	.61
26	.4133	.4207	.4306	.4432	.4585	.4768	.4980	.5225	.5505	.5822	.61
28	.4135	.4209	.4310	.4437	.4591	.4774	.4988	.5234	.5515	.5834	.61
30	9.4137	9.4212	9.4314	9.4441	9.4597	9.4781	9.4996	9.5243	9.5525	9.5845	9.62
32	.4139	.4215	.4317	.4446	.4602	.4788	.5003	.5252	.5535	.5856	.62
34	.4141	.4218	.4321	.4451	.4608	.4794	.5011	.5261	.5545	.5868	.62
36	.4144	.4221	.4325	.4456	.4614	.4801	.5019	.5269	.5555	.5879	.62
38	.4146	.4224	.4329	.4460	.4620	.4808	.5027	.5278	.5565	.5891	.62
40	9.4148	9.4227	9.4333	9.4465	9.4625	9.4815	9.5035	9.5287	9.5576	9.5902	9.62
42	.4150	.4231	.4337	.4470	.4631	.4821	.5042	.5296	.5586	.5914	.62
44	.4152	.4234	.4341	.4475	.4637	.4828	.5050	.5305	.5596	.5926	.62
46	.4155	.4237	.4345	.4480	.4643	.4835	.5058	.5315	.5606	.5937	.63
48	.4157	.4240	.4349	.4485	.4649	.4842	.5066	.5324	.5617	.5949	.63
50	9.4159	9.4243	9.4353	9.4490	9.4655	9.4849	9.5074	9.5333	9.5627	9.5961	9.63
52	.4162	.4246	.4357	.4494	.4661	.4856	.5082	.5342	.5638	.5973	.63
54	.4164	.4250	.4361	.4500	.4667	.4863	.5091	.5351	.5648	.5985	.63
56	.4167	.4253	.4366	.4505	.4673	.4870	.5099	.5361	.5659	.5997	.63
58	.4169	.4256	.4370	.4510	.4679	.4877	.5107	.5370	.5669	.6009	.63

	Logarithm of A.										
Min.	**13 h.**	**14 h.**	**15 h.**	**16 h.**	**17 h.**	**18 h.**	**19 h.**	**20 h.**	**21 h.**	**22 h.**	**23**
0	9.6406	9.6841	9.7333	9.7895	9.8539	9.9287	0.0172	0.1249	0.2623	0.4523	0.76
2	.6419	.6856	.7351	.7915	.8562	.9314	.0204	.1290	.2676	.4601	.78
4	.6433	.6872	.7369	.7935	.8585	.9341	.0237	.1330	.2729	.4680	.80
6	.6447	.6887	.7386	.7955	.8608	.9368	.0270	.1371	.2783	.4761	.81
8	.6461	.6903	.7404	.7975	.8632	.9396	.0303	.1412	.2838	.4842	.83
10	9.6474	9.6919	9.7422	9.7996	9.8655	9.9424	0.0336	0.1454	0.2893	0.4926	0.85
12	.6488	.6934	.7440	.8016	.8679	.9451	.0370	.1496	.2949	.5010	.86
14	.6502	.6950	.7458	.8037	.8703	.9479	.0403	.1538	.3005	.5097	.88
16	.6516	.6966	.7476	.8058	.8727	.9508	.0437	.1581	.3063	.5184	.90
18	.6530	.6982	.7494	.8078	.8751	.9536	.0472	.1623	.3120	.5274	.92
20	9.6545	9.6998	9.7512	9.8099	9.8775	9.9564	0.0506	0.1667	0.3179	0.5365	0.95
22	.6559	.7014	.7531	.8120	.8799	.9593	.0541	.1711	.3238	.5458	.97
24	.6573	.7030	.7549	.8141	.8824	.9622	.0576	.1755	.3298	.5553	.99
26	.6588	.7047	.7568	.8162	.8848	.9651	.0611	.1799	.3359	.5649	1.02
28	.6602	.7063	.7586	.8184	.8873	.9680	.0646	.1844	.3420	.5748	.04
30	9.6616	9.7079	9.7605	9.8205	9.8898	9.9709	0.0682	0.1889	0.3482	0.5848	1.07
32	.6631	.7096	.7624	.8227	.8923	.9739	.0718	.1935	.3545	.5951	.10
34	.6645	.7112	.7642	.8248	.8948	.9769	.0754	.1981	.3609	.6056	.14
36	.6660	.7129	.7661	.8270	.8973	.9798	.0790	.2028	.3674	.6164	.17
38	.6675	.7146	.7680	.8292	.8999	.9829	.0827	.2075	.3739	.6273	.21
40	9.6690	9.7162	9.7699	9.8314	9.9024	9.9859	0.0864	0.2122	0.3805	0.6386	1.25
42	.6704	.7179	.7718	.8336	.9050	.9889	.0901	.2170	.3873	.6501	.30
44	.6719	.7196	.7738	.8358	.9075	.9920	.0939	.2218	.3941	.6619	.35
46	.6734	.7213	.7757	.8380	.9101	.9951	.0976	.2267	.4010	.6740	.41
48	.6749	.7230	.7776	.8402	.9127	.9982	.1015	.2316	.4080	.6865	.48
50	9.6764	9.7247	9.7796	9.8425	9.9154	0.0013	0.1053	0.2366	0.4151	0.6993	1.56
52	.6779	.7264	.7815	.8447	.9180	.0044	.1092	.2416	.4223	.7124	.65
54	.6795	.7281	.7835	.8470	.9206	.0076	.1131	.2467	.4297	.7259	.78
56	.6810	.7299	.7855	.8493	.9233	.0108	.1170	.2518	.4371	.7398	.96
58	.6825	.7316	.7875	.8516	.9260	.0140	.1209	.2570	.4446	.7541	2.26

	Logarithm of B.										B neg.
Min.	**2 h.**	**3 h.**	**4 h.**	**5 h.**	**6 h.**	**7 h.**	**8 h.**	**9 h.**	**10 h.**	**11 h.**	**12 h.**
0	9.3959	9.3828	9.3635	9.3369	9.3010	9.2530	9.1874	9.0943	8.9509	8.6837	Inf.
2	.3955	.3822	.3627	.3358	.2996	.2511	.1848	.0906	.9447	.6701	7.2431
4	.3952	.3817	.3620	.3348	.2982	.2492	.1822	.0867	.9384	.6560	.5453
6	.3948	.3811	.3612	.3337	.2968	.2473	.1796	.0828	.9320	.6414	.7226
8	.3944	.3806	.3604	.3327	.2954	.2454	.1769	.0789	.9254	.6262	.8488
10	9.3941	9.3800	9.3596	9.3316	9.2940	9.2434	9.1742	9.0749	8.9187	8.6103	7.9469
12	.3937	.3794	.3588	.3305	.2925	.2415	.1715	.0708	.9118	.5937	8.0273
14	.3933	.3789	.3580	.3294	.2911	.2395	.1687	.0667	.9048	.5764	.0955
16	.3929	.3783	.3572	.3283	.2896	.2375	.1659	.0625	.8977	.5583	.1547
18	.3925	.3777	.3564	.3272	.2881	.2355	.1630	.0583	.8903	.5392	.2071
20	9.3921	9.3771	9.3555	9.3261	9.2866	9.2334	9.1602	9.0540	8.8829	8.5192	8.2541
22	.3917	.3765	.3547	.3249	.2850	.2313	.1573	.0496	.8752	.4981	.2967
24	.3913	.3759	.3538	.3238	.2835	.2292	.1543	.0452	.8674	.4758	.3357
26	.3909	.3752	.3530	.3226	.2819	.2271	.1513	.0406	.8594	.4521	.3717
28	.3905	.3746	.3521	.3214	.2804	.2250	.1483	.0360	.8512	.4270	.4051
30	9.3900	9.3740	9.3512	9.3203	9.2788	9.2228	9.1453	9.0314	8.8427	8.4001	8.4363
32	.3896	.3733	.3503	.3191	.2772	.2206	.1422	.0266	.8341	.3713	.4657
34	.3892	.3727	.3494	.3178	.2756	.2184	.1390	.0218	.8253	.3403	.4932
36	.3887	.3720	.3485	.3166	.2739	.2162	.1359	.0169	.8162	.3067	.5192
38	.3882	.3713	.3476	.3154	.2723	.2140	.1327	.0119	.8068	.2701	.5440
40	9.3878	9.3707	9.3467	9.3142	9.2706	9.2117	9.1294	9.0069	8.7972	8.2299	8.5675
42	.3873	.3700	.3457	.3129	.2689	.2094	.1261	.0017	.7873	.1853	.5899
44	.3868	.3693	.3448	.3116	.2672	.2070	.1228	8.9965	.7772	.1354	.6114
46	.3863	.3686	.3438	.3103	.2655	.2047	.1194	.9911	.7668	.0786	.6320
48	.3859	.3679	.3429	.3091	.2638	.2023	.1159	.9857	.7560	.0128	.6517
50	9.3854	9.3672	9.3419	9.3078	9.2620	9.1999	9.1125	8.9802	8.7449	7.9348	8.6707
52	.3849	.3665	.3409	.3064	.2602	.1974	.1089	.9745	.7335	.8391	.6890
54	.3843	.3657	.3399	.3051	.2584	.1950	.1054	.9688	.7217	.7154	.7067
56	.3838	.3650	.3389	.3038	.2566	.1925	.1017	.9630	.7094	.5405	.7237
58	.3833	.3643	.3379	.3024	.2548	.1900	.0981	.9570	.6968	.2407	.7402

Logarithm of B negative.

Min.	**13 h.**	**14 h.**	**15 h.**	**16 h.**	**17 h.**	**18 h.**	**19 h.**	**20 h.**	**21 h.**	**22 h.**	**23 h.**
0	8.7563	9.0971	9.3162	9.4884	9.6383	9.7782	9.9167	0.0625	0.2279	0.4372	0.7652
2	.7718	.1057	.3225	.4937	.6431	.7827	.9213	.0676	.2339	.4455	.7807
4	.7868	.1141	.3287	.4990	.6478	.7873	.9260	.0727	.2401	.4540	.7967
6	.8015	.1224	.3350	.5042	.6526	.7919	.9307	.0779	.2462	.4625	.8133
8	.8158	.1306	.3411	.5094	.6573	.7965	.9355	.0830	.2524	.4711	.8305
10	8.8296	9.1387	9.3472	9.5146	9.6621	9.8011	9.9402	0.0882	0.2587	0.4799	0.8483
12	.8432	.1468	.3533	.5197	.6668	.8057	.9449	.0935	.2650	.4889	.8667
14	.8564	.1547	.3593	.5248	.6715	.8103	.9497	.0987	.2714	.4980	.8860
16	.8692	.1625	.3653	.5300	.6762	.8149	.9544	.1040	.2778	.5072	.9060
18	.8818	.1703	.3713	.5351	.6809	.8195	.9592	.1093	.2843	.5165	.9270
20	8.8941	9.1779	9.3772	9.5401	9.6856	9.8241	9.9640	0.1146	0.2909	0.5261	0.9489
22	.9062	.1855	.3831	.5452	.6903	.8287	.9687	.1200	.2975	.5358	.9719
24	.9180	.1930	.3889	.5502	.6949	.8333	.9735	.1253	.3041	.5457	.9961
26	.9295	.2004	.3947	.5553	.6996	.8379	.9784	.1308	.3109	.5557	1.0216
28	.9408	.2078	.4005	.5603	.7043	.8425	.9832	.1362	.3177	.5660	.0487
30	8.9519	9.2150	9.4062	9.5653	9.7089	9.8471	9.9880	0.1417	0.3245	0.5764	1.0774
32	.9627	.2222	.4119	.5702	.7136	.8517	.9929	.1472	.3315	.5871	.1081
34	.9734	.2293	.4175	.5752	.7182	.8563	.9977	.1527	.3385	.5979	.1409
36	.9839	.2364	.4232	.5801	.7228	.8609	0.0026	.1582	.3456	.6090	.1764
38	.9942	.2434	.4288	.5850	.7275	.8655	.0075	.1638	.3527	.6204	.2149
40	9.0043	9.2503	9.4343	9.5900	9.7321	9.8701	0.0124	0.1695	0.3599	0.6319	1.2569
42	.0142	.2571	.4399	.5948	.7367	.8748	.0173	.1751	.3673	.6438	.3033
44	.0240	.2639	.4454	.5997	.7413	.8794	.0223	.1808	.3747	.6559	.3552
46	.0336	.2706	.4509	.6046	.7459	.8840	.0272	.1866	.3822	.6684	.4138
48	.0431	.2773	.4563	.6094	.7505	.8887	.0322	.1924	.3897	.6811	.4814
50	9.0524	9.2839	9.4617	9.6143	9.7552	9.8933	0.0372	0.1982	0.3974	0.6942	1.5612
52	.0616	.2905	.4671	.6191	.7598	.8980	.0422	.2040	.4052	.7076	.6587
54	.0707	.2970	.4725	.6239	.7644	.9026	.0473	.2099	.4130	.7214	.7843
56	.0796	.3034	.4778	.6287	.7690	.9073	.0523	.2159	.4210	.7355	.9610
58	.0884	.3098	.4831	.6335	.7736	.9120	.0574	.2219	.4291	.7501	2.2627

Geograph. Latitude.		Angle of Vertical.		Diff.	Logarithm of Earth's Radius.	Diff.	Deg. of Meridian. English Feet.	Diff.	Deg. of Parallel. English Feet.	Diff.
°	′	′	″	″						
0	0	0	0.00	24.02	0.0000000	4	362748.33	1.10	365185.71	55.24
1	0	0	24.02	24.00	9.9999996	14	749.43	3.32	5130.47	165.73
2	0	0	48.02	23.93	9982	21	752.75	5.53	4964.74	276.17
3	0	1	11.95	23.85	9961	31	758.28	7.72	4688.57	386.52
4	0	1	35.80	23.74	9930	39	766.00	9.91	4302.05	496.76
5	0	1	59.54	23.58	9891	48	775.91	12.10	3805.29	606.86
6	0	2	23.12	23.42	9.9999843	57	362788.01	14.26	363198.43	716.79
7	0	2	46.54	23.22	9786	65	802.27	16.41	2481.64	826.51
8	0	3	9.76	22.98	9721	73	818.68	18.54	1655.13	936.00
9	0	3	32.74	22.73	9648	82	837.22	20.64	0719.13	1045.21
10	0	3	55.47	22.45	9566	90	857.86	22.73	359673.92	1154.13
11	0	4	17.92	22.14	9476	99	880.59	24.78	8519.79	1262.72
12	0	4	40.06	21.79	9.9999377	106	362905.37	26.81	357257.07	1370.94
13	0	5	1.85	21.43	9271	114	932.18	28.80	5886.13	1478.77
14	0	5	23.28	21.05	9157	122	960.98	30.76	4407.36	1586.17
15	0	5	44.33	20.62	9035	130	991.74	32.68	2821.19	1693.12
16	0	6	4.95	20.19	8905	137	363024.42	34.57	1128.07	1799.57
17	0	6	25.14	19.72	8768	144	058.99	36.41	349328.50	1905.52
18	0	6	44.86	19.23	9.9998624	152	363095.40	38.21	347422.98	2010.91
19	0	7	4.09	18.71	8472	158	133.61	39.96	5412.07	2115.71
20	0	7	22.80	18.19	8314	165	173.57	41.66	3296.36	2219.91
21	0	7	40.99	17.62	8149	172	215.23	43.33	1076.45	2323.47
22	0	7	58.61	17.05	7977	178	258.56	44.93	338752.98	2426.36
23	0	8	15.66	16.44	7799	185	303.49	46.47	6326.62	2528.54
24	0	8	32.10	15.83	9.9997614	190	363349.96	47.97	333798.08	2629.98
25	0	8	47.93	15.19	7424	196	397.93	49.41	1168.10	2730.66
26	0	9	3.12	14.53	7228	201	447.34	50.79	328437.44	2830.55
27	0	9	17.65	13.85	7027	207	498.13	52.11	5606.89	2929.62
28	0	9	31.50	13.16	6820	212	550.24	53.36	2677.27	3027.83
29	0	9	44.66	12.46	6608	216	603.60	54.54	319649.44	3125.15
30	0	9	57.12	2.00	9.9996392	37	363658.14	9.20	316524.29	530.26
	10		59.12	1.99	6355	36	667.34	9.24	5994.03	532.93
	20	10	1.11	1.96	6319	37	676.58	9.26	5461.10	535.60
	30		3.07	1.95	6282	37	685.84	9.30	4925.50	538.27
	40		5.02	1.92	6245	37	695.14	9.32	4387.23	540.92
	50		6.94	1.91	6208	37	704.46	9.35	3846.31	543.59
31	0	10	8.85	1.88	9.9996171	37	363713.81	9.39	313302.72	546.24
	10		10.73	1.86	6134	38	723.20	9.41	2756.48	548.88
	20		12.59	1.85	6096	37	732.61	9.44	2207.60	551.52
	30		14.44	1.82	6059	38	742.05	9.47	1656.08	554.17
	40		16.26	1.80	6021	37	751.52	9.50	1101.91	556.80
	50		18.06	1.78	5984	38	761.02	9.52	0545.11	559.43
32	0	10	19.84	1.76	9.9995946	38	363770.54	9.56	309985.68	562.05
	10		21.60	1.74	5908	38	780.10	9.58	9423.63	564.68
	20		23.34	1.71	5870	38	789.68	9.61	8858.95	567.29
	30		25.05	1.70	5832	38	799.29	9.63	8291.66	569.90
	40		26.75	1.68	5794	39	808.92	9.66	7721.76	572.51
	50		28.43	1.65	5755	38	818.58	9.69	7149.25	575.11
33	0	10	30.08	1.63	9.9995717	39	363828.27	9.71	306574.14	577.71
	10		31.71	1.61	5678	38	837.98	9.74	5996.43	580.30
	20		33.32	1.59	5640	39	847.72	9.76	5416.13	582.89
	30		34.91	1.57	5601	39	857.48	9.78	4833.24	585.47
	40		36.48	1.55	5562	39	867.26	9.81	4247.77	588.05
	50		38.03	1.52	5523	39	877.07	9.84	3659.72	590.62
34	0	10	39.55	1.51	9.9995484	39	363886.91	9.86	303069.10	593.19
	10		41.06	1.48	5445	39	896.77	9.88	2475.91	595.75
	20		42.54	1.46	5406	39	906.65	9.90	1880.16	598.32
	30		44.00	1.44	5367	40	916.55	9.93	1281.84	600.87
	40		45.44	1.42	5327	39	926.48	9.95	0680.97	603.42
	50		46.86	1.39	5288	40	936.43	9.97	0077.55	605.95
35	0	10	48.25		9.9995248		363946.40		299471.60	

Geograph. Latitude.		Angle of Vertical.		Diff.	Logarithm of Earth's Radius.	Diff.	Deg. of Meridian. English Feet.	Diff	Deg. of Parallel. English Feet.	Diff.
°	'	'	"	"						
35	0	10	48.25	1.38	9.9995248	40	363946.40	9.99	299471.60	608.51
	10		49.63	1.35	5208	39	956.39	10.02	8863.09	611.03
	20		50.98	1.33	5169	40	966.41	10.03	8252.06	613.57
	30		52.31	1.31	5129	40	976.44	10.05	7638.49	616.09
	40		53.62	1.28	5089	40	986.49	10.08	7022.40	618.60
	50		54.90	1.26	5049	40	996.57	10.10	6403.80	621.13
36	0	10	56.16	1.25	9.9995009	40	364006.67	10.11	295782.67	623.63
	10		57.41	1.22	4969	40	016.78	10.13	5159.04	626.14
	20		58.63	1.19	4929	41	026.91	10.16	4532.90	628.63
	30		59.82	1.18	4888	40	037.07	10.17	3904.27	631.13
	40	11	1.00	1.15	4848	41	047.24	10.19	3273.14	633.62
	50		2.15	1.13	4807	40	057.43	10.21	2639.52	636.10
37	0	11	3.28	1.11	9.9994767	41	364067.64	10.22	292003.42	638.58
	10		4.39	1.08	4726	40	077.86	10.24	1364.84	641.05
	20		5.47	1.07	4686	41	088.10	10.26	0723.79	643.51
	30		6.54	1.04	4645	41	098.36	10.28	0080.28	645.98
	40		7.58	1.01	4604	41	108.64	10.29	289434.30	648.44
	50		8.59	1.00	4563	41	118.93	10.31	8785.86	650.89
38	0	11	9.59	.97	9.9994522	41	364129.24	10.32	288134.97	653.34
	10		10.56	.95	4481	41	139.56	10.34	7481.63	655.78
	20		11.51	.93	4440	41	149.90	10.35	6825.85	658.21
	30		12.44	.90	4399	41	160.25	10.37	6167.64	660.65
	40		13.34	.88	4358	41	170.62	10.38	5506.99	663.06
	50		14.22	.86	4317	41	181.00	10.40	4843.93	665.49
39	0	11	15.08	.84	9.9994276	42	364191.40	10.40	284178.44	667.90
	10		15.92	.81	4234	41	201.80	10.42	3510.54	670.31
	20		16.73	.79	4193	41	212.22	10.44	2840.23	672.71
	30		17.52	.77	4152	42	222.66	10.45	2167.52	675.11
	40		18.29	.75	4110	41	233.11	10.46	1492.41	677.50
	50		19.04	.72	4069	42	243.57	10.47	0814.91	679.90
40	0	11	19.76	.70	9.9994027	42	364254.04	10.48	280135.01	682.26
	10		20.46	.67	3985	41	264.52	10.49	279452.75	684.65
	20		21.13	.66	3944	42	275.01	10.50	8768.10	687.01
	30		21.79	.63	3902	42	285.51	10.52	8081.09	689.38
	40		22.42	.60	3860	41	296.03	10.52	7391.71	691.74
	50		23.02	.59	3819	42	306.55	10.53	6699.97	694.08
41	0	11	23.61	.56	9.9993777	42	364317.08	10.54	276005.89	696.43
	10		24.17	.53	3735	42	327.62	10.55	5309.46	698.78
	20		24.70	.52	3693	42	338.17	10.56	4610.68	701.12
	30		25.22	.49	3651	42	348.73	10.57	3909.56	703.43
	40		25.71	.47	3609	42	359.30	10.57	3206.13	705.77
	50		26.18	.44	3567	42	369.87	10.58	2500.36	708.08
42	0	11	26.62	.42	9.9993525	42	364380.45	10.59	271792.28	710.39
	10		27.04	.40	3483	42	391.04	10.60	1081.89	712.70
	20		27.44	.38	3441	42	401.64	10.60	0369.19	715.00
	30		27.82	.35	3399	42	412.24	10.61	269654.19	717.29
	40		28.17	.33	3357	42	422.85	10.61	8936.90	719.58
	50		28.50	.30	3315	42	433.46	10.62	8217.32	721.87
43	0	11	28.80	.28	9.9993273	43	364444.08	10.62	267495.45	724.14
	10		29.08	.26	3230	42	454.70	10.63	6771.31	726.41
	20		29.34	.24	3188	42	465.33	10.63	6044.90	728.68
	30		29.58	.21	3146	42	475.96	10.63	5316.22	730.93
	40		29.79	.19	3104	42	486.59	10.64	4585.29	733.19
	50		29.98	.16	3062	43	497.23	10.64	3852.10	735.43
44	0	11	30.14	.15	9.9993019	42	364507.87	10.65	263116.67	737.67
	10		30.29	.12	2977	42	518.52	10.64	2379.00	739.91
	20		30.41	.09	2935	43	529.16	10.65	1639.09	742.13
	30		30.50	.07	2892	42	539.81	10.65	0896.96	744.36
	40		30.57	.05	2850	42	550.46	10.66	0152.60	746.57
	50		30.62	.03	2808	42	561.12	10.65	259406.03	748.78
45	0		30.65		9.9992766		364571.77		258657.25	

Geograph. Latitude.		Angle of Vertical.		Diff.	Logarithm of Earth's Radius.	Diff.	Deg. of Meridian. English Feet.	Diff.	Deg. of Parallel. English Feet.	Diff.
°	′	′	″	″						
45	0	11	30.65	.00	9.9992766	43	364571.77	10.65	258657.25	750.99
	10		30.65	.02	2723	42	582.42	10.66	7906.26	753.18
	20		30.63	.05	2681	42	593.08	10.65	7153.08	755.37
	30		30.58	.07	2639	43	603.73	10.65	6397.71	757.56
	40		30.51	.09	2596	42	614.38	10.65	5640.15	759.73
	50		30.42	.11	2554	42	625.03	10.65	4880.42	761.91
46	0	11	30.31	.14	9.9992512	42	364635.68	10.65	254118.51	764.08
	10		30.17	.16	2470	43	646.33	10.65	3354.43	766.23
	20		30.01	.19	2427	42	656.98	10.64	2588.20	768.38
	30		29.82	.21	2385	42	667.62	10.64	1819.82	770.53
	40		29.61	.23	2343	43	678.26	10.64	1049.29	772.67
	50		29.38	.26	2300	42	688.90	10.64	0276.62	774.81
47	0	11	29.12	.27	9.9992258	42	364699.54	10.63	249501.81	776.93
	10		28.85	.31	2216	42	710.17	10.63	8724.88	779.05
	20		28.54	.32	2174	42	720.80	10.62	7945.83	781.17
	30		28.22	.35	2132	43	731.42	10.61	7164.66	783.27
	40		27.87	.37	2089	42	742.03	10.62	6381.39	785.38
	50		27.50	.40	2047	42	752.65	10.60	5596.01	787.47
48	0	11	27.10	.41	9.9992005	42	364763.25	10.60	244808.54	789.56
	10		26.69	.45	1963	42	773.85	10.60	4018.98	791.64
	20		26.24	.46	1921	42	784.45	10.59	3227.34	793.72
	30		25.78	.49	1879	42	795.04	10.58	2433.62	795.78
	40		25.29	.51	1837	42	805.62	10.57	1637.84	797.85
	50		24.78	.54	1795	42	816.19	10.56	0839.99	799.90
49	0	11	24.24	.55	9.9991752	42	364826.75	10.56	240040.09	801.95
	10		23.69	.58	1711	42	837.31	10.55	239238.14	803.99
	20		23.11	.61	1669	42	847.86	10.54	8434.15	806.03
	30		22.50	.63	1627	41	858.40	10.53	7628.12	808.05
	40		21.87	.65	1586	42	868.93	10.52	6820.07	810.08
	50		21.22	.67	1544	42	879.45	10.51	6009.99	812.09
50	0	11	20.55	.70	9.9991502	42	364889.96	10.50	235197.90	814.10
	10		19.85	.72	1460	41	900.46	10.49	4383.80	816.10
	20		19.13	.74	1419	42	910.95	10.48	3567.70	818.10
	30		18.39	.76	1377	42	921.43	10.47	2749.60	820.08
	40		17.63	.79	1335	41	931.90	10.46	1929.52	822.06
	50		16.84	.82	1294	42	942.36	10.44	1107.46	824.04
51	0	11	16.02	.83	9.9991252	41	364952.80	10.43	230283.42	826.00
	10		15.19	.86	1211	41	963.23	10.42	229457.42	827.97
	20		14.33	.88	1170	42	973.65	10.41	8629.45	829.91
	30		13.45	.90	1128	41	984.06	10.39	7799.54	831.87
	40		12.55	.93	1087	41	994.45	10.38	6967.67	833.80
	50		11.62	.95	1046	41	365004.83	10.37	6133.87	835.74
52	0	11	10.67	.97	9.9991005	42	365015.20	10.35	225298.13	837.66
	10		9.70	.99	0963	41	025.55	10.33	4460.47	839.57
	20		8.71	1.02	0922	41	035.88	10.32	3620.90	841.49
	30		7.69	1.03	0881	41	046.20	10.31	2779.41	843.40
	40		6.66	1.06	0840	40	056.51	10.28	1936.01	845.29
	50		5.60	1.09	0800	41	066.79	10.27	1090.72	847.18
53	0	11	4.51	1.11	9.9990759	41	365077.06	10.26	220243.54	849.07
	10		3.40	1.13	0718	41	087.32	10.24	219394.47	850.93
	20		2.27	1.15	0677	40	097.56	10.22	8543.54	852.81
	30		1.12	1.18	0637	41	107.78	10.20	7690.73	854.67
	40	10	59.94	1.20	0596	40	117.98	10.19	6836.06	856.53
	50		58.74	1.22	0556	41	128.17	10.17	5979.53	858.37
54	0	10	57.52	1.24	9.9990515	40	365138.34	10.15	215121.16	860.21
	10		56.28	1.26	0475	40	148.49	10.12	4260.95	862.05
	20		55.02	1.29	0435	40	158.61	10.11	3398.90	863.86
	30		53.73	1.31	0395	40	168.72	10.09	2535.04	865.69
	40		52.42	1.33	0355	40	178.81	10.07	1669.35	867.50
	50		51.09	1.35	0315	40	188.88	10.05	210801.85	869.30
55	0	10	49.74		9.9990275		365198.93		209932.55	

Geograph. Latitude.		Angle of Vertical.		Diff.	Logarithm of Earth's Radius.	Diff.	Deg. of Meridian. English Feet.	Diff.	Deg. of Parallel. English Feet.	Diff.
°	'	'	"	"						
55	0	10	49.74	1.38	9.9990275	40	365198.93	10.03	209932.55	871.11
	10		48.36	1.39	0235	40	208.96	10.01	9061.44	872.88
	20		46.97	1.42	0195	40	218.97	9.99	8188.56	874.67
	30		45.55	1.44	0155	39	228.96	9.96	7313.89	876.45
	40		44.11	1.46	0116	40	238.92	9.94	6437.44	878.21
	50		42.65	1.49	0076	39	248.86	9.92	5559.23	879.97
56	0	10	41.16	1.51	9.9990037	39	365258.78	9.90	204679.26	881.72
	10		39.65	1.52	9.9989998	40	268.68	9.87	3797.54	883.47
	20		38.13	1.55	9958	39	278.55	9.85	2914.07	885.20
	30		36.58	1.57	9919	39	288.40	9.83	2028.87	886.94
	40		35.01	1.60	9880	39	298.23	9.80	1141.93	888.65
	50		33.41	1.61	9841	39	308.03	9.77	200253.28	890.38
57	0	10	31.80	1.64	9.9989802	38	365317.80	9.76	199362.90	892.08
	10		30.16	1.66	9764	39	327.56	9.73	8470.82	893.78
	20		28.50	1.67	9725	39	337.29	9.70	7577.04	895.47
	30		26.83	1.70	9686	38	346.99	9.67	6681.57	897.15
	40		25.13	1.73	9648	38	356.66	9.65	5784.42	898.84
	50		23.40	1.74	9610	39	366.31	9.62	4885.58	900.50
58	0	10	21.66	1.76	9.9989571	38	365375.93	9.60	193985.08	902.17
	10		19.90	1.79	9533	38	385.53	9.57	3082.91	903.82
	20		18.11	1.80	9495	38	395.10	9.54	2179.09	905.46
	30		16.31	1.83	9457	38	404.64	9.52	1273.63	907.11
	40		14.48	1.85	9419	37	414.16	9.49	0366.52	908.74
	50		12.63	1.86	9382	38	423.65	9.45	189457.78	910.35
59	0	10	10.77	1.89	9.9989344	37	365433.10	9.43	188547.43	911.98
	10		8.88	1.91	9307	38	442.53	9.40	7635.45	913.59
	20		6.97	1.93	9269	37	451.93	9.37	6721.86	915.19
	30		5.04	1.96	9232	37	461.30	9.34	5806.67	916.77
	40		3.08	1.97	9195	37	470.64	9.31	4889.90	918.37
	50		1.11	1.99	9158	37	479.95	9.28	3971.53	919.94
60	0	9	59.12	12.38	9.9989121	219	365489.23	55.02	183051.59	5552.34
61	0	9	46.74	13.09	8902	214	544.25	53.85	177499.25	5607.15
62	0	9	33.65	13.80	8688	209	598.10	52.60	171892.10	5660.26
63	0	9	19.85	14.49	8479	204	650.70	51.30	166231.84	5711.63
64	0	9	5.36	15.15	8275	199	702.00	49.94	160520.21	5761.26
65	0	8	50.21	15.81	8077	193	751.94	48.50	154758.95	5809.12
66	0	8	34.40	16.43	9.9987884	187	365800.44	47.01	148949.83	5855.20
67	0	8	17.97	17.05	7697	180	847.45	45.46	143094.63	5899.49
68	0	8	0.92	17.63	7517	175	892.91	43.86	137195.14	5941.96
69	0	7	43.29	18.21	7342	168	936.77	42.20	131253.18	5982.61
70	0	7	25.08	18.75	7174	161	978.97	40.48	125270.57	6021.43
71	0	7	6.33	19.27	7013	154	366019.45	38.72	119249.14	6058.36
72	0	6	47.06	19.78	9.9986859	146	366058.17	36.91	113190.78	6093.46
73	0	6	27.28	20.25	6713	140	095.08	35.06	107097.32	6126.66
74	0	6	7.03	20.70	6573	132	130.14	33.16	100970.66	6157.96
75	0	5	46.33	21.13	6441	124	163.30	31.21	94812.70	6187.38
76	0	5	25.20	21.53	6317	116	194.51	29.23	88625.32	6214.88
77	0	5	3.67	21.90	6201	108	223.74	27.22	82410.44	6240.47
78	0	4	41.77	22.24	9.9986093	100	366250.96	25.17	76169.97	6264.10
79	0	4	19.53	22.57	5993	92	276.13	23.08	69905.87	6285.80
80	0	3	56.96	22.86	5901	83	299.21	20.98	63620.07	6305.54
81	0	3	34.10	23.12	5818	75	320.19	18.83	57314.53	6323.36
82	0	3	10.98	23.35	5743	67	339.02	16.67	50991.17	6339.19
83	0	2	47.63	23.56	5676	57	355.69	14.50	44651.98	6353.05
84	0	2	24.07	23.74	9.9985619	49	366370.19	12.30	38298.93	6364.96
85	0	2	0.33	23.89	5570	40	382.49	10.08	31933.97	6374.89
86	0	1	36.44	24.01	5530	32	392.57	7.86	25559.08	6382.81
87	0	1	12.43	24.09	5498	22	400.43	5.61	19176.27	6388.78
88	0	0	48.34	24.16	5476	13	406.04	3.38	12787.49	6392.75
89	0	0	24.18	24.18	5463	5	409.42	1.12	6394.74	6394.74
90	0	0	0.00		9.9985458		366410.54		0000.00	

TABLE XIII.

Augmentation of the Moon's Semi-diameter, on account of her apparent Altitude.

App. Alt.	Horizontal Semi-diameter.					
	′ ″ 14 30	′ ″ 15 0	′ ″ 15 30	′ ″ 16 0	′ ″ 16 30	′ ″ 17 0
	″	″	″	″	″	″
0	0.10	0.12	0.13	0.14	0.15	0.17
2	0.58	0.62	0.66	0.71	0.76	0.81
4	1.05	1.12	1.20	1.28	1.37	1.46
6	1.51	1.62	1.74	1.86	1.98	2.10
8	1.98	2.12	2.27	2.42	2.58	2.75
10	2.44	2.62	2.80	2.99	3.18	3.39
12	2.90	3.11	3.33	3.56	3.78	4.02
14	3.36	3.61	3.86	4.11	4.37	4.66
16	3.82	4.10	4.38	4.67	4.97	5.28
18	4.28	4.58	4.89	5.22	5.56	5.90
20	4.72	5.06	5.41	5.76	6.14	6.52
22	5.16	5.53	5.91	6.30	6.71	7.13
24	5.60	5.99	6.41	6.83	7.27	7.72
26	6.03	6.45	6.90	7.35	7.83	8.31
28	6.45	6.91	7.38	7.87	8.37	8.89
30	6.86	7.35	7.85	8.37	8.91	9.46
32	7.27	7.78	8.32	8.87	9.44	10.02
34	7.67	8.21	8.77	9.35	9.95	10.57
36	8.06	8.62	9.22	9.83	10.46	11.11
38	8.43	9.03	9.65	10.29	10.95	11.63
40	8.80	9.42	10.07	10.74	11.43	12.14
42	9.16	9.80	10.48	11.17	11.89	12.63
44	9.51	10.17	10.88	11.60	12.34	13.11
46	9.84	10.54	11.26	12.01	12.78	13.57
48	10.16	10.88	11.63	12.40	13.20	14.02
50	10.48	11.22	11.99	12.78	13.60	14.45
52	10.78	11.54	12.33	13.15	13.99	14.86
54	11.07	11.84	12.65	13.50	14.36	15.25
56	11.34	12.14	12.97	13.83	14.72	15.63
58	11.60	12.42	13.27	14.15	15.05	15.99
60	11.84	12.68	13.55	14.44	15.37	16.32
62	12.07	12.93	13.81	14.73	15.67	16.64
64	12.29	13.16	14.06	14.99	15.95	16.94
66	12.49	13.37	14.29	15.24	16.21	17.22
68	12.68	13.58	14.50	15.46	16.45	17.47
70	12.85	13.76	14.70	15.67	16.67	17.71
72	13.00	13.92	14.88	15.86	16.88	17.92
74	13.14	14.07	15.04	16.03	17.06	18.12
76	13.27	14.21	15.18	16.18	17.22	18.29
78	13.38	14.32	15.30	16.31	17.36	18.43
80	13.47	14.42	15.40	16.42	17.47	18.56
82	13.54	14.50	15.49	16.51	17.57	18.66
84	13.60	14.56	15.56	16.59	17.65	18.74
86	13.64	14.61	15.60	16.64	17.70	18.80
88	13.67	14.63	15.63	16.67	17.73	18.83
90	13.67	14.63	15.64	16.68	17.74	18.85

TABLE XIV.

Reduction of the Moon's equatorial Parallax.

Latitude.	Moon's Equatorial Parallax.			Latitude.	Moon's Equatorial Parallax.		
	53′	57′	61′		53′	57′	61′
°	″	″	″	°	″	″	″
0	0.00	0.00	0.00	45	5.29	5.69	6.09
1	0.00	0.00	0.00	46	5.48	5.89	6.30
2	0.01	0.01	0.01	47	5.66	6.09	6.52
3	0.03	0.03	0.03	48	5.85	6.29	6.73
4	0.05	0.06	0.06	49	6.03	6.49	6.94
5	0.08	0.09	0.09	50	6.22	6.69	7.15
6	0.12	0.12	0.13	51	6.40	6.88	7.36
7	0.16	0.17	0.18	52	6.58	7.08	7.57
8	0.20	0.22	0.24	53	6.76	7.27	7.78
9	0.26	0.28	0.30	54	6.94	7.46	7.98
10	0.32	0.34	0.37	55	7.11	7.65	8.19
11	0.38	0.41	0.44	56	7.29	7.84	8.39
12	0.46	0.49	0.52	57	7.46	8.02	8.58
13	0.54	0.57	0.61	58	7.63	8.20	8.78
14	0.62	0.66	0.71	59	7.79	8.38	8.97
15	0.71	0.76	0.81	60	7.96	8.56	9.16
16	0.80	0.86	0.92	61	8.12	8.73	9.34
17	0.90	0.97	1.04	62	8.27	8.90	9.52
18	1.01	1.08	1.16	63	8.42	9.06	9.70
19	1.12	1.20	1.29	64	8.57	9.22	9.87
20	1.24	1.33	1.42	65	8.72	9.38	10.04
21	1.36	1.46	1.56	66	8.86	9.53	10.20
22	1.48	1.59	1.70	67	9.00	9.67	10.35
23	1.61	1.73	1.85	68	9.13	9.81	10.50
24	1.75	1.88	2.01	69	9.26	9.95	10.65
25	1.89	2.03	2.17	70	9.38	10.09	10.79
26	2.03	2.18	2.33	71	9.50	10.21	10.93
27	2.18	2.34	2.50	72	9.61	10.33	11.06
28	2.33	2.50	2.68	73	9.71	10.45	11.18
29	2.48	2.67	2.86	74	9.82	10.56	11.30
30	2.64	2.84	3.04	75	9.91	10.66	11.41
31	2.80	3.01	3.23	76	10.00	10.76	11.51
32	2.97	3.19	3.42	77	10.09	10.85	11.61
33	3.14	3.37	3.61	78	10.17	10.93	11.70
34	3.31	3.55	3.80	79	10.24	11.01	11.78
35	3.48	3.74	4.00	80	10.31	11.08	11.86
36	3.65	3.93	4.20	81	10.37	11.15	11.93
37	3.83	4.12	4.41	82	10.42	11.21	11.99
38	4.01	4.31	4.61	83	10.47	11.26	12.05
39	4.19	4.50	4.82	84	10.51	11.31	12.10
40	4.37	4.70	5.03	85	10.55	11.35	12.14
41	4.55	4.90	5.24	86	10.58	11.38	12.17
42	4.74	5.09	5.45	87	10.60	11.40	12.20
43	4.92	5.29	5.66	88	10.62	11.42	12.22
44	5.11	5.49	5.88	89	10.63	11.43	12.23
45	5.29	5.69	6.09	90	10.63	11.43	12.23

Parallax of the Sun and Planets at different Altitudes.

Alt.	Horizontal Parallax.																	Alt.
	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	20″	30″	8″.4	8″.5	8″.6	8″.7	8″.8	
°	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	20.0	30.0	8.40	8.50	8.60	8.70	8.80	0
2	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	20.0	30.0	8.39	8.49	8.59	8.69	8.79	2
4	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	20.0	29.9	8.38	8.48	8.58	8.68	8.78	4
6	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	9.9	19.9	29.8	8.35	8.45	8.55	8.65	8.75	6
8	1.0	2.0	3.0	4.0	5.0	5.9	6.9	7.9	8.9	9.9	19.8	29.7	8.32	8.42	8.52	8.62	8.71	8
10	1.0	2.0	3.0	3.9	4.9	5.9	6.9	7.9	8.9	9.8	19.7	29.5	8.27	8.37	8.47	8.57	8.67	10
12	1.0	2.0	2.9	3.9	4.9	5.9	6.8	7.8	8.8	9.8	19.6	29.3	8.22	8.31	8.41	8.51	8.61	12
14	1.0	1.9	2.9	3.9	4.9	5.8	6.8	7.8	8.7	9.7	19.4	29.1	8.15	8.25	8.34	8.44	8.54	14
16	1.0	1.9	2.9	3.8	4.8	5.8	6.7	7.7	8.7	9.6	19.2	28.8	8.07	8.17	8.27	8.36	8.46	16
18	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.6	8.6	9.5	19.0	28.5	7.99	8.08	8.18	8.27	8.37	18
20	0.9	1.9	2.8	3.8	4.7	5.6	6.6	7.5	8.5	9.4	18.8	28.2	7.89	7.99	8.08	8.18	8.27	20
22	0.9	1.9	2.8	3.7	4.6	5.6	6.5	7.4	8.3	9.3	18.5	27.8	7.79	7.88	7.97	8.07	8.16	22
24	0.9	1.8	2.7	3.7	4.6	5.5	6.4	7.3	8.2	9.1	18.3	27.4	7.67	7.77	7.86	7.95	8.04	24
26	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	18.0	27.0	7.55	7.64	7.73	7.82	7.91	26
28	0.9	1.8	2.6	3.5	4.4	5.3	6.2	7.1	7.9	8.8	17.7	26.5	7.42	7.51	7.59	7.68	7.77	28
30	0.9	1.7	2.6	3.5	4.3	5.2	6.1	6.9	7.8	8.7	17.3	26.0	7.27	7.36	7.45	7.53	7.62	30
32	0.8	1.7	2.5	3.4	4.2	5.1	5.9	6.8	7.6	8.5	17.0	25.4	7.12	7.21	7.29	7.38	7.46	32
34	0.8	1.7	2.5	3.3	4.1	5.0	5.8	6.6	7.5	8.3	16.6	24.9	6.96	7.05	7.13	7.21	7.30	34
36	0.8	1.6	2.4	3.2	4.0	4.9	5.7	6.5	7.3	8.1	16.2	24.3	6.80	6.88	6.96	7.04	7.12	36
38	0.8	1.6	2.4	3.2	3.9	4.7	5.5	6.3	7.1	7.9	15.8	23.6	6.62	6.70	6.78	6.86	6.93	38
40	0.8	1.5	2.3	3.1	3.8	4.6	5.4	6.1	6.9	7.7	15.3	23.0	6.43	6.51	6.59	6.66	6.74	40
42	0.7	1.5	2.2	3.0	3.7	4.5	5.2	5.9	6.7	7.4	14.9	22.3	6.24	6.32	6.39	6.47	6.54	42
44	0.7	1.4	2.2	2.9	3.6	4.3	5.0	5.8	6.5	7.2	14.4	21.6	6.04	6.11	6.19	6.26	6.33	44
46	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	6.9	13.9	20.8	5.84	5.90	5.97	6.04	6.11	46
48	0.7	1.3	2.0	2.7	3.3	4.0	4.7	5.4	6.0	6.7	13.4	20.1	5.62	5.69	5.75	5.82	5.89	48
50	0.6	1.3	1.9	2.6	3.2	3.9	4.5	5.1	5.8	6.4	12.9	19.3	5.40	5.46	5.53	5.59	5.66	50
52	0.6	1.2	1.8	2.5	3.1	3.7	4.3	4.9	5.5	6.2	12.3	18.5	5.17	5.23	5.29	5.36	5.42	52
54	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3	5.9	11.8	17.6	4.94	5.00	5.05	5.11	5.17	54
56	0.6	1.1	1.7	2.2	2.8	3.4	3.9	4.5	5.0	5.6	11.2	16.8	4.70	4.75	4.81	4.86	4.92	56
58	0.5	1.1	1.6	2.1	2.6	3.2	3.7	4.2	4.8	5.3	10.6	15.9	4.45	4.50	4.56	4.61	4.66	58
60	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	10.0	15.0	4.20	4.25	4.30	4.35	4.40	60
62	0.5	0.9	1.4	1.9	2.3	2.8	3.3	3.8	4.2	4.7	9.4	14.1	3.94	3.99	4.04	4.08	4.13	62
64	0.4	0.9	1.3	1.8	2.2	2.6	3.1	3.5	3.9	4.4	8.8	13.2	3.68	3.73	3.77	3.81	3.86	64
66	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.7	4.1	8.1	12.2	3.42	3.46	3.50	3.54	3.58	66
68	0.4	0.7	1.1	1.5	1.9	2.2	2.6	3.0	3.4	3.7	7.5	11.2	3.15	3.18	3.22	3.26	3.30	68
70	0.3	0.7	1.0	1.4	1.7	2.1	2.4	2.7	3.1	3.4	6.8	10.3	2.87	2.91	2.94	2.98	3.01	70
72	0.3	0.6	0.9	1.2	1.5	1.9	2.2	2.5	2.8	3.1	6.2	9.3	2.60	2.63	2.66	2.69	2.72	72
74	0.3	0.6	0.8	1.1	1.4	1.7	1.9	2.2	2.5	2.8	5.5	8.3	2.32	2.34	2.37	2.40	2.43	74
76	0.2	0.5	0.7	1.0	1.2	1.5	1.7	1.9	2.2	2.4	4.8	7.3	2.03	2.06	2.08	2.10	2.13	76
78	0.2	0.4	0.6	0.8	1.0	1.2	1.5	1.7	1.9	2.1	4.2	6.2	1.75	1.77	1.79	1.81	1.83	78
80	0.2	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.7	3.5	5.2	1.46	1.48	1.49	1.51	1.53	80
82	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.3	1.4	2.8	4.2	1.17	1.18	1.20	1.21	1.22	82
84	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	2.1	3.1	0.88	0.89	0.90	0.91	0.92	84
86	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.6	0.7	1.4	2.1	0.59	0.59	0.60	0.61	0.61	86
88	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.3	0.7	1.0	0.29	0.30	0.30	0.30	0.31	88
90	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.00	0.00	0.00	0.00	90

Hour Angle.	Moon's true Declination, 0°.			Moon's true Declination, 5°.			Moon's true Declination, 10°.		
	Horizontal Parallax.			Horizontal Parallax.			Horizontal Parallax.		
	53′	57′	61′	53′	57′	61′	53′	57′	61′
Min.	s.	s.	s.	s.	s.	s.	s.	s.	s.
5	3.47	3.73	3.99	3.48	3.75	4.01	3.52	3.79	4.06
10	6.93	7.46	7.99	6.96	7.49	8.02	7.04	7.58	8.11
15	10.39	11.19	11.98	10.43	11.23	12.03	10.55	11.36	12.17
20	13.85	14.91	15.96	13.90	14.96	16.03	14.06	15.14	16.21
25	17.30	18.62	19.94	17.36	18.69	20.02	17.57	18.91	20.25
30	20.74	22.32	23.91	20.82	22.41	24.00	21.06	22.67	24.28
35	24.17	26.01	27.86	24.26	26.11	27.97	24.54	26.42	28.30
40	27.59	29.69	31.80	27.69	29.81	31.93	28.02	30.16	32.30
45	30.99	33.36	35.73	31.11	33.49	35.87	31.47	33.88	36.29
50	34.38	37.01	39.64	34.51	37.15	39.79	34.92	37.58	40.25
55	37.75	40.64	43.52	37.90	40.79	43.69	38.34	41.27	44.20
60	41.11	44.25	47.39	41.27	44.42	47.57	41.75	44.94	48.13
65	44.44	47.83	51.23	44.61	48.02	51.43	45.13	48.58	52.03
70	47.75	51.40	55.05	47.94	51.60	55.26	48.50	52.20	55.91
75	51.04	54.94	58.84	51.24	55.15	59.07	51.84	55.80	59.76
80	54.30	58.45	62.60	54.51	58.68	62.84	55.15	59.36	63.58
85	57.54	61.93	66.33	57.76	62.17	66.59	58.44	62.90	67.37
90	60.75	65.39	70.03	60.98	65.64	70.30	61.70	66.41	71.12
95	63.93	68.81	73.69	64.17	69.07	73.98	64.92	69.88	74.84
100	67.08	72.20	77.32	67.33	72.47	77.62	68.12	73.32	78.53
105	70.19	75.55	80.91	70.46	75.84	81.22	71.28	76.72	82.17
110	73.27	78.86	84.46	73.55	79.16	84.78	74.41	80.09	85.77
115	76.31	82.14	87.97	76.61	82.45	88.30	77.50	83.42	89.34
120	79.32	85.37	91.43	79.63	85.70	91.78	80.56	86.70	92.85
125	82.28	88.56	94.85	82.60	88.90	95.21	83.57	89.95	96.33
130	85.21	91.71	98.22	85.54	92.07	98.60	86.54	93.14	99.75
135	88.10	94.82	101.54	88.44	95.18	101.93	89.47	96.29	103.12
140	90.94	97.88	104.82	91.29	98.25	105.22	92.36	99.40	106.45
145	93.74	100.89	108.04	94.10	101.27	108.45	95.20	102.46	109.72
150	96.49	103.85	111.21	96.86	104.24	111.63	97.99	105.46	112.94
155	99.20	106.76	114.32	99.58	107.17	114.76	100.74	108.42	116.10
160	101.85	109.61	117.38	102.24	110.03	117.83	103.44	111.32	119.21
170	107.01	115.16	123.32	107.43	115.61	123.80	108.68	116.96	125.24
180	111.97	120.49	129.02	112.40	120.96	129.52	113.71	122.37	131.03
190	116.70	125.58	134.47	117.15	126.07	134.99	118.52	127.54	136.57
200	121.21	130.43	139.66	121.67	130.93	140.20	123.09	132.46	141.83
210	125.48	135.02	144.57	125.96	135.54	145.13	127.43	137.12	146.82
220	129.51	139.35	149.20	130.01	139.89	149.78	131.52	141.52	151.52
230	133.29	143.41	153.54	133.80	143.96	154.13	135.36	145.64	155.93
240	136.80	147.19	157.59	137.33	147.76	158.19	138.93	149.48	160.03
250	140.06	150.69	161.32	140.60	151.27	161.94	142.23	153.03	163.83
260	143.04	153.89	164.75	143.59	154.48	165.38	145.26	156.28	167.30
270	145.75	156.80	167.85	146.31	157.40	168.50	148.00	159.23	170.46
280	148.17	159.40	170.63	148.74	160.01	171.29	150.47	161.87	173.28
290	150.31	161.70	173.09	150.89	162.32	173.75	152.64	164.20	175.77
300	152.16	163.68	175.20	152.75	164.31	175.87	154.52	166.22	177.92
310	153.72	165.35	176.99	154.31	165.99	177.66	156.10	167.91	179.72
320	154.99	166.71	178.43	155.58	167.34	179.11	157.38	169.28	181.19
330	155.95	167.74	179.53	156.55	168.38	180.22	158.36	170.33	182.30
340	156.62	168.45	180.29	157.22	169.10	180.98	159.04	171.05	183.07
350	156.99	168.85	180.70	157.59	169.49	181.39	159.42	171.45	183.49
360	157.06	168.92	180.77	157.66	169.56	181.46	159.49	171.52	183.56
370				157.43	169.31	181.18	159.25	171.26	183.27
380							158.72	170.68	182.64

Hour Angle.	Moon's true Declination, 15°.			Moon's true Declination, 20°.			Moon's true Declination, 25°.		
	Horizontal Parallax.			Horizontal Parallax.			Horizontal Parallax.		
	53′	57′	61′	53′	57′	61′	53′	57′	61′
Min.	s.	s.	s.	s.	s.	s.	s.	s.	s.
5	3.59	3.86	4.14	3.69	3.97	4.26	3.83	4.12	4.41
10	7.17	7.73	8.28	7.38	7.94	8.51	7.66	8.24	8.83
15	10.76	11.59	12.41	11.07	11.91	12.76	11.48	12.36	13.24
20	14.34	15.44	16.54	14.74	15.87	17.00	15.30	16.47	17.64
25	17.91	19.28	20.65	18.42	19.83	21.24	19.11	20.57	22.03
30	21.48	23.12	24.76	22.08	23.77	25.46	22.91	24.66	26.41
35	25.03	26.94	28.86	25.74	27.71	29.68	26.70	28.74	30.78
40	28.57	30.75	32.94	29.38	31.62	33.87	30.47	32.80	35.14
45	32.10	34.55	37.01	33.00	35.53	38.05	34.23	36.85	39.47
50	35.61	38.33	41.05	36.61	39.41	42.21	37.98	40.88	43.79
55	39.10	42.09	45.08	40.20	43.28	46.36	41.70	44.89	48.09
60	42.57	45.83	49.08	43.78	47.12	50.47	45.41	48.88	52.36
65	46.03	49.54	53.06	47.32	50.94	54.57	49.09	52.84	56.60
70	49.45	53.23	57.02	50.85	54.74	58.63	52.75	56.78	60.82
75	52.86	56.90	60.94	54.35	58.51	62.67	56.38	60.69	65.01
80	56.24	60.54	64.84	57.83	62.25	66.67	59.98	64.57	69.16
85	59.59	64.14	68.70	61.28	65.96	70.65	63.56	68.42	73.28
90	62.91	67.72	72.53	64.69	69.63	74.58	67.10	72.23	77.37
95	66.21	71.26	76.32	68.08	73.28	78.48	70.61	76.01	81.41
100	69.47	74.77	80.08	71.43	76.88	82.34	74.09	79.75	85.42
105	72.69	78.24	83.80	74.74	80.45	86.17	77.53	83.45	89.38
110	75.88	81.67	87.47	78.02	83.98	89.95	80.93	87.11	93.30
115	79.03	85.06	91.10	81.26	87.47	93.68	84.29	90.73	97.17
120	82.14	88.41	94.69	84.46	90.91	97.37	87.61	94.30	101.00
125	85.22	91.72	98.23	87.62	94.31	101.01	90.88	97.82	104.77
130	88.25	94.98	101.72	90.74	97.67	104.60	94.11	101.30	108.50
135	91.24	98.20	105.17	93.81	100.97	108.14	97.30	104.73	112.17
140	94.18	101.36	108.55	96.84	104.23	111.62	100.44	108.11	115.78
145	97.08	104.48	111.89	99.81	107.43	115.05	103.53	111.43	119.34
150	99.93	107.55	115.17	102.74	110.58	118.42	106.57	114.70	122.84
155	102.73	110.56	118.40	105.62	113.68	121.74	109.55	117.91	126.27
160	105.48	113.52	121.56	108.45	116.72	124.99	112.48	121.06	129.65
170	110.82	119.27	127.72	113.94	122.63	131.32	118.18	127.19	136.21
180	115.95	124.78	133.62	119.21	128.30	137.39	123.65	133.07	142.50
190	120.85	130.05	139.26	124.25	133.72	143.19	128.87	138.69	148.51
200	125.52	135.07	144.63	129.05	138.87	148.70	133.84	144.03	154.23
210	129.94	139.82	149.71	133.59	143.76	153.93	138.55	149.10	159.65
220	134.11	144.31	154.51	137.88	148.36	158.85	143.00	153.87	164.75
230	138.02	148.51	159.00	141.90	152.68	163.47	147.16	158.35	169.54
240	141.66	152.42	163.18	145.64	156.70	167.77	151.03	162.51	173.99
250	145.02	156.03	167.05	149.10	160.42	171.74	154.62	166.36	178.11
260	148.11	159.35	170.59	152.27	163.82	175.38	157.90	169.89	181.88
270	150.91	162.35	173.80	155.14	166.91	178.68	160.89	173.09	185.30
280	153.42	165.05	176.68	157.72	169.67	181.63	163.55	175.95	188.36
290	155.63	167.42	179.22	159.99	172.11	184.24	165.91	178.48	191.06
300	157.55	169.48	181.41	161.96	174.22	186.49	167.94	180.66	193.38
310	159.16	171.20	183.25	163.61	175.99	188.38	169.65	182.49	195.34
320	160.46	172.68	184.74	164.95	177.43	189.91	171.04	183.98	196.92
330	161.46	173.67	185.87	165.98	178.52	191.07	172.10	185.11	198.12
340	162.15	174.40	186.65	166.68	179.28	191.87	172.83	185.89	198.95
350	162.53	174.80	187.08	167.07	179.69	192.30	173.23	186.31	199.39
360	162.60	174.87	187.14	167.14	179.75	192.37	173.30	186.38	199.45
370	162.37	174.61	186.85	166.90	179.48	192.07	173.04	186.09	199.14
380	161.82	174.02	186.21	166.33	178.87	191.40	172.45	185.45	198.44
390	160.97	173.09	185.21	165.45	177.91	190.37	171.53	184.45	197.37
400	159.81	171.84	183.87	164.26	176.62	188.99	170.29	183.11	195.93
410				162.75	175.00	187.24	168.73	181.43	194.12
420				160.94	173.04	185.14	166.85	179.40	191.94

Hour Angle.	Moon's true Declination, 0°.			Moon's true Declination, 5° N.			Moon's true Declination, 10° N.		
	Horizontal Parallax.			Horizontal Parallax.			Horizontal Parallax.		
	53′	57′	61′	53′	57′	61′	53′	57′	61′
m.	″	″	″	″	″	″	″	″	″
0	2160.1	2325.2	2490.5	1945.9	2094.7	2243.8	1716.3	1847.7	1979.3
20	2160.0	2325.1	2490.4	1946.6	2095.5	2244.6	1717.8	1849.3	1981.1
40	2159.8	2324.7	2490.0	1948.8	2097.8	2247.0	1722.4	1854.2	1986.3
60	2159.3	2324.2	2489.3	1952.3	2101.5	2251.0	1729.9	1862.3	1994.9
80	2158.6	2323.4	2488.4	1957.2	2106.8	2256.6	1740.4	1873.5	2006.9
100	2157.8	2322.4	2487.3	1963.5	2113.5	2263.7	1753.7	1887.8	2022.2
120	2156.8	2321.3	2486.0	1971.0	2121.5	2272.2	1769.7	1905.0	2040.5
140	2155.6	2319.9	2484.4	1979.7	2130.8	2282.1	1788.3	1925.0	2061.8
160	2154.3	2318.3	2482.6	1989.6	2141.4	2293.3	1809.4	1947.6	2086.0
180	2152.8	2316.6	2480.7	2000.6	2153.1	2305.7	1832.8	1972.6	2112.7
200	2151.2	2314.8	2478.6	2012.5	2165.8	2319.2	1858.2	1999.9	2141.8
220	2149.4	2312.8	2476.3	2025.3	2179.4	2333.7	1885.4	2029.1	2173.0
240	2147.6	2310.7	2473.8	2038.9	2193.9	2349.1	1914.4	2060.1	2206.1
260	2145.7	2308.4	2471.3	2053.1	2209.1	2365.2	1944.7	2092.6	2240.8
280	2143.7	2306.1	2468.6	2067.9	2224.9	2381.9	1976.2	2126.4	2276.8
300	2141.7	2303.8	2465.9	2083.2	2241.1	2399.1	2008.7	2161.2	2313.9
320	2139.6	2301.3	2463.1	2098.7	2257.7	2416.7	2041.8	2196.7	2351.7
340	2137.5	2298.9	2460.3	2114.5	2274.4	2434.4	2075.3	2232.6	2389.9
360	2135.3	2296.4	2457.5	2130.3	2291.3	2452.3	2109.0	2268.7	2428.4
380							2142.6	2304.6	2466.7

MOON'S PARALLAX IN DECLINATION

Hour Angle.	Moon's true Declination, 0°.			Moon's true Declination, 5° S.			Moon's true Declination, 10° S.		
	Horizontal Parallax.			Horizontal Parallax.			Horizontal Parallax.		
	53′	57′	61′	53′	57′	61′	53′	57′	61′
m.	″	″	″	″	″	″	″	″	″
0	2160.1	2325.2	2490.5	2357.3	2537.3	2717.4	2536.1	2729.4	2923.0
20	2160.0	2325.1	2490.4	2356.4	2536.3	2716.4	2534.4	2727.6	2921.0
40	2159.8	2324.7	2490.0	2353.8	2533.4	2713.2	2529.3	2722.1	2915.1
60	2159.3	2324.2	2489.3	2349.3	2528.5	2708.0	2520.9	2712.9	2905.3
80	2158.6	2323.4	2488.4	2343.1	2521.7	2700.7	2509.1	2700.2	2891.6
100	2157.8	2322.4	2487.3	2335.2	2513.1	2691.4	2494.2	2684.1	2874.3
120	2156.8	2321.8	2486.0	2325.6	2502.8	2680.2	2476.3	2664.7	2853.4
140	2155.6	2319.9	2484.4	2314.5	2490.8	2667.2	2455.5	2642.2	2829.1
160	2154.3	2318.3	2482.6	2302.1	2477.2	2652.5	2431.9	2616.7	2801.7
180	2152.8	2316.6	2480.7	2288.2	2462.1	2636.2	2405.8	2588.5	2771.3
200	2151.2	2314.8	2478.6	2273.1	2445.8	2618.5	2377.4	2557.7	2738.2
220	2149.4	2312.8	2476.3	2256.9	2428.2	2599.5	2346.9	2524.7	2702.7
240	2147.6	2310.7	2473.8	2239.7	2409.5	2579.4	2314.5	2489.7	2665.0
260	2145.7	2308.4	2471.3	2221.7	2389.9	2558.3	2280.6	2453.0	2625.5
280	2143.7	2306.1	2468.6	2203.0	2369.6	2536.4	2245.3	2414.9	2584.5
300	2141.7	2303.8	2465.9	2183.7	2348.7	2513.8	2209.0	2375.7	2542.4
320	2139.6	2301.3	2463.1	2164.0	2327.4	2490.8	2171.9	2335.7	2499.3
340	2137.5	2298.9	2460.3	2144.1	2305.8	2467.5			
360	2135.3	2296.4	2457.5						

Hour Angle.	Moon's true Declination, 15° N.			Moon's true Declination, 20° N.			Moon's true Declination, 25° N.		
	Horizontal Parallax.			Horizontal Parallax.			Horizontal Parallax.		
	53′	57′	61′	53′	57′	61′	53′	57′	61′
m.	″	″	″	″	″	″	″	″	″
0	1473.2	1586.0	1699.0	1218.3	1311.7	1405.2	953.8	1026.9	1100.2
20	1475.5	1588.5	1701.7	1221.4	1315.0	1408.8	957.6	1031.1	1104.7
40	1482.4	1595.9	1709.7	1230.7	1325.0	1419.5	969.2	1043.5	1118.0
60	1493.9	1608.3	1722.9	1246.0	1341.5	1437.2	988.3	1064.1	1140.1
80	1509.8	1625.4	1741.3	1267.4	1364.5	1461.8	1014.8	1092.7	1170.7
100	1530.1	1647.2	1764.6	1294.5	1393.6	1493.0	1048.6	1129.0	1209.6
120	1554.5	1673.5	1792.7	1327.1	1428.8	1530.6	1089.2	1172.8	1256.5
140	1582.9	1704.0	1825.3	1365.1	1469.6	1574.3	1136.5	1223.6	1310.9
160	1615.0	1738.5	1862.2	1408.0	1515.8	1623.8	1189.9	1281.1	1372.5
180	1650.6	1776.7	1903.1	1455.6	1566.9	1678.5	1249.1	1344.8	1440.7
200	1689.3	1818.3	1947.6	1507.3	1622.6	1738.1	1313.6	1414.2	1515.0
220	1730.9	1863.0	1995.3	1562.9	1682.3	1802.0	1382.7	1488.6	1594.7
240	1775.0	1910.4	2045.9	1621.9	1745.7	1869.8	1456.1	1567.5	1679.2
260	1821.3	1960.0	2098.9	1683.7	1812.2	1940.9	1533.1	1650.3	1767.8
280	1869.3	2011.6	2154.1	1747.9	1881.2	2014.7	1613.1	1736.3	1859.8
300	1918.8	2064.7	2210.8	1814.1	1952.3	2090.6	1695.4	1824.8	1954.4
320	1969.3	2118.9	2268.6	1881.6	2024.8	2168.1	1779.5	1915.2	2051.0
340	2020.4	2173.7	2327.1	1949.9	2098.2	2246.5	1864.6	2006.6	2148.8
360	2071.7	2228.8	2385.9	2018.6	2171.9	2325.3	1950.1	2098.4	2246.9
380	2122.9	2283.7	2444.5	2087.1	2245.4	2403.8	2035.3	2190.0	2344.8
400	2173.6	2338.0	2502.5	2154.8	2318.1	2481.4	2119.7	2280.6	2441.6
420				2221.4	2389.5	2557.6	2202.5	2369.5	2536.5
440							2283.2	2456.1	2629.0

FOR CAMBRIDGE OBSERVATORY, LAT. 42° 22′ 48″.6.

Hour Angle.	Moon's true Declination, 15° S.			Moon's true Declination, 20° S.			Moon's true Declination, 25° S.		
	Horizontal Parallax.			Horizontal Parallax.			Horizontal Parallax.		
	53′	57′	61′	53′	57′	61′	53′	57′	61′
m.	″	″	″	″	″	″	″	″	″
0	2694.9	2900.1	3105.6	2832.8	3048.2	3263.8	2948.6	3172.5	3396.6
20	2692.5	2897.5	3102.7	2829.5	3044.7	3260.1	2944.6	3168.2	3392.0
40	2685.0	2889.4	3094.0	2819.8	3034.2	3248.8	2932.7	3155.4	3378.2
60	2672.7	2876.1	3079.7	2803.8	3016.9	3230.2	2913.0	3134.1	3355.3
80	2655.6	2857.6	3059.9	2781.4	2992.7	3204.2	2885.6	3104.5	3323.6
100	2633.9	2834.2	3034.6	2753.0	2962.1	3171.3	2850.8	3067.0	3283.3
120	2607.7	2805.9	3004.2	2718.8	2925.1	3131.6	2808.9	3021.7	3234.7
140	2577.3	2773.0	2968.9	2679.1	2882.2	3085.5	2760.1	2969.1	3178.2
160	2542.9	2735.9	2928.9	2634.1	2883.7	3033.4	2705.0	2909.7	3114.4
180	2504.8	2694.7	2884.7	2584.3	2780.0	2975.7	2643.9	2843.8	3043.7
200	2463.3	2649.8	2836.5	2530.1	2721.5	2912.9	2577.4	2772.0	2966.7
220	2418.7	2601.7	2784.8	2471.9	2658.7	2845.5	2506.0	2695.1	2884.1
240	2371.5	2550.7	2730.0	2410.2	2592.1	2774.0	2430.3	2613.4	2796.5
260	2321.9	2497.2	2672.5	2345.4	2522.2	2699.0	2350.9	2527.8	2704.7
280	2270.4	2441.6	2612.9	2277.1	2449.6	2621.1			
300	2217.4	2384.5	2551.5						

Table XVII.

Parallactic Angles for the Latitude of Washington Observatory.

Dec.	Hour Angle.										
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°
°	°	°	°	°	°	°	°	°	°	°	°
29 N.	37.8	54.4	60.7	62.9	63.1	62.1	60.3	58.0	55.0	51.4	47.3
28	35.3	52.1	59.0	61.6	62.1	61.4	59.8	57.6	54.7	51.3	47.3
27	33.1	50.0	57.3	60.3	61.1	60.7	59.3	57.2	54.5	51.2	47.2
26	31.0	48.0	55.7	59.1	60.2	60.0	58.7	56.8	54.2	51.0	47.2
25	29.2	46.1	54.2	58.0	59.3	59.3	58.2	56.5	54.0	50.9	47.2
24	27.6	44.4	52.8	56.8	58.4	58.6	57.7	56.1	53.8	50.8	47.2
23	26.2	42.7	51.4	55.7	57.6	57.9	57.3	55.8	53.6	50.7	47.2
22	24.9	41.2	50.1	54.7	56.8	57.3	56.8	55.5	53.4	50.6	47.2
21	23.7	39.8	48.8	53.6	55.9	56.7	56.3	55.2	53.2	50.6	
20	22.6	38.4	47.6	52.6	55.2	56.1	55.9	54.8	53.0	50.5	
19	21.7	37.1	46.4	51.7	54.4	55.5	55.5	54.6	52.8	50.4	
18	20.8	36.0	45.3	50.7	53.7	54.9	55.1	54.3	52.7	50.4	
17	19.9	34.8	44.3	49.8	52.9	54.4	54.7	54.0	52.5	50.3	
16	19.2	33.8	43.2	49.0	52.2	53.9	54.3	53.7	52.4	50.3	
15	18.5	32.8	42.3	48.2	51.6	53.3	53.9	53.5	52.3	50.3	
14	17.8	31.9	41.4	47.4	50.9	52.8	53.5	53.3	52.1	50.2	
13	17.2	31.0	40.5	46.6	50.3	52.3	53.2	53.0	52.0	50.2	
12	16.7	30.2	39.6	45.8	49.7	51.9	52.8	52.8	51.9	50.2	
11	16.2	29.4	38.8	45.1	49.1	51.4	52.5	52.6	51.8		
10	15.7	28.7	38.0	44.4	48.5	51.0	52.2	52.4	51.7		
9	15.2	28.0	37.3	43.7	48.0	50.5	51.9	52.2	51.6		
8	14.8	27.3	36.6	43.1	47.4	50.1	51.6	52.0	51.6		
7	14.4	26.7	35.9	42.5	46.9	49.7	51.3	51.8	51.5		
6	14.0	26.1	35.3	41.9	46.4	49.3	51.0	51.7	51.4		
5	13.7	25.5	34.7	41.3	45.9	48.9	50.7	51.5	51.4		
4	13.4	25.0	34.1	40.7	45.4	48.6	50.5	51.4	51.4		
3	13.1	24.5	33.5	40.2	45.0	48.2	50.2	51.2	51.3		
2	12.8	24.0	33.0	39.7	44.5	47.9	50.0	51.1	51.3		
1 N.	12.5	23.5	32.5	39.2	44.1	47.5	49.8	51.0	51.3		
0	12.2	23.1	32.0	38.7	43.7	47.2	49.5	50.9	51.3		
1 S.	12.0	22.7	31.5	38.3	43.3	46.9	49.3	50.8			
2	11.7	22.3	31.0	37.8	42.9	46.6	49.1	50.7			
3	11.5	21.9	30.6	37.4	42.6	46.3	49.0	50.6			
4	11.3	21.6	30.2	37.0	42.2	46.1	48.8	50.5			
5	11.1	21.2	29.8	36.6	41.9	45.8	48.6	50.5			
6	10.9	20.9	29.4	36.2	41.6	45.6	48.5	50.4			
7	10.7	20.6	29.0	35.9	41.2	45.3	48.3	50.3			
8	10.6	20.3	28.7	35.5	40.9	45.1	48.2	50.3			
9	10.4	20.0	28.3	35.2	40.7	44.9	48.0	50.3			
10	10.2	19.8	28.0	34.9	40.4	44.7	47.9	50.2			
11	10.1	19.5	27.7	34.6	40.1	44.5	47.8	50.2			
12	10.0	19.3	27.4	34.3	39.9	44.3	47.7	50.2			
13	9.8	19.0	27.1	34.0	39.6	44.1	47.6	50.2			
14	9.7	18.8	26.9	33.7	39.4	43.9	47.5				
15	9.6	18.6	26.6	33.5	39.2	43.8	47.5				
16	9.5	18.4	26.4	33.2	39.0	43.6	47.4				
17	9.4	18.2	26.1	33.0	38.8	43.5	47.3				
18	9.3	18.0	25.9	32.8	38.6	43.4	47.3				
19	9.2	17.8	25.7	32.5	38.4	43.3	47.2				
20	9.1	17.7	25.5	32.3	38.2	43.1	47.2				
21	9.0	17.5	25.3	32.2	38.1	43.0	47.2				
22	8.9	17.3	25.1	32.0	37.9	43.0	47.2				
23	8.8	17.2	24.9	31.8	37.8	42.9	47.2				
24	8.7	17.1	24.8	31.6	37.6	42.8	47.2				
25	8.6	16.9	24.6	31.5	37.5	42.7					
26	8.6	16.8	24.4	31.3	37.4	42.7					
27	8.5	16.7	24.3	31.2	37.3	42.6					
28	8.4	16.6	24.2	31.1	37.2	42.6					
29 S.	8.4	16.5	24.0	30.9	37.1	42.6					

TABLE XVIII.

Correction to be added to the Moon's Declination in computing an Occultation or Eclipse.

Dec.	Difference of Right Ascension.																	
	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	65′	70′	75′	80′	85′	90′
° ′	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0 30	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.5	0.5	0.6
1 0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.9	1.0	1.1	1.
1 30	0.0	0.0	0.1	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0	1.1	1.3	1.5	1.7	1.
2 0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.1	1.3	1.5	1.7	1.9	2.2	2.
2 30	0.0	0.0	0.1	0.2	0.2	0.3	0.5	0.6	0.8	1.0	1.2	1.4	1.6	1.9	2.1	2.4	2.8	3.1
3 0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.9	1.1	1.4	1.6	1.9	2.2	2.6	2.9	3.3	3.7
3 30	0.0	0.1	0.1	0.2	0.3	0.5	0.7	0.9	1.1	1.3	1.6	1.9	2.2	2.6	3.0	3.4	3.8	4.3
4 0	0.0	0.1	0.1	0.2	0.4	0.5	0.7	1.0	1.2	1.5	1.8	2.2	2.6	3.0	3.4	3.9	4.4	4.9
4 30	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.4	1.7	2.1	2.5	2.9	3.3	3.8	4.4	4.9	5.5
5 0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.5	1.9	2.3	2.7	3.2	3.7	4.3	4.8	5.5	6.1
5 30	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.3	1.7	2.1	2.5	3.0	3.5	4.1	4.7	5.3	6.0	6.7
6 0	0.0	0.1	0.2	0.4	0.6	0.8	1.1	1.4	1.8	2.3	2.7	3.3	3.8	4.4	5.1	5.8	6.6	7.3
6 30	0.0	0.1	0.2	0.4	0.6	0.9	1.2	1.6	2.0	2.5	3.0	3.5	4.1	4.8	5.5	6.3	7.1	8.0
7 0	0.0	0.1	0.2	0.4	0.7	1.0	1.3	1.7	2.1	2.6	3.2	3.8	4.5	5.2	5.9	6.8	7.7	8.6
7 30	0.0	0.1	0.3	0.4	0.7	1.0	1.4	1.8	2.3	2.8	3.4	4.1	4.8	5.5	6.3	7.2	8.2	9.1
8 0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.4	3.0	3.6	4.3	5.1	5.9	6.8	7.7	8.7	9.7
8 30	0.0	0.1	0.3	0.5	0.8	1.1	1.6	2.0	2.6	3.2	3.9	4.6	5.4	6.3	7.2	8.2	9.2	10.3
9 0	0.0	0.1	0.3	0.5	0.8	1.2	1.7	2.2	2.8	3.4	4.1	4.9	5.7	6.6	7.6	8.6	9.8	10.9
9 30	0.0	0.1	0.3	0.6	0.9	1.3	1.8	2.3	2.9	3.5	4.3	5.1	6.0	6.9	8.0	9.1	10.3	11.5
10 0	0.0	0.1	0.3	0.6	0.9	1.3	1.8	2.4	3.0	3.7	4.5	5.4	6.3	7.3	8.4	9.6	10.8	12.1
10 30	0.0	0.2	0.3	0.6	1.0	1.4	1.9	2.5	3.2	3.9	4.7	5.6	6.6	7.7	8.8	10.0	11.3	12.6
11 0	0.0	0.2	0.4	0.7	1.0	1.5	2.0	2.6	3.3	4.1	5.0	5.9	6.9	8.0	9.2	10.5	11.8	13.2
11 30	0.0	0.2	0.4	0.7	1.1	1.5	2.1	2.7	3.4	4.2	5.2	6.1	7.2	8.3	9.6	10.9	12.3	13.8
12 0	0.0	0.2	0.4	0.7	1.1	1.6	2.2	2.8	3.6	4.4	5.4	6.4	7.5	8.7	10.0	11.4	12.8	14.4
12 30	0.1	0.2	0.4	0.7	1.1	1.6	2.2	2.9	3.7	4.6	5.6	6.6	7.8	9.0	10.4	11.8	13.3	14.9
13 0	0.1	0.2	0.4	0.8	1.2	1.7	2.3	3.1	3.9	4.8	5.8	6.9	8.1	9.4	10.8	12.2	13.8	15.5
13 30	0.1	0.2	0.4	0.8	1.2	1.8	2.4	3.2	4.0	5.0	6.0	7.1	8.4	9.7	11.1	12.6	14.3	16.0
14 0	0.1	0.2	0.5	0.8	1.3	1.8	2.5	3.3	4.2	5.1	6.2	7.4	8.7	10.0	11.5	13.1	14.8	16.6
14 30	0.1	0.2	0.5	0.8	1.3	1.9	2.6	3.4	4.3	5.3	6.4	7.6	8.9	10.3	11.9	13.5	15.3	17.1
15 0	0.1	0.2	0.5	0.9	1.4	2.0	2.7	3.5	4.5	5.5	6.6	7.9	9.2	10.7	12.3	14.0	15.8	17.7
15 30	0.1	0.2	0.5	0.9	1.4	2.0	2.8	3.6	4.6	5.6	6.8	8.1	9.5	11.0	12.7	14.4	16.3	18.2
16 0	0.1	0.2	0.5	0.9	1.5	2.1	2.9	3.7	4.7	5.8	7.0	8.3	9.8	11.3	13.0	14.8	16.7	18.7
16 30	0.1	0.2	0.5	0.9	1.5	2.1	2.9	3.8	4.8	5.9	7.2	8.6	10.0	11.6	13.4	15.2	17.2	19.2
17 0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	5.0	6.1	7.4	8.8	10.3	12.0	13.8	15.6	17.7	19.8
17 30	0.1	0.2	0.6	1.0	1.6	2.3	3.1	4.0	5.1	6.3	7.6	9.0	10.6	12.3	14.1	16.0	18.1	20.8
18 0	0.1	0.3	0.6	1.0	1.6	2.3	3.1	4.1	5.2	6.4	7.8	9.2	10.9	12.6	14.5	16.4	18.6	20.8
18 30	0.1	0.3	0.6	1.0	1.6	2.4	3.2	4.2	5.3	6.6	8.0	9.5	11.1	12.9	14.8	16.8	19.0	21.3
19 0	0.1	0.3	0.6	1.1	1.7	2.4	3.3	4.3	5.5	6.7	8.2	9.7	11.3	13.2	15.2	17.2	19.5	21.8
19 30	0.1	0.3	0.6	1.1	1.7	2.5	3.4	4.4	5.6	6.9	8.3	9.9	11.6	13.5	15.5	17.6	19.9	22.2
20 0	0.1	0.3	0.6	1.1	1.8	2.5	3.5	4.5	5.7	7.0	8.5	10.1	11.9	13.7	15.8	18.0	20.3	22.7
20 30	0.1	0.3	0.6	1.1	1.8	2.6	3.5	4.6	5.8	7.1	8.7	10.3	12.1	14.0	16.1	18.3	20.7	23.2
21 0	0.1	0.3	0.7	1.2	1.8	2.6	3.6	4.7	5.9	7.3	8.9	10.5	12.4	14.3	16.5	18.7	21.1	23.6
21 30	0.1	0.3	0.7	1.2	1 9	2.7	3.6	4.8	6.0	7.4	9.0	10.7	12.6	14.6	16.8	19.1	21.5	24.1
22 0	0.1	0.3	0.7	1.2	1.9	2.7	3.7	4.8	6.2	7.6	9.2	10.9	12.8	14.9	17.1	19.4	21.9	24.6
22 30	0.1	0.3	0.7	1.2	1.9	2.8	3.8	4.9	6.3	7.7	9.4	11.1	13.0	15.1	17.4	19.8	22.3	25.0
23 0	0.1	0.3	0.7	1.3	2.0	2.8	3.9	5.0	6.4	7.8	9.5	11.3	13.3	15.4	17.7	20.1	22.7	25.4
23 30	0.1	0.3	0.7	1.3	2.0	2.9	3.9	5.1	6.5	8.0	9.7	11.5	13.6	15.7	18.0	20.5	23.1	25.9
24 0	0.1	0.3	0.7	1.3	2.0	2.9	4.0	5.2	6.6	8.1	9.9	11.7	13.8	15.9	18.3	20.8	23.5	26.3
24 30	0.1	0.3	0.7	1.3	2.1	3.0	4.0	5.3	6.7	8.2	10.0	11.9	13.9	16.1	18.5	21.1	23.9	26.7
25 0	0.1	0.3	0.7	1.3	2.1	3.0	4.1	5.3	6.8	8.4	10.1	12.0	14.1	16.4	18.8	21.4	24.2	27.1
25 30	0.1	0.3	0.8	1.3	2.1	3.0	4.2	5.4	6.9	8.5	10.3	12.2	14.3	16.6	19.1	21.7	24.5	27.5
26 0	0.1	0.3	0.8	1.4	2.2	3.1	4.3	5.5	7.0	8.6	10.4	12.4	14.5	16.8	19.3	22.0	24.9	27.9
26 30	0.1	0.3	0.8	1.4	2.2	3.1	4.3	5.6	7.1	8.7	10.6	12.6	14.7	17.1	19.6	22.3	25.2	28.2
27 0	0.1	0.3	0.8	1.4	2.2	3.2	4.4	5.6	7.1	8.8	10.7	12.7	14.9	17.3	19.9	22.6	25.5	28.6
27 30	0.1	0.4	0.8	1.4	2.2	3.2	4.4	5.7	7.2	8.9	10.8	12.9	15.1	17.5	20.1	22.9	25.8	28.9
28 0	0.1	0.4	0.8	1.4	2.3	3.3	4.5	5.8	7.3	9.0	10.9	13.0	15.3	17.7	20.4	23.2	26.2	29.3
28 30	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.9	7.4	9.1	11.1	13.2	15.5	17.9	20.6	23.4	26.5	29.6
29 0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.9	7.5	9.3	11.2	13.3	15.6	18.1	20.8	23.7	26.8	30.0

Star's Dec.	Latitude of the Place.																											
	4°		8°		10°		12°		14°		16°		18°		20°		22°		24°		26°		28°		30°		32°	
°	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.
29 N.	6	9	6	18	6	22	6	27	6	32	6	37	6	41	6	47	6	52	6	57	7	3	7	8	7	15	7	21
28	6	9	6	17	6	21	6	26	6	30	6	35	6	40	6	45	6	50	6	55	7	0	7	6	7	11	7	18
27	6	8	6	16	6	21	6	25	6	29	6	34	6	38	6	43	6	48	6	52	6	58	7	3	7	8	7	14
26	6	8	6	16	6	20	6	24	6	28	6	32	6	36	6	41	6	45	6	50	6	55	7	0	7	5	7	11
25	6	7	6	15	6	19	6	23	6	27	6	31	6	35	6	39	6	43	6	48	6	53	6	57	7	2	7	8
24	6	7	6	14	6	18	6	22	6	25	6	29	6	33	6	37	6	41	6	46	6	50	6	55	7	0	7	5
23	6	7	6	14	6	17	6	21	6	24	6	28	6	32	6	36	6	39	6	44	6	48	6	52	6	57	7	2
22	6	6	6	13	6	16	6	20	6	23	6	27	6	30	6	34	6	38	6	41	6	45	6	50	6	54	6	58
21	6	6	6	12	6	16	6	19	6	22	6	25	6	29	6	32	6	36	6	39	6	43	6	47	6	51	6	56
20	6	6	6	12	6	15	6	18	6	21	6	24	6	27	6	30	6	34	6	37	6	41	6	45	6	49	6	53
19	6	6	6	11	6	14	6	17	6	20	6	23	6	26	6	29	6	32	6	35	6	39	6	42	6	46	6	50
18	6	5	6	10	6	13	6	16	6	19	6	21	6	24	6	27	6	30	6	33	6	36	6	40	6	43	6	47
17	6	5	6	10	6	12	6	15	6	17	6	20	6	23	6	26	6	28	6	31	6	34	6	37	6	41	6	44
16	6	5	6	9	6	12	6	14	6	16	6	19	6	21	6	24	6	27	6	29	6	32	6	35	6	38	6	41
15	6	4	6	9	6	11	6	13	6	15	6	18	6	20	6	22	6	25	6	27	6	30	6	33	6	36	6	39
14	6	4	6	8	6	10	6	12	6	14	6	16	6	19	6	21	6	23	6	25	6	28	6	30	6	33	6	36
13	6	4	6	7	6	9	6	11	6	13	6	15	6	17	6	19	6	21	6	24	6	26	6	28	6	31	6	33
12	6	3	6	7	6	9	6	10	6	12	6	14	6	16	6	18	6	20	6	22	6	24	6	26	6	28	6	31
11	6	3	6	6	6	8	6	9	6	11	6	13	6	14	6	16	6	18	6	20	6	22	6	24	6	26	6	28
10	6	3	6	6	6	7	6	9	6	10	6	12	6	13	6	15	6	16	6	18	6	20	6	22	6	23	6	25
9	6	3	6	5	6	6	6	8	6	9	6	10	6	12	6	13	6	15	6	16	6	18	6	19	6	21	6	23
8	6	2	6	5	6	6	6	7	6	8	6	9	6	10	6	12	6	13	6	14	6	16	6	17	6	19	6	20
7	6	2	6	4	6	5	6	6	6	7	6	8	6	9	6	10	6	11	6	13	6	14	6	15	6	16	6	18
6	6	2	6	3	6	4	6	5	6	6	6	7	6	8	6	9	6	10	6	11	6	12	6	13	6	14	6	15
5	6	1	6	3	6	4	6	4	6	5	6	6	6	7	6	7	6	8	6	9	6	10	6	11	6	12	6	13
4	6	1	6	2	6	3	6	3	6	4	6	5	6	5	6	6	6	6	6	7	6	8	6	9	6	9	6	10
3	6	1	6	2	6	2	6	3	6	3	6	3	6	4	6	4	6	5	6	5	6	6	6	6	6	7	6	8
2	6	1	6	1	6	1	6	2	6	2	6	2	6	3	6	3	6	3	6	4	6	4	6	4	6	5	6	5
1 N.	6	0	6	1	6	1	6	1	6	1	6	1	6	1	6	1	6	2	6	2	6	2	6	2	6	2	6	2
0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0
1 S.	6	0	5	59	5	59	5	59	5	59	5	59	5	59	5	59	5	58	5	58	5	58	5	58	5	58	5	58
2	5	59	5	59	5	59	5	58	5	58	5	58	5	57	5	57	5	57	5	56	5	56	5	56	5	55	5	55
3	5	59	5	58	5	58	5	57	5	57	5	57	5	56	5	56	5	55	5	55	5	54	5	54	5	53	5	52
4	5	59	5	58	5	57	5	57	5	56	5	55	5	55	5	54	5	54	5	53	5	52	5	51	5	51	5	50
5	5	59	5	57	5	56	5	56	5	55	5	54	5	53	5	53	5	52	5	51	5	50	5	49	5	48	5	47
6	5	58	5	57	5	56	5	55	5	54	5	53	5	52	5	51	5	50	5	49	5	48	5	47	5	46	5	45
7	5	58	5	56	5	55	5	54	5	53	5	52	5	51	5	50	5	49	5	47	5	46	5	45	5	44	5	42
8	5	58	5	55	5	54	5	53	5	52	5	51	5	50	5	48	5	47	5	46	5	44	5	43	5	41	5	40
9	5	57	5	55	5	54	5	52	5	51	5	50	5	48	5	47	5	45	5	44	5	42	5	41	5	39	5	37
10	5	57	5	54	5	53	5	51	5	50	5	48	5	47	5	45	5	44	5	42	5	40	5	38	5	37	5	35
11	5	57	5	54	5	52	5	51	5	49	5	47	5	46	5	44	5	42	5	40	5	38	5	36	5	34	5	32
12	5	57	5	53	5	51	5	50	5	48	5	46	5	44	5	42	5	40	5	38	5	36	5	34	5	32	5	29
13	5	56	5	53	5	51	5	49	5	47	5	45	5	43	5	41	5	39	5	36	5	34	5	32	5	29	5	27
14	5	56	5	52	5	50	5	48	5	46	5	44	5	41	5	39	5	37	5	35	5	32	5	30	5	27	5	24
15	5	56	5	51	5	49	5	47	5	45	5	42	5	40	5	38	5	35	5	33	5	30	5	27	5	24	5	21
16	5	55	5	51	5	48	5	46	5	44	5	41	5	39	5	36	5	33	5	31	5	28	5	25	5	22	5	19
17	5	55	5	50	5	48	5	45	5	43	5	40	5	37	5	34	5	32	5	28	5	26	5	23	5	19	5	16
18	5	55	5	50	5	47	5	44	5	41	5	39	5	36	5	33	5	30	5	27	5	24	5	20	5	17	5	13
19	5	54	5	49	5	46	5	43	5	40	5	37	5	34	5	31	5	28	5	25	5	21	5	18	5	14	5	10
20	5	54	5	48	5	45	5	42	5	39	5	36	5	33	5	30	5	26	5	23	5	19	5	15	5	11	5	7
21	5	54	5	48	5	44	5	41	5	38	5	35	5	31	5	28	5	24	5	21	5	17	5	13	5	9	5	4
22	5	54	5	47	5	44	5	40	5	37	5	33	5	30	5	26	5	22	5	19	5	15	5	10	5	6	5	2
23	5	53	5	46	5	43	5	39	5	36	5	32	5	28	5	24	5	21	5	16	5	12	5	8	5	3	4	58
24	5	53	5	46	5	42	5	38	5	35	5	31	5	27	5	23	5	19	5	14	5	10	5	5	5	0	4	55
25	5	53	5	45	5	41	5	37	5	33	5	29	5	25	5	21	5	17	5	12	5	7	5	3	4	58	4	52
26	5	52	5	44	5	40	5	36	5	32	5	28	5	24	5	19	5	15	5	10	5	5	5	0	4	55	4	49
27	5	52	5	44	5	39	5	35	5	31	5	26	5	22	5	17	5	12	5	8	5	2	4	57	4	52	4	46
28	5	51	5	43	5	39	5	34	5	30	5	25	5	20	5	15	5	10	5	5	5	0	4	54	4	49	4	42
29 S.	5	51	5	42	5	38	5	33	5	28	5	23	5	19	5	13	5	8	5	3	4	57	4	52	4	45	4	39

Star's Dec.	Latitude of the Place.													
	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	58°	60°
°	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
29 N.	7 28	7 35	7 43	7 51	8 0	8 10	8 20	8 32	8 45	9 1	9 19	9 41	10 10	10 55
28	7 24	7 31	7 38	7 46	7 55	8 4	8 14	8 25	8 37	8 52	9 8	9 28	9 53	10 28
27	7 20	7 27	7 34	7 41	7 49	7 58	8 7	8 18	8 30	8 43	8 58	9 16	9 39	10 8
26	7 17	7 23	7 30	7 37	7 44	7 52	8 1	8 11	8 22	8 35	8 49	9 5	9 25	9 51
25	7 13	7 19	7 25	7 32	7 39	7 47	7 55	8 5	8 15	8 27	8 40	8 54	9 13	9 35
24	7 10	7 16	7 21	7 28	7 35	7 42	7 50	7 59	8 8	8 19	8 31	8 45	9 2	9 22
23	7 7	7 12	7 17	7 23	7 30	7 37	7 44	7 53	8 2	8 12	8 23	8 36	8 51	9 9
22	7 3	7 8	7 14	7 19	7 25	7 32	7 39	7 47	7 55	8 5	8 15	8 27	8 41	8 58
21	7 0	7 5	7 10	7 15	7 21	7 27	7 34	7 41	7 49	7 58	8 8	8 19	8 32	8 47
20	6 57	7 1	7 6	7 11	7 17	7 22	7 29	7 35	7 43	7 51	8 0	8 11	8 22	8 36
19	6 54	6 58	7 2	7 7	7 12	7 18	7 24	7 30	7 37	7 45	7 53	8 3	8 14	8 26
18	6 51	6 55	6 59	7 3	7 8	7 13	7 19	7 25	7 31	7 38	7 46	7 55	8 5	8 17
17	6 48	6 51	6 55	6 59	7 4	7 9	7 14	7 19	7 25	7 32	7 40	7 48	7 57	8 8
16	6 45	6 48	6 52	6 56	7 0	7 4	7 9	7 14	7 20	7 26	7 33	7 41	7 49	7 59
15	6 42	6 45	6 48	6 52	6 56	7 0	7 4	7 9	7 14	7 20	7 27	7 34	7 42	7 51
14	6 39	6 42	6 45	6 48	6 52	6 56	7 0	7 4	7 9	7 14	7 20	7 27	7 34	7 42
13	6 36	6 39	6 42	6 45	6 48	6 52	6 55	6 59	7 4	7 9	7 14	7 20	7 27	7 34
12	6 33	6 36	6 38	6 41	6 44	6 47	6 51	6 55	6 59	7 3	7 8	7 13	7 20	7 26
11	6 30	6 32	6 35	6 38	6 40	6 43	6 46	6 50	6 54	6 58	7 2	7 7	7 12	7 19
10	6 27	6 29	6 32	6 34	6 37	6 39	6 42	6 45	6 49	6 52	6 56	7 1	7 6	7 11
9	6 25	6 26	6 28	6 31	6 33	6 35	6 38	6 41	6 44	6 47	6 50	6 54	6 59	7 4
8	6 22	6 23	6 25	6 27	6 29	6 31	6 33	6 36	6 39	6 41	6 45	6 48	6 52	6 56
7	6 19	6 20	6 22	6 24	6 25	6 27	6 29	6 31	6 34	6 36	6 39	6 42	6 45	6 49
6	6 16	6 18	6 19	6 20	6 22	6 23	6 25	6 27	6 29	6 31	6 33	6 36	6 39	6 42
5	6 14	6 15	6 16	6 17	6 18	6 19	6 21	6 22	6 24	6 26	6 28	6 30	6 32	6 35
4	6 11	6 12	6 13	6 13	6 14	6 15	6 17	6 18	6 19	6 21	6 22	6 24	6 26	6 28
3	6 8	6 9	6 9	6 10	6 11	6 12	6 12	6 13	6 14	6 15	6 17	6 18	6 19	6 21
2	6 5	6 6	6 6	6 7	6 7	6 8	6 8	6 9	6 10	6 10	6 11	6 12	6 13	6 14
1 N.	6 3	6 3	6 3	6 3	6 4	6 4	6 4	6 4	6 5	6 5	6 6	6 6	6 6	6 7
0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
1 S.	5 57	5 57	5 57	5 57	5 56	5 56	5 56	5 56	5 55	5 55	5 54	5 54	5 54	5 53
2	5 55	5 54	5 54	5 53	5 53	5 52	5 52	5 51	5 50	5 50	5 49	5 48	5 47	5 46
3	5 52	5 51	5 51	5 50	5 49	5 48	5 48	5 47	5 46	5 45	5 43	5 42	5 41	5 39
4	5 49	5 48	5 47	5 47	5 46	5 45	5 43	5 42	5 41	5 39	5 38	5 36	5 34	5 32
5	5 46	5 45	5 44	5 43	5 42	5 41	5 39	5 38	5 36	5 34	5 32	5 30	5 28	5 25
6	5 44	5 42	5 41	5 40	5 38	5 37	5 35	5 33	5 31	5 29	5 27	5 24	5 21	5 18
7	5 41	5 40	5 38	5 36	5 35	5 33	5 31	5 29	5 26	5 24	5 21	5 18	5 15	5 11
8	5 38	5 37	5 35	5 33	5 31	5 29	5 27	5 24	5 21	5 19	5 15	5 12	5 8	5 4
9	5 35	5 34	5 32	5 29	5 27	5 25	5 22	5 19	5 16	5 13	5 10	5 6	5 1	4 56
10	5 33	5 31	5 28	5 26	5 23	5 21	5 18	5 15	5 11	5 8	5 4	4 59	4 54	4 49
11	5 30	5 28	5 25	5 22	5 20	5 17	5 14	5 10	5 6	5 2	4 58	4 53	4 48	4 41
12	5 27	5 24	5 22	5 19	5 16	5 13	5 9	5 5	5 1	4 57	4 52	4 47	4 40	4 34
13	5 24	5 21	5 18	5 15	5 12	5 8	5 5	5 1	4 56	4 51	4 46	4 40	4 33	4 26
14	5 21	5 18	5 15	5 12	5 8	5 4	5 0	4 56	4 51	4 46	4 40	4 33	4 26	4 18
15	5 18	5 15	5 12	5 8	5 4	5 0	4 56	4 51	4 46	4 40	4 33	4 26	4 18	4 9
16	5 15	5 12	5 8	5 4	5 0	4 56	4 51	4 46	4 40	4 34	4 27	4 19	4 11	4 1
17	5 12	5 9	5 5	5 1	4 56	4 51	4 46	4 41	4 35	4 28	4 20	4 12	4 3	3 52
18	5 9	5 5	5 1	4 57	4 52	4 47	4 41	4 35	4 29	4 22	4 14	4 5	3 55	3 43
19	5 6	5 2	4 58	4 53	4 48	4 42	4 36	4 30	4 23	4 15	4 7	3 57	3 46	3 34
20	5 3	4 59	4 54	4 49	4 43	4 38	4 31	4 25	4 17	4 9	4 0	3 49	3 38	3 24
21	5 0	4 55	4 50	4 45	4 39	4 33	4 26	4 19	4 11	4 2	3 52	3 41	3 28	3 13
22	4 57	4 52	4 46	4 41	4 35	4 28	4 21	4 13	4 5	3 55	3 45	3 33	3 19	3 2
23	4 53	4 48	4 43	4 37	4 30	4 23	4 16	4 7	3 58	3 48	3 37	3 24	3 9	2 51
24	4 50	4 44	4 39	4 32	4 25	4 18	4 10	4 1	3 52	3 41	3 29	3 15	2 58	2 38
25	4 47	4 41	4 35	4 28	4 21	4 13	4 5	3 55	3 45	3 33	3 20	3 6	2 47	2 25
26	4 43	4 37	4 30	4 23	4 16	4 8	3 59	3 49	3 38	3 25	3 11	2 55	2 35	2 9
27	4 40	4 33	4 26	4 19	4 11	4 2	3 53	3 42	3 30	3 17	3 2	2 44	2 21	1 52
28	4 36	4 29	4 22	4 14	4 5	3 56	3 46	3 35	3 23	3 8	2 52	2 32	2 7	1 32
29 S.	4 32	4 25	4 17	4 9	4 0	3 56	3 40	3 28	3 15	2 59	2 41	2 19	1 50	1 5

To convert Millimeters into English Inches.

Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.
1	0.0394	59	2.3229	117	4.6064	662	26.0635	720	28.3470	778	30.6305
2	.0787	60	.3622	118	.6458	663	.1028	721	.3864	779	.6698
3	.1181	61	.4016	119	.6851	664	.1422	722	.4257	780	.7092
4	.1575	62	.4410	120	.7245	665	.1816	723	.4651	781	.7486
5	.1969	63	.4804	130	5.1182	666	.2209	724	.5045	782	.7879
6	.2362	64	.5197	140	.5119	667	.2603	725	.5438	783	.8273
7	.2756	65	.5591	150	.9056	668	.2997	726	.5832	784	.8667
8	.3150	66	.5985	160	6.2993	669	.3391	727	.6226	785	.9060
9	.3543	67	.6378	170	.6930	670	.3784	728	.6620	786	.9454
10	.3937	68	.6772	180	7.0867	671	.4178	729	.7013	787	.9848
11	.4331	69	.7166	190	.4805	672	.4572	730	.7407	788	31.0242
12	.4724	70	.7560	200	.8742	673	.4965	731	.7800	789	.0635
13	.5118	71	.7953	210	8.2679	674	.5359	732	.8194	790	.1029
14	.5512	72	.8347	220	.6616	675	.5753	733	.8588	791	.1423
15	.5906	73	.8741	230	9.0553	676	.6147	734	.8982	792	.1817
16	.6299	74	.9134	240	.4490	677	.6540	735	.9375	793	.2210
17	.6693	75	.9528	250	.8427	678	.6934	736	.9769	794	.2604
18	.7087	76	.9922	260	10.2364	679	.7328	737	29.0163	795	.2998
19	.7480	77	3.0316	270	.6301	680	.7721	738	.0556	796	.3391
20	.7874	78	.0709	280	11.0238	681	.8115	739	.0950	797	.3785
21	.8268	79	.1103	290	.4175	682	.8509	740	.1344	798	.4179
22	.8662	80	.1497	300	.8112	683	.8902	741	.1738	799	.4573
23	.9055	81	.1890	310	12.2049	684	.9296	742	.2131	800	.4966
24	.9449	82	.2284	320	.5987	685	.9690	743	.2525	810	31.8903
25	.9843	83	.2678	330	.9924	686	27.0084	744	.2919	820	32.2840
26	1.0236	84	.3071	340	13.3861	687	.0477	745	.3312	830	.6778
27	.0630	85	.3465	350	.7798	688	.0871	746	.3706	840	33.0715
28	.1024	86	.3859	360	14.1735	689	.1265	747	.4100	850	.4652
29	.1418	87	.4253	370	.5672	690	.1658	748	.4494	860	.8589
30	.1811	88	.4646	380	.9609	691	.2052	749	.4887	870	34.2526
31	.2205	89	.5040	390	15.3546	692	.2446	750	.5281	880	.6463
32	.2599	90	.5434	400	.7483	693	.2840	751	.5675	890	35.0400
33	.2992	91	.5827	410	16.1420	694	.3233	752	.6068	900	.4337
34	.3386	92	.6221	420	.5357	695	.3627	753	.6462	910	.8274
35	.3780	93	.6615	430	.9294	696	.4021	754	.6856	920	36.2211
36	.4173	94	.7009	440	17.3232	697	.4414	755	.7249	930	.6148
37	.4567	95	.7402	450	.7169	698	.4808	756	.7643	940	37.0085
38	.4961	96	.7796	460	18.1106	699	.5202	757	.8037	950	.4023
39	.5355	97	.8190	470	.5043	700	.5596	758	.8431	960	.7960
40	.5748	98	.8583	480	.8980	701	.5989	759	.8824	970	38.1897
41	.6142	99	.8977	490	19.2917	702	.6383	760	.9218	980	.5834
42	.6536	100	.9371	500	.6854	703	.6777	761	.9612	990	.9771
43	.6929	101	.9764	510	20.0791	704	.7170	762	30.0005	1000	39.3708
44	.7323	102	4.0158	520	.4728	705	.7564	763	.0399		
45	.7717	103	.0552	530	.8665	706	.7958	764	.0793		
46	.8111	104	.0946	540	21.2602	707	.8351	765	.1187		
47	.8504	105	.1339	550	.6539	708	.8745	766	.1580		
48	.8898	106	.1733	560	22.0477	709	.9139	767	.1974		
49	.9292	107	.2127	570	.4414	710	.9533	768	.2368		
50	.9685	108	.2520	580	.8351	711	.9927	769	.2761		
51	2.0079	109	.2914	590	23.2288	712	28.0320	770	.3155		
52	.0473	110	.3308	600	.6225	713	.0714	771	.3549		
53	.0867	111	.3702	610	24.0162	714	.1108	772	.3942		
54	.1260	112	.4095	620	.4099	715	.1501	773	.4336		
55	.1654	113	.4489	630	.8036	716	.1895	774	.4730		
56	.2048	114	.4883	640	25.1973	717	.2289	775	.5124		
57	.2441	115	.5276	650	.5910	718	.2683	776	.5517		
58	.2835	116	.5670	660	.9847	719	.3076	777	.5911		

Proportional Parts.

Millimeters.	English Inches.
0.1	0.0039
0.2	0.0079
0.3	0.0118
0.4	0.0157
0.5	0.0197
0.6	0.0236
0.7	0.0276
0.8	0.0315
0.9	0.0354
1.0	0.0394

One millimeter equals 0.03937079 English inch.

TABLE XXI.

To convert English Inches into Millimeters.

glish ches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.	English Inches.	Millimeters.
.1	2.54	5.9	149.86	11.7	297.17	17.5	444.49	23.3	591.81	29.1	739.13
.2	5.08	6.0	152.40	.8	299.71	.6	447.03	.4	594.35	.2	741.67
.3	7.62	.1	154.94	.9	302.25	.7	449.57	.5	596.89	.3	744.21
.4	10.16	.2	157.48	12.0	304.79	.8	452.11	.6	599.43	.4	746.75
.5	12.70	.3	160.02	.1	307.83	.9	454.65	.7	601.97	.5	749.29
.6	15.24	.4	162.56	.2	309.37	18.0	457.19	.8	604.51	.6	751.83
.7	17.78	.5	165.10	.3	312.41	.1	459.73	.9	607.05	.7	754.37
.8	20.32	.6	167.64	.4	314.95	.2	462.27	24.0	609.59	.8	756.91
.9	22.86	.7	170.18	.5	317.49	.3	464.81	.1	612.13	.9	759.45
.0	25.40	.8	172.72	.6	320.03	.4	467.35	.2	614.67	30.0	761.99
.1	27.94	.9	175.26	.7	322.57	.5	469.89	.3	617.21	.1	764.53
.2	30.48	7.0	177.80	.8	325.11	.6	472.43	.4	619.75	.2	767.07
.3	33.02	.1	180.34	.9	327.65	.7	474.97	.5	622.29	.3	769.61
.4	35.56	.2	182.88	13.0	330.19	.8	477.51	.6	624.83	.4	772.15
.5	38.10	.3	185.42	.1	332.73	.9	480.05	.7	627.37	.5	774.69
.6	40.64	.4	187.96	.2	335.27	19.0	482.59	.8	629.91	.6	777.23
.7	43.18	.5	190.50	.3	337.81	.1	485.13	.9	632.45	.7	779.77
.8	45.72	.6	193.04	.4	340.35	.2	487.67	25.0	634.99	.8	782.31
.9	48.26	.7	195.58	.5	342.89	.3	490.21	.1	637.53	.9	784.85
.0	50.80	.8	198.12	.6	345.43	.4	492.75	.2	640.07	31.0	787.39
.1	53.34	.9	200.66	.7	347.97	.5	495.29	.3	642.61	.1	789.93
.2	55.88	8.0	203.20	.8	350.51	.6	497.83	.4	645.15	.2	792.47
.3	58.42	.1	205.74	.9	353.05	.7	500.37	.5	647.69	.3	795.01
.4	60.96	.2	208.28	14.0	355.59	.8	502.91	.6	650.23	.4	797.55
.5	63.50	.3	210.82	.1	358.13	.9	505.45	.7	652.77	.5	800.09
.6	66.04	.4	213.36	.2	360.67	20.0	507.99	.8	655.31	.6	802.63
.7	68.58	.5	215.90	.3	363.21	.1	510.53	.9	657.85	.7	805.17
.8	71.12	.6	218.44	.4	365.75	.2	513.07	26.0	660.39	.8	807.71
.9	73.66	.7	220.98	.5	368.29	.3	515.61	.1	662.93	.9	810.25
.0	76.20	.8	223.52	.6	370.83	.4	518.15	.2	665.47	32.0	812.79
.1	78.74	.9	226.06	.7	373.37	.5	520.69	.3	668.01	.1	815.33
.2	81.28	9.0	228.60	.8	375.91	.6	523.23	.4	670.55	.2	817.87
.3	83.82	.1	231.14	.9	378.45	.7	525.77	.5	673.09	.3	820.41
.4	86.36	.2	233.68	15.0	380.99	.8	528.31	.6	675.63	.4	822.95
.5	88.90	.3	236.22	.1	383.53	.9	530.85	.7	678.17	.5	825.49
.6	91.44	.4	238.76	.2	386.07	21.0	533.39	.8	680.71	.6	828.03
.7	93.98	.5	241.30	.3	388.61	.1	535.93	.9	683.25	.7	830.56
.8	96.52	.6	243.84	.4	391.15	.2	538.47	27.0	685.79	.8	833.10
.9	99.06	.7	246.38	.5	393.69	.3	541.01	.1	688.33	.9	835.64
.0	101.60	.8	248.92	.6	396.23	.4	543.55	.2	690.87	33.0	838.18
.1	104.14	.9	251.46	.7	398.77	.5	546.09	.3	693.41	.1	840.72
.2	106.68	10.0	254.00	.8	401.31	.6	548.63	.4	695.95	.2	843.26
.3	109.22	.1	256.54	.9	403.85	.7	551.17	.5	698.49	.3	845.80
.4	111.76	.2	259.08	16.0	406.39	.8	553.71	.6	701.03		
.5	114.30	.3	261.62	.1	408.93	.9	556.25	.7	703.57		
.6	116.84	.4	264.16	.2	411.47	22.0	558.79	.8	706.11	Proportional Parts.	
.7	119.38	.5	266.70	.3	414.01	.1	561.33	.9	708.65	English Inches.	Millimeters.
.8	121.92	.6	269.24	.4	416.55	.2	563.87	28.0	711.19		
.9	124.46	.7	271.78	.5	419.09	.3	566.41	.1	713.73	0.01	0.254
.0	127.00	.8	274.32	.6	421.63	.4	568.95	.2	716.27	0.02	0.508
.1	129.54	.9	276.85	.7	424.17	.5	571.49	.3	718.81	0.03	0.762
.2	132.08	11.0	279.39	.8	426.71	.6	574.03	.4	721.35	0.04	1.016
.3	134.62	.1	281.93	.9	429.25	.7	576.57	.5	723.89	0.05	1.270
.4	137.16	.2	284.47	17.0	431.79	.8	579.11	.6	726.43	0.06	1.524
.5	139.70	.3	287.01	.1	434.33	.9	581.65	.7	728.97	0.07	1.778
.6	142.24	.4	289.55	.2	436.87	23.0	584.19	.8	731.51	0.08	2.032
.7	144.78	.5	292.09	.3	439.41	.1	586.73	.9	734.05	0.09	2.286
.8	147.32	.6	294.63	.4	441.95	.2	589.27	29.0	736.59	0.10	2.540

One English inch equals 25.39954 millimeters.

PART I.

Argument, the observed Height of the Barometer at either Station.

Inches.	Feet.	Diff.	Inches.	Feet.	Diff.	Inches.	Feet.	Diff.	Inches.	Feet.	Diff.
11.0	1396.9		16.0	11186.3		21.0	18291.0		26.0	23871.0	
11.1	1633.3	236.4	16.1	11349.1	162.8	21.1	18415.1	124.1	26.1	23971.3	100.
11.2	1867.6	234.3	16.2	11510.9	161.8	21.2	18538.7	123.6	26.2	24071.2	99.
11.3	2099.9	232.3	16.3	11671.7	160.8	21.3	18661.6	122.9	26.3	24170.7	99.
11.4	2330.1	230.2	16.4	11831.5	159.8	21.4	18784.0	122.4	26.4	24269.8	99.
11.5	2558.3	228.2	16.5	11990.3	158.8	21.5	18905.8	121.8	26.5	24368.6	98.
11.6	2784.5	226.2	16.6	12148.2	157.9	21.6	19027.0	121.2	26.6	24467.0	98.
11.7	3008.7	224.2	16.7	12305.1	156.9	21.7	19147.7	120.7	26.7	24565.1	98.
11.8	3231.1	222.4	16.8	12461.0	155.9	21.8	19267.8	120.1	26.8	24662.7	97.
11.9	3451.6	220.5	16.9	12616.1	155.1	21.9	19387.4	119.6	26.9	24760.0	97.
12.0	3670.2	218.6	17.0	12770.2	154.1	22.0	19506.4	119.0	27.0	24857.0	97.
12.1	3887.0	216.8	17.1	12923.5	153.3	22.1	19624.9	118.5	27.1	24953.6	96.
12.2	4102.0	215.0	17.2	13075.8	152.3	22.2	19742.9	118.0	27.2	25049.8	96.
12.3	4315.3	213.3	17.3	13227.3	151.5	22.3	19860.3	117.4	27.3	25145.7	95.
12.4	4526.9	211.6	17.4	13377.9	150.6	22.4	19977.2	116.9	27.4	25241.2	95.
12.5	4736.7	209.8	17.5	13527.6	149.7	22.5	20093.6	116.4	27.5	25336.4	95.
12.6	4944.9	208.2	17.6	13676.5	148.9	22.6	20209.4	115.8	27.6	25431.2	94.
12.7	5151.4	206.5	17.7	13824.5	148.0	22.7	20324.8	115.4	27.7	25525.7	94.
12.8	5356.4	205.0	17.8	13971.7	147.2	22.8	20439.6	114.8	27.8	25619.9	94.
12.9	5559.7	203.3	17.9	14118.0	146.3	22.9	20554.0	114.4	27.9	25713.7	93.
13.0	5761.4	201.7	18.0	14263.6	145.6	23.0	20667.8	113.8	28.0	25807.1	93.
13.1	5961.6	200.2	18.1	14408.3	144.7	23.1	20781.1	113.3	28.1	25900.3	93.
13.2	6160.3	198.7	18.2	14552.3	144.0	23.2	20894.0	112.9	28.2	25993.1	92.
13.3	6357.5	197.2	18.3	14695.4	143.1	23.3	21006.4	112.4	28.3	26085.6	92.
13.4	6553.2	195.7	18.4	14837.8	142.4	23.4	21118.3	111.9	28.4	26177.7	92.
13.5	6747.5	194.3	18.5	14979.4	141.6	23.5	21229.7	111.4	28.5	26269.6	91.
13.6	6940.3	192.8	18.6	15120.3	140.9	23.6	21340.6	110.9	28.6	26361.1	91.
13.7	7131.7	191.4	18.7	15260.3	140.0	23.7	21451.1	110.5	28.7	26452.3	91.
13.8	7321.7	190.0	18.8	15399.7	139.4	23.8	21561.1	110.0	28.8	26543.2	90.
13.9	7510.3	188.6	18.9	15538.3	138.6	23.9	21670.6	109.5	28.9	26633.7	90.
14.0	7697.6	187.3	19.0	15676.2	137.9	24.0	21779.7	109.1	29.0	26724.0	90.
14.1	7883.6	186.0	19.1	15813.3	137.1	24.1	21888.4	108.7	29.1	26813.9	89.
14.2	8068.2	184.6	19.2	15949.8	136.5	24.2	21996.6	108.2	29.2	26903.5	89.
14.3	8251.5	183.3	19.3	16085.5	135.7	24.3	22104.3	107.7	29.3	26992.8	89.
14.4	8433.6	182.1	19.4	16220.5	135.0	24.4	22211.6	107.3	29.4	27081.9	89.
14.5	8614.4	180.8	19.5	16354.8	134.3	24.5	22318.4	106.8	29.5	27170.6	88.
14.6	8794.0	179.6	19.6	16488.5	133.7	24.6	22424.8	106.4	29.6	27259.0	88.
14.7	8972.3	178.3	19.7	16621.4	132.9	24.7	22530.8	106.0	29.7	27347.1	88.
14.8	9149.5	177.2	19.8	16753.7	132.3	24.8	22636.4	105.6	29.8	27434.9	87.
14.9	9325.5	176.0	19.9	16885.3	131.6	24.9	22741.5	105.1	29.9	27522.5	87.
15.0	9500.3	174.8	20.0	17016.3	131.0	25.0	22846.3	104.8	30.0	27609.7	87.
15.1	9673.8	173.5	20.1	17146.6	130.3	25.1	22950.6	104.3	30.1	27696.6	86.
15.2	9846.2	172.4	20.2	17276.3	129.7	25.2	23054.4	103.8	30.2	27783.3	86.
15.3	10017.5	171.3	20.3	17405.3	129.0	25.3	23157.9	103.5	30.3	27869.7	86.
15.4	10187.7	170.2	20.4	17533.7	128.4	25.4	23261.0	103.1	30.4	27955.7	86.
15.5	10356.8	169.1	20.5	17661.4	127.7	25.5	23363.6	102.6	30.5	28041.5	85.
15.6	10524.8	168.0	20.6	17788.6	127.2	25.6	23465.9	102.3	30.6	28127.1	85.
15.7	10691.8	167.0	20.7	17915.1	126.5	25.7	23567.7	101.8	30.7	28212.3	85.
15.8	10857.7	165.9	20.8	18041.0	125.9	25.8	23669.2	101.5	30.8	28297.3	85.
15.9	11022.5	164.8	20.9	18166.3	125.3	25.9	23770.3	101.1	30.9	28382.0	84.
16.0	11186.3	163.8	21.0	18291.0	124.7	26.0	23871.0	100.7	31.0	28466.4	84.

PART II.

Correction due to T—T′, or the Difference of the Temperatures of the Barometers at the two Stations.

This correction is Negative when the temperature at the upper station is lowest, and vice versâ.

T—T′.	Correction.	T—T′.	Correction.	T—T′.	Correction.	T—T′.	Correction.	T—T′.	Correction.	T—T′.	Correction.
Fah't.	Feet.	Fah't.	Feet.	Fah't.	Feet.	Fah't.	Feet.	Fah't.	Feet.	Fah't.	Feet.
1°	2.3	14°	32.8	27°	63.2	40°	93.6	53°	124.1	66°	154.5
2	4.7	15	35.1	28	65.5	41	96.0	54	126.4	67	156.8
3	7.0	16	37.5	29	67.9	42	98.3	55	128.7	68	159.2
4	9.4	17	39.8	30	70.2	43	100.7	56	131.1	69	161.5
5	11.7	18	42.1	31	72.6	44	103.0	57	133.4	70	163.9
6	14.0	19	44.5	32	74.9	45	105.3	58	135.8	71	166.2
7	16.4	20	46.8	33	77.3	46	107.7	59	138.1	72	168.6
8	18.7	21	49.2	34	79.6	47	110.0	60	140.4	73	170.9
9	21.1	22	51.5	35	81.9	48	112.4	61	142.8	74	173.3
10	23.4	23	53.8	36	84.3	49	114.7	62	145.1	75	175.6
11	25.8	24	56.2	37	86.6	50	117.0	63	147.5	76	177.9
12	28.1	25	58.5	38	89.0	51	119.4	64	149.8	77	180.3
13	30.4	26	60.9	39	91.3	52	121.7	65	152.2	78	182.6

PART III.

Correction due to the Change of Gravity from the Latitude of 45° to the Latitude of the Place of Observation.

Positive from Lat. 0° to 45°; Negative from Lat. 45° to 90°.

PART IV.

Correction for Decrease of Gravity on a Vertical.

Always Positive.

PART V.

Correction due to the Height of the lower Station.

Always Positive.

App. Alt.	Latitude.						Correction for Decrease of Gravity on a Vertical.	Height of Barometer at lower Station.							App. Alt.
	0°	10°	20°	30°	40°	45°		16 in.	18 in.	20 in.	22 in.	24 in.	26 in.	28 in.	
	90°	80°	70°	60°	50°										
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1000	2.6	2.5	2.0	1.3	0.5	0	2.5	1.6	1.3	1.0	0.8	0.6	0.4	0.2	1000
2000	5.3	5.0	4.1	2.6	0.9	0	5.2	3.1	2.5	2.0	1.5	1.1	0.7	0.3	2000
3000	7.9	7.5	6.1	4.0	1.4	0	7.9	4.7	3.8	3.0	2.3	1.7	1.1	0.5	3000
4000	10.6	10.0	8.1	5.3	1.8	0	10.8	6.3	5.1	4.0	3.1	2.2	1.4	0.7	4000
5000	13.2	12.4	10.1	6.6	2.3	0	13.7	7.8	6.4	5.0	3.8	2.8	1.8	0.8	5000
6000	15.9	14.9	12.2	7.9	2.8	0	16.7	9.4	7.6	6.0	4.6	3.3	2.1	1.0	6000
7000	18.5	17.4	14.2	9.3	3.2	0	19.9	11.0	8.9	7.1	5.4	3.9	2.5	1.2	7000
8000	21.2	19.9	16.2	10.6	3.7	0	23.1	12.5	10.2	8.1	6.2	4.4	2.8	1.3	8000
9000	23.8	22.4	18.3	11.9	4.1	0	26.4	14.1	11.4	9.1	6.9	5.0	3.2	1.5	9000
10000	26.5	24.9	20.3	13.2	4.6	0	29.8	15.7	12.7	10.1	7.7	5.5	3.5	1.7	10000
11000	29.1	27.4	22.3	14.6	5.1	0	33.3	17.2	14.0	11.1	8.5	6.1	3.9	1.8	11000
12000	31.8	29.9	24.4	15.9	5.5	0	36.9	18.8	15.3	12.1	9.2	6.6	4.2	2.0	12000
13000	34.4	32.4	26.4	17.2	6.0	0	40.6	20.4	16.5	13.1	10.0	7.2	4.6	2.2	13000
14000	37.1	34.9	28.4	18.5	6.4	0	44.4	21.9	17.8	14.1	10.8	7.7	4.9	2.3	14000
15000	39.7	37.3	30.4	19.9	6.9	0	48.3	23.5	19.1	15.1	11.5	8.3	5.3	2.5	15000
16000	42.4	39.8	32.5	21.2	7.4	0	52.3	25.1	20.3	16.1	12.3	8.8	5.6	2.7	16000
17000	45.0	42.3	34.5	22.5	7.8	0	56.4	26.6	21.6	17.1	13.1	9.4	6.0	2.8	17000
18000	47.7	44.8	36.5	23.8	8.3	0	60.5	28.2	22.9	18.1	13.8	9.9	6.3	3.0	18000
19000	50.3	47.3	38.6	25.2	8.7	0	64.8	29.8	24.1	19.2	14.6	10.5	6.7	3.2	19000
20000	53.0	49.8	40.6	26.5	9.2	0	69.2	31.3	25.4	20.2	15.4	11.0	7.0	3.3	20000
21000	55.6	52.3	42.6	27.8	9.7	0	73.6	32.9	26.7	21.2	16.1	11.6	7.4	3.5	21000
22000	58.3	54.8	44.7	29.1	10.1	0	78.2	34.5	28.0	22.2	16.9	12.1	7.7	3.7	22000
23000	60.9	57.3	46.7	30.5	10.6	0	82.9	36.0	29.2	23.2	17.7	12.7	8.1	3.8	23000
24000	63.6	59.8	48.7	31.8	11.0	0	87.6	37.6	30.5	24.2	18.5	13.2	8.4	4.0	24000
25000	66.2	62.2	50.7	33.1	11.5	0	92.5	39.1	31.8	25.2	19.2	13.8	8.8	4.1	25000

Parts of the Unit of Time.	Bessel's Coefficients for				Parts of the Unit of Time.	Bessel's Coefficients for			
	2d Diff.	3d Diff.	4th Diff.	5th Diff.		2d Diff.	3d Diff.	4th Diff.	5th Di
	$t \cdot \frac{t-1}{2}$.	$t \cdot \frac{t-1}{2} \cdot \frac{t-\frac{1}{2}}{3}$.	$t \ldots \frac{t-2}{4}$.	$t \ldots \frac{t-\frac{1}{2}}{5}$.		$t \cdot \frac{t-1}{2}$.	$t \cdot \frac{t-1}{2} \cdot \frac{t-\frac{1}{2}}{3}$.	$t \ldots \frac{t-2}{4}$.	$t \ldots t-$
0.01	—.00495	.00081	.00083	—.00008	0.51	—.12495	—.00042	.02343	.000
.02	.00980	.00157	.00165	.00016	.52	.12480	.00083	.02340	.000
.03	.01455	.00228	.00246	.00023	.53	.12455	.00125	.02334	.000
.04	.01920	.00294	.00326	.00030	.54	.12420	.00166	.02327	.000
.05	.02375	.00356	.00405	.00036	.55	.12375	.00206	.02318	.000
.06	.02820	.00414	.00483	.00043	.56	.12320	.00246	.02306	.000
.07	.03255	.00467	.00560	.00048	.57	.12255	.00286	.02293	.000
.08	.03680	.00515	.00636	.00053	.58	.12180	.00325	.02278	.000
.09	.04095	.00560	.00711	.00058	.59	.12095	.00363	.02260	.000
.10	.04500	.00600	.00784	.00063	.60	.12000	.00400	.02240	.000
.11	.04895	.00636	.00856	.00067	.61	.11895	.00436	.02218	.000
.12	.05280	.00669	.00927	.00070	.62	.11780	.00471	.02194	.000
.13	.05655	.00697	.00996	.00074	.63	.11655	.00505	.02169	.000
.14	.06020	.00723	.01064	.00077	.64	.11520	.00538	.02141	.000
.15	.06375	.00744	.01130	.00079	.65	.11375	.00569	.02111	.000
.16	.06720	.00762	.01195	.00081	.66	.11220	.00598	.02080	.000
.17	.07055	.00776	.01259	.00083	.67	.11055	.00626	.02046	.000
.18	.07380	.00787	.01321	.00085	.68	.10880	.00653	.02010	.000
.19	.07695	.00795	.01381	.00086	.69	.10695	.00677	.01973	.000
.20	.08000	.00800	.01440	.00086	.70	.10500	.00700	.01934	.000
.21	.08295	.00802	.01497	.00087	.71	.10295	.00721	.01893	.000
.22	.08580	.00801	.01553	.00087	.72	.10080	.00739	.01850	.000
.23	.08855	.00797	.01606	.00087	.73	.09855	.00756	.01805	.000
.24	.09120	.00790	.01658	.00086	.74	.09620	.00770	.01758	.000
.25	.09375	.00781	.01709	.00085	.75	.09375	.00781	.01709	.000
.26	.09620	.00770	.01758	.00084	.76	.09120	.00790	.01658	.000
.27	.09855	.00756	.01805	.00083	.77	.08855	.00797	.01606	.000
.28	.10080	.00739	.01850	.00081	.78	.08580	.00801	.01553	.000
.29	.10295	.00721	.01893	.00080	.79	.08295	.00802	.01497	.000
.30	.10500	.00700	.01934	.00077	.80	.08000	.00800	.01440	.000
.31	.10695	.00677	.01973	.00075	.81	.07695	.00795	.01381	.000
.32	.10880	.00653	.02010	.00072	.82	.07380	.00787	.01321	.000
.33	.11055	.00626	.02046	.00070	.83	.07055	.00776	.01259	.000
.34	.11220	.00598	.02080	.00067	.84	.06720	.00762	.01195	.000
.35	.11375	.00569	.02111	.00063	.85	.06375	.00744	.01130	.000
.36	.11520	.00538	.02141	.00060	.86	.06020	.00723	.01064	.000
.37	.11655	.00505	.02169	.00056	.87	.05655	.00697	.00996	.000
.38	.11780	.00471	.02194	.00053	.88	.05280	.00669	.00927	.000
.39	.11895	.00436	.02218	.00049	.89	.04895	.00636	.00856	.000
.40	.12000	.00400	.02240	.00045	.90	.04500	.00600	.00784	.000
.41	.12095	.00363	.02260	.00041	.91	.04095	.00560	.00711	.000
.42	.12180	.00325	.02278	.00036	.92	.03680	.00515	.00636	.000
.43	.12255	.00286	.02293	.00032	.93	.03255	.00467	.00560	.000
.44	.12320	.00246	.02306	.00028	.94	.02820	.00414	.00483	.000
.45	.12375	.00206	.02318	.00023	.95	.02375	.00356	.00405	.000
.46	.12420	.00166	.02327	.00019	.96	.01920	.00294	.00326	.000
.47	.12455	.00125	.02334	.00014	.97	.01455	.00228	.00246	.000
.48	.12480	.00083	.02340	.00009	.98	.00980	.00157	.00165	.000
.49	.12495	.00042	.02343	.00005	.99	.00495	.00081	.00083	.000
.50	—.12500	.00000	.02344	—.00000	1.00	—.00000	—.00000	.00000	.000

Parts of the Unit of Time.	Binomial Coefficients for				Parts of the Unit of Time.	Binomial Coefficients for			
	2d Diff.	3d Diff.	4th Diff.	5th Diff.		2d Diff.	3d Diff.	4th Diff.	5th Diff.
	$t.\frac{t-1}{2}$.	$t.\frac{t-1}{2}.\frac{t-2}{3}$.	$t\ldots\frac{t-3}{4}$.	$t..\frac{t-4}{5}$.		$t.\frac{t-1}{2}$.	$t.\frac{t-1}{2}.\frac{t-2}{3}$.	$t\ldots\frac{t-3}{4}$.	$t\ldots\frac{t-4}{5}$.
0.01	—.00495	.00328	—.00245	.00196	0.51	—.12495	.06206	—.03863	.02696
.02	.00980	.00647	.00482	.00384	.52	.12480	.06157	.03817	.02657
.03	.01455	.00955	.00709	.00563	.53	.12455	.06103	.03769	.02615
.04	.01920	.01254	.00928	.00735	.54	.12420	.06044	.03717	.02572
.05	.02375	.01544	.01139	.00899	.55	.12375	.05981	.03664	.02528
.06	.02820	.01824	.01340	.01056	.56	.12320	.05914	.03607	.02482
.07	.03255	.02094	.01534	.01206	.57	.12255	.05842	.03549	.02434
.08	.03680	.02355	.01719	.01348	.58	.12180	.05765	.03488	.02386
.09	.04095	.02607	.01897	.01483	.59	.12095	.05685	.03425	.02336
.10	.04500	.02850	.02066	.01612	.60	.12000	.05600	.03360	.02285
.11	.04895	.03084	.02228	.01733	.61	.11895	.05511	.03293	.02233
.12	.05280	.03309	.02382	.01849	.62	.11780	.05419	.03224	.02180
.13	.05655	.03525	.02529	.01958	.63	.11655	.05322	.03154	.02125
.14	.06020	.03732	.02669	.02060	.64	.11520	.05222	.03081	.02071
.15	.06375	.03931	.02801	.02157	.65	.11375	.05119	.03007	.02015
.16	.06720	.04122	.02926	.02247	.66	.11220	.05012	.02932	.01958
.17	.07055	.04304	.03045	.02332	.67	.11055	.04901	.02855	.01901
.18	.07380	.04477	.03156	.02412	.68	.10880	.04787	.02777	.01844
.19	.07695	.04643	.03261	.02485	.69	.10695	.04670	.02697	.01785
.20	.08000	.04800	.03360	.02554	.70	.10500	.04550	.02616	.01727
.21	.08295	.04949	.03452	.02617	.71	.10295	.04427	.02534	.01668
.22	.08580	.05091	.03538	.02674	.72	.10080	.04301	.02451	.01608
.23	.08855	.05224	.03618	.02728	.73	.09855	.04172	.02368	.01548
.24	.09120	.05350	.03692	.02776	.74	.09620	.04040	.02283	.01488
.25	.09375	.05469	.03760	.02820	.75	.09375	.03906	.02197	.01428
.26	.09620	.05580	.03822	.02859	.76	.09120	.03770	.02111	.01368
.27	.09855	.05683	.03879	.02894	.77	.08855	.03631	.02024	.01308
.28	.10080	.05779	.03930	.02924	.78	.08580	.03489	.01937	.01247
.29	.10295	.05868	.03976	.02950	.79	.08295	.03346	.01848	.01187
.30	.10500	.05950	.04016	.02972	.80	.08000	.03200	.01760	.01126
.31	.10695	.06025	.04052	.02990	.81	.07695	.03052	.01671	.01066
.32	.10880	.06093	.04082	.03004	.82	.07380	.02903	.01582	.01006
.33	.11055	.06154	.04108	.03015	.83	.07055	.02751	.01493	.00946
.34	.11220	.06208	.04129	.03022	.84	.06720	.02598	.01403	.00887
.35	.11375	.06256	.04145	.03026	.85	.06375	.02444	.01314	.00828
.36	.11520	.06298	.04156	.03026	.86	.06020	.02288	.01224	.00769
.37	.11655	.06333	.04164	.03023	.87	.05655	.02130	.01134	.00710
.38	.11780	.06361	.04167	.03017	.88	.05280	.01971	.01045	.00652
.39	.11895	.06384	.04165	.03007	.89	.04895	.01811	.00955	.00594
.40	.12000	.06400	.04160	.02995	.90	.04500	.01650	.00866	.00537
.41	.12095	.06410	.04151	.02980	.91	.04095	.01488	.00777	.00480
.42	.12180	.06415	.04138	.02962	.92	.03680	.01325	.00689	.00424
.43	.12255	.06413	.04121	.02942	.93	.03255	.01161	.00601	.00369
.44	.12320	.06406	.04100	.02919	.94	.02820	.00996	.00513	.00314
.45	.12375	.06394	.04076	.02894	.95	.02375	.00831	.00426	.00260
.46	.12420	.06376	.04049	.02866	.96	.01920	.00666	.00339	.00206
.47	.12455	.06352	.04018	.02836	.97	.01455	.00500	.00254	.00154
.48	.12480	.06323	.03984	.02804	.98	.00980	.00333	.00168	.00102
.49	.12495	.06289	.03946	.02770	.99	.00495	.00167	.00084	.00050
.50	—.12500	.06250	—.03906	.02734	1.00	—.00000	.00000	—.00000	.00000

Logarithms of the Coefficients for Interpolation by Bessel's Formula.

Argument for T = 12 hours.	Logarithms of the Coefficients for				
	First Differences.	Second Differences.	Third Differences.	Fourth Differences.	Fifth Differences.
h. m.					
0 0	—∞	—∞	—∞	—∞	—∞
5	7.8416375	7.53758*n*	6.75336	6.76092	5.75485*n*
10	8.1426675	7.83556*n*	7.04518	7.06038	6.04814*n*
15	8.3187588	8.00859*n*	7.21195	7.23484	6.21636*n*
20	8.4436975	8.13043*n*	7.32746	7.35811	6.33328*n*
25	8.5406075	8.22423*n*	7.41482	7.45330	6.42204*n*
30	8.6197888	8.30028*n*	7.48434	7.53071	6.49292*n*
35	8.6867355	8.36406*n*	7.54149	7.59584	6.55142*n*
40	8.7447275	8.41887*n*	7.58957	7.65197	6.60082*n*
45	8.7958800	8.46682*n*	7.63068	7.70121	6.64322*n*
50	8.8416375	8.50935*n*	7.66626	7.74501	6.68007*n*
55	8.8830302	8.54749*n*	7.69734	7.78439	6.71239*n*
1 0	8.9208188	8.58200*n*	7.72467	7.82013	6.74095*n*
5	8.9555809	8.61346*n*	7.74883	7.85279	6.76631*n*
10	8.9877655	8.64232*n*	7.77026	7.88282	6.78891*n*
15	9.0177288	8.66893*n*	7.78932	7.91058	6.80912*n*
20	9.0457575	8.69358*n*	7.80628	7.93636	6.82721*n*
25	9.0720864	8.71650*n*	7.82138	7.96039	6.84342*n*
30	9.0969100	8.73789*n*	7.83480	7.98286	6.85792*n*
35	9.1203911	8.75791*n*	7.84670	8.00394	6.87089*n*
40	9.1426675	8.77670*n*	7.85722	8.02377	6.88244*n*
45	9.1638568	8.79437*n*	7.86646	8.04246	6.89270*n*
50	9.1840602	8.81103*n*	7.87451	8.06011	6.90175*n*
55	9.2033653	8.82676*n*	7.88147	8.07681	6.90968*n*
2 0	9.2218487	8.84164*n*	7.88739	8.09264	6.91655*n*
5	9.2395775	8.85573*n*	7.89235	8.10766	6.92243*n*
10	9.2566109	8.86910*n*	7.89637	8.12194	6.92737*n*
15	9.2730013	8.88179*n*	7.89952	8.13553	6.93141*n*
20	9.2887955	8.89386*n*	7.90183	8.14846	6.93458*n*
25	9.3040355	8.90534*n*	7.90333	8.16078	6.93692*n*
30	9.3187588	8.91627*n*	7.90404	8.17253	6.93845*n*
35	9.3329992	8.92669*n*	7.90399	8.18375	6.93920*n*
40	9.3467875	8.93661*n*	7.90319	8.19446	6.93919*n*
45	9.3601514	8.94608*n*	7.90166	8.20469	6.93842*n*
50	9.3731164	8.95512*n*	7.89942	8.21446	6.93692*n*
55	9.3857055	8.96374*n*	7.89646	8.22381	6.93468*n*
3 0	9.3979400	8.97197*n*	7.89279	8.23274	6.93171*n*
5	9.4098392	8.97983*n*	7.88841	8.24128	6.92801*n*
10	9.4214211	8.98733*n*	7.88333	8.24944	6.92359*n*
15	9.4327021	8.99450*n*	7.87753	8.25724	6.91842*n*
20	9.4436975	9.00134*n*	7.87100	8.26470	6.91252*n*
25	9.4544214	9.00787*n*	7.86374	8.27183	6.90586*n*
30	9.4648868	9.01409*n*	7.85573	8.27864	6.89843*n*
35	9.4751060	9.02003*n*	7.84695	8.28514	6.89020*n*
40	9.4850902	9.02570*n*	7.83737	8.29134	6.88116*n*
45	9.4948500	9.03109*n*	7.82697	8.29725	6.87129*n*
50	9.5043953	9.03623*n*	7.81572	8.30289	6.86053*n*
55	9.5137354	9.04111*n*	7.80357	8.30826	6.84887*n*
4 0	9.5228787	9.04576*n*	7.79048	8.31336	6.83624*n*

Logarithms of the Coefficients for Interpolation by Bessel's Formula.

Argument for T=12 hours.	Logarithms of the Coefficients for				
	First Differences.	Second Differences.	Third Differences.	Fourth Differences.	Fifth Differences.
h. *m.*					
4 0	9.5228787	9.04576*n*	7.79048	8.31336	6.83624*n*
5	9.5318336	9.05016*n*	7.77641	8.31821	6.82261*n*
10	9.5406075	9.05434*n*	7.76128	8.32282	6.80791*n*
15	9.5492077	9.05830*n*	7.74503	8.32718	6.79206*n*
20	9.5576409	9.06204*n*	7.72758	8.33130	6.77500*n*
25	9.5659134	9.06556*n*	7.70883	8.33519	6.75661*n*
30	9.5740313	9.06888*n*	7.68867	8.33886	6.73680*n*
35	9.5820002	9.07200*n*	7.66696	8.34230	6.71542*n*
40	9.5898255	9.07492*n*	7.64355	8.34553	6.69232*n*
45	9.5975124	9.07764*n*	7.61825	8.34854	6.66730*n*
50	9.6050655	9.08017*n*	7.59082	8.35134	6.64014*n*
55	9.6124895	9.08252*n*	7.56098	8.35394	6.61055*n*
5 0	9.6197887	9.08468*n*	7.52837	8.35633	6.57818*n*
5	9.6269674	9.08665*n*	7.49256	8.35853	6.54259*n*
10	9.6340292	9.08845*n*	7.45297	8.36052	6.50319*n*
15	9.6409781	9.09007*n*	7.40883	8.36232	6.45923*n*
20	9.6478175	9.09152*n*	7.35912	8.36392	6.40968*n*
25	9.6545509	9.09279*n*	7.30240	8.36533	6.35310*n*
30	9.6611814	9.09388*n*	7.23655	8.36655	6.28737*n*
35	9.6677123	9.09481*n*	7.15830	8.36758	6.20922*n*
40	9.6741464	9.09557*n*	7.06214	8.36842	6.11315*n*
45	9.6804866	9.09616*n*	6.93779	8.36907	5.98886*n*
50	9.6867355	9.09657*n*	6.76212	8.36954	5.81324*n*
55	9.6928959	9.09683*n*	6.46134	8.36982	5.51249*n*
6 0	9.6989700	9.09691*n*	—∞	8.36991	—∞
5	9.7049604	9.09683*n*	6.46134*n*	8.36982	5.51249
10	9.7108692	9.09657*n*	6.76212*n*	8.36954	5.81324
15	9.7166988	9.09616*n*	6.93779*n*	8.36907	5.98886
20	9.7224511	9.09557*n*	7.06214*n*	8.36842	6.11315
25	9.7281282	9.09481*n*	7.15830*n*	8.36758	6.20922
30	9.7337321	9.09388*n*	7.23655*n*	8.36655	6.28737
35	9.7392646	9.09279*n*	7.30240*n*	8.36533	6.35310
40	9.7447275	9.09152*n*	7.35912*n*	8.36392	6.40968
45	9.7501225	9.09007*n*	7.40883*n*	8.36232	6.45923
50	9.7554514	9.08845*n*	7.45297*n*	8.36052	6.50319
55	9.7607156	9.08665*n*	7.49256*n*	8.35853	6.54259
7 0	9.7659167	9.08468*n*	7.52837*n*	8.35633	6.57818
5	9.7710564	9.08252*n*	7.56098*n*	8.35394	6.61055
10	9.7761360	9.08017*n*	7.59082*n*	8.35134	6.64014
15	9.7811568	9.07764*n*	7.61825*n*	8.34854	6.66730
20	9.7861202	9.07492*n*	7.64355*n*	8.34553	6.69232
25	9.7910275	9.07200*n*	7.66696*n*	8.34230	6.71542
30	9.7958800	9.06888*n*	7.68867*n*	8.33886	6.73680
35	9.8006789	9.06556*n*	7.70883*n*	8.33519	6.75661
40	9.8054253	9.06204*n*	7.72758*n*	8.33130	6.77500
45	9.8101205	9.05830*n*	7.74503*n*	8.32718	6.79206
50	9.8147654	9.05434*n*	7.76128*n*	8.32282	6.80791
55	9.8193611	9.05016*n*	7.77641*n*	8.31821	6.82261
8 0	9.8239087	9.04576*n*	7.79048*n*	8.31336	6.83624

Logarithms of the Coefficients for Interpolation by Bessel's Formula.

Argument for T = 12 hours.		Logarithms of the Coefficients for				
		First Differences.	Second Differences.	Third Differences.	Fourth Differences.	Fifth Differences.
h.	m.					
8	0	9.8239087	9.04576n	7.79048n	8.31336	6.83624
	5	9.8284092	9.04111n	7.80357n	8.30826	6.84887
	10	9.8328636	9.03623n	7.81572n	8.30289	6.86053
	15	9.8372727	9.03109n	7.82697n	8.29725	6.87129
	20	9.8416375	9.02570n	7.83737n	8.29134	6.88116
	25	9.8459589	9.02003n	7.84695n	8.28514	6.89020
	30	9.8502377	9.01409n	7.85573n	8.27864	6.89843
	35	9.8544747	9.00787n	7.86374n	8.27183	6.90586
	40	9.8586709	9.00134n	7.87100n	8.26470	6.91252
	45	9.8628268	8.99450n	7.87753n	8.25724	6.91842
	50	9.8669434	8.98733n	7.88333n	8.24944	6.92359
	55	9.8710213	8.97983n	7.88841n	8.24128	6.92801
9	0	9.8750613	8.97197n	7.89279n	8.23274	6.93171
	5	9.8790640	8.96374n	7.89646n	8.22381	6.93468
	10	9.8830302	8.95512n	7.89942n	8.21446	6.93692
	15	9.8869605	8.94608n	7.90166n	8.20469	6.93842
	20	9.8908555	8.93661n	7.90319n	8.19446	6.93919
	25	9.8947160	8.92669n	7.90399n	8.18375	6.93920
	30	9.8985424	8.91627n	7.90404n	8.17253	6.93845
	35	9.9023354	8.90534n	7.90333n	8.16078	6.93692
	40	9.9060955	8.89386n	7.90183n	8.14846	6.93458
	45	9.9098234	8.88179n	7.89952n	8.13553	6.93141
	50	9.9135195	8.86910n	7.89637n	8.12194	6.92737
	55	9.9171845	8.85573n	7.89235n	8.10766	6.92243
10	0	9.9208187	8.84164n	7.88739n	8.09264	6.91655
	5	9.9244229	8.82676n	7.88147n	8.07681	6.90968
	10	9.9279973	8.81103n	7.87451n	8.06011	6.90175
	15	9.9315426	8.79437n	7.86646n	8.04246	6.89270
	20	9.9350592	8.77670n	7.85722n	8.02377	6.88244
	25	9.9385475	8.75791n	7.84670n	8.00394	6.87089
	30	9.9420080	8.73789n	7.83480n	7.98286	6.85792
	35	9.9454412	8.71650n	7.82138n	7.96039	6.84342
	40	9.9488475	8.69358n	7.80628n	7.93636	6.82721
	45	9.9522272	8.66893n	7.78932n	7.91058	6.80912
	50	9.9555809	8.64232n	7.77026n	7.88282	6.78891
	55	9.9589088	8.61346n	7.74883n	7.85279	6.76631
11	0	9.9622115	8.58200n	7.72467n	7.82013	6.74095
	5	9.9654892	8.54749n	7.69734n	7.78439	6.71239
	10	9.9687423	8.50935n	7.66626n	7.74501	6.68007
	15	9.9719713	8.46682n	7.63068n	7.70121	6.64322
	20	9.9751764	8.41887n	7.58957n	7.65197	6.60082
	25	9.9783581	8.36406n	7.54149n	7.59584	6.55142
	30	9.9815166	8.30028n	7.48434n	7.53071	6.49292
	35	9.9846523	8.22423n	7.41482n	7.45330	6.42204
	40	9.9877655	8.13043n	7.32746n	7.35811	6.33328
	45	9.9908566	8.00859n	7.21195n	7.23484	6.21636
	50	9.9939258	7.83556n	7.04518n	7.06038	6.04814
	55	9.9969736	7.53758n	6.75336n	6.76092	5.75485
12	0	0.0000000	—∞	—∞	—∞	—∞

TABLE XXV.

To compare the Centesimal Thermometer with Fahrenheit's.

Centes.	Fahrenheit.	Cent.	Fahrenheit.	Cent.	Fahrenheit.	Cent.	Fahrenh't.	Cent.	Fahrenh't.
°	°	°	°	°	°	°	°	°	°
+100	+212.0	+71	+159.8	+42	+107.6	+13	+55.4	−16	+ 3.2
99	210.2	70	158.0	41	105.8	12	53.6	−17	+ 1.4
98	208.4	69	156.2	40	104.0	11	51.8	−18	− 0.4
97	206.6	68	154.4	39	102.2	10	50.0	−19	− 2.2
96	204.8	67	152.6	38	100.4	9	48.2	−20	− 4.0
95	203.0	66	150.8	37	98.6	8	46.4	−21	− 5.8
94	201.2	65	149.0	36	96.8	7	44.6	−22	− 7.6
93	199.4	64	147.2	35	95.0	6	42.8	−23	− 9.4
92	197.6	63	145.4	34	93.2	5	41.0	−24	−11.2
91	195.8	62	143.6	33	91.4	4	39.2	−25	−13.0
90	194.0	61	141.8	32	89.6	3	37.4	−26	−14.8
89	192.2	60	140.0	31	87.8	2	35.6	−27	−16.6
88	190.4	59	138.2	30	86.0	+ 1	33.8	−28	−18.4
87	188.6	58	136.4	29	84.2	0	32.0	−29	−20.2
86	186.8	57	134.6	28	82.4	− 1	30.2	−30	−22.0
85	185.0	56	132.8	27	80.6	− 2	28.4	−31	−23.8
84	183.2	55	131.0	26	78.8	− 3	26.6	−32	−25.6
83	181.4	54	129.2	25	77.0	− 4	24.8	−33	−27.4
82	179.6	53	127.4	24	75.2	− 5	23.0	−34	−29.2
81	177.8	52	125.6	23	73.4	− 6	21.2	−35	−31.0
80	176.0	51	123.8	22	71.6	− 7	19.4	−36	−32.8
79	174.2	50	122.0	21	69.8	− 8	17.6	−37	−34.6
78	172.4	49	120.2	20	68.0	− 9	15.8	−38	−36.4
77	170.6	48	118.4	19	66.2	−10	14.0	−39	−38.2
76	168.8	47	116.6	18	64.4	−11	12.2	−40	−40.0
75	167.0	46	114.8	17	62.6	−12	10.4	−41	−41.8
74	165.2	45	113.0	16	60.8	−13	8.6	−42	−43.6
73	163.4	44	111.2	15	59.0	−14	6.8	−43	−45.4
+ 72	+161.6	+43	+109.4	+14	+ 57.2	−15	+ 5.0	−44	−47.2

Proportional Parts.	
C.	Fah't.
°	°
0.1	0.18
0.2	0.36
0.3	0.54
0.4	0.72
0.5	0.90
0.6	1.08
0.7	1.26
0.8	1.44
0.9	1.62
1.0	1.80

$x°$ Centesimal $=(32° + \frac{9}{5}x°)$ Fahrenheit.

TABLE XXVI.

To compare Reaumur's Thermometer with Fahrenheit's.

Reaum.	Fahrenheit.	R'm'r.	Fahrenheit.	R'm'r.	Fahrenheit.	R'm'r.	Fahrenh't.	R'm'r.	Fahrenh't.
°	°	°	°	°	°	°	°	°	°
+ 80	+212.0	+57	+160.25	+34	+108.5	+11	+56.75	−12	+ 5.0
79	209.75	56	158.0	33	106.25	10	54.5	−13	+ 2.75
78	207.5	55	155.75	32	104.0	9	52.25	−14	+ 0.5
77	205.25	54	153.5	31	101.75	8	50.0	−15	− 1.75
76	203.0	53	151.25	30	99.5	7	47.75	−16	− 4.0
75	200.75	52	149.0	29	97.25	6	45.5	−17	− 6.25
74	198.5	51	146.75	28	95.0	5	43.25	−18	− 8.5
73	196.25	50	144.5	27	92.75	4	41.0	−19	−10.75
72	194.0	49	142.25	26	90.5	3	38.75	−20	−13.0
71	191.75	48	140.0	25	88.25	2	36.5	−21	−15.25
70	189.5	47	137.75	24	86.0	+ 1	34.25	−22	−17.5
69	187.25	46	135.5	23	83.75	0	32.0	−23	−19.75
68	185.0	45	133.25	22	81.5	− 1	29.75	−24	−22.0
67	182.75	44	131.0	21	79.25	− 2	27.5	−25	−24.25
66	180.5	43	128.75	20	77.0	− 3	25.25	−26	−26.5
65	178.25	42	126.5	19	74.75	− 4	23.0	−27	−28.75
64	176.0	41	124.25	18	72.5	− 5	20.75	−28	−31.0
63	173.75	40	122.0	17	70.25	− 6	18.5	−29	−33.25
62	171.5	39	119.75	16	68.0	− 7	16.25	−30	−35.5
61	169.25	38	117.5	15	65.75	− 8	14.0	−31	−37.75
60	167.0	37	115.25	14	63.5	− 9	11.75	−32	−40.0
59	164.75	36	113.0	13	61.25	−10	9.5	−33	−42.25
+ 58	+162.5	+35	+110.75	+12	+ 59.0	−11	+ 7.25	−34	−44.5

Proportional Parts.	
R.	Fah't.
°	°
0.1	0.225
0.2	0.45
0.3	0.675
0.4	0.90
0.5	1.125
0.6	1.35
0.7	1.575
0.8	1.80
0.9	2.025
1.0	2.25

$x°$ Reaumur $=(32° + \frac{9}{4}x°)$ Fahrenheit.

Height of Barometer corresponding to Temperature of

Fah't Degrees.	English Inches.	Diff.	Fah't Degrees.	English Inches.	Diff.	Fah't Degrees.	English Inches.	Diff.	Fah't Degrees.	Engli Inche	
185.0	17.049	.037	190.8	19.326	.042	196.6	21.851	.046	202.4	24.6	
.1	.086	.037	.9	.368	.041	.7	.897	.046	.5	.69	
.2	.123	.038	191.0	.409	.041	.8	.943	.046	.6	.74	
.3	.161	.037	.1	.450	.042	.9	.989	.046	.7	.79	
.4	.198	.038	.2	.492	.042	197.0	22.035	.046	.8	.85	
.5	.236	.037	.3	.534	.041	.1	.081	.047	.9	.901	
.6	.273	.037	.4	.575	.042	.2	.128	.046	203.0	.952	
.7	.310	.038	.5	.617	.042	.3	.174	.047	.1	25.003	
.8	.348	.038	.6	.659	.042	.4	.221	.046	.2	.055	
.9	.386	.038	.7	.701	.042	.5	.267	.047	.3	.106	
186.0	.424	.038	.8	.743	.042	.6	.314	.047	.4	.158	
.1	.462	.038	.9	.785	.042	.7	.361	.046	.5	.210	.
.2	.500	.038	192.0	.827	.042	.8	.407	.047	.6	.261	.
.3	.538	.038	.1	.869	.043	.9	.454	.047	.7	.313	.
.4	.576	.039	.2	.912	.042	198.0	.501	.047	.8	.365	.
.5	.615	.038	.3	.954	.042	.1	.548	.047	.9	.417	.0
.6	.653	.038	.4	.996	.043	.2	.595	.047	204.0	.469	.0
.7	.691	.039	.5	20.039	.043	.3	.642	.047	.1	.521	.0
.8	.730	.038	.6	.082	.042	.4	.689	.047	.2	.573	.0
.9	.768	.039	.7	.124	.043	.5	.736	.048	.3	.626	.0
187.0	.807	.039	.8	.167	.043	.6	.784	.047	.4	.678	.0
.1	.846	.038	.9	.210	.043	.7	.831	.048	.5	.730	.05
.2	.884	.039	193.0	.253	.043	.8	.879	.047	.6	.783	.05
.3	.923	.039	.1	.296	.043	.9	.926	.048	.7	.836	.05
.4	.962	.039	.2	.339	.043	199.0	.974	.048	.8	.888	.05
.5	18.001	.039	.3	.382	.044	.1	23.022	.048	.9	.941	.053
.6	.040	.039	.4	.426	.043	.2	.070	.048	205.0	.994	.053
.7	.979	.039	.5	.469	.043	.3	.118	.048	.1	26.047	.053
.8	.118	.040	.6	.512	.044	.4	.166	.048	.2	.100	.053
.9	.158	.039	.7	.556	.043	.5	.214	.048	.3	.153	.053
188.0	.197	.039	.8	.599	.044	.6	.262	.049	.4	.206	.053
.1	.236	.040	.9	.643	.044	.7	.311	.048	.5	.259	.054
.2	.276	.039	194.0	.687	.044	.8	.359	.048	.6	.313	.053
.3	.315	.040	.1	.731	.044	.9	.407	.049	.7	.366	.054
.4	.355	.040	.2	.775	.044	200.0	.456	.049	.8	.420	.053
.5	.395	.039	.3	.819	.044	.1	.505	.049	.9	.473	.054
.6	.434	.040	.4	.863	.044	.2	.553	.049	206.0	.527	.054
.7	.474	.040	.5	.907	.044	.3	.602	.049	.1	.581	.054
.8	.514	.040	.6	.951	.045	.4	.651	.049	.2	.635	.054
.9	.554	.040	.7	.996	.044	.5	.700	.049	.3	.689	.054
189.0	.594	.040	.8	21.040	.044	.6	.749	.049	.4	.743	.054
.1	.634	.040	.9	.084	.045	.7	.798	.049	.5	.797	.055
.2	.674	.040	195.0	.129	.045	.8	.847	.050	.6	.852	.054
.3	.714	.041	.1	.174	.044	.9	.897	.049	.7	.906	.055
.4	.755	.040	.2	.218	.045	201.0	.946	.050	.8	.961	.054
.5	.795	.040	.3	.263	.045	.1	.996	.049	.9	27.015	.055
.6	.835	.041	.4	.308	.045	.2	24.045	.050	207.0	.070	.055
.7	.876	.041	.5	.353	.045	.3	.095	.050	.1	.125	.055
.8	.917	.040	.6	.398	.045	.4	.145	.050	.2	.180	.055
.9	.957	.041	.7	.443	.045	.5	.195	.050	.3	.235	.055
190.0	.998	.041	.8	.488	.045	.6	.245	.050	.4	.290	.055
.1	19.039	.041	.9	.533	.045	.7	.295	.050	.5	.345	.055
.2	.080	.041	196.0	.578	.045	.8	.345	.050	.6	.400	.056
.3	.121	.041	.1	.623	.046	.9	.395	.050	.7	.456	.055
.4	.162	.041	.2	.669	.045	202.0	.445	.050	.8	.511	.055
.5	.203	.041	.3	.714	.046	.1	.495	.051	.9	.566	.056
.6	.244	.041	.4	.760	.046	.2	.546	.050	208.0	.622	.056
.7	.285	.041	.5	.806	.045	.3	.596	.051	.1	.678	.055
.8	.326		.6	.851		.4	.647		.2	.733	

CATALOGUE OF 1500 STARS.

No.	B.A.C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
1	4	21	Andromedæ,	α	1	0	0	38.57*	+3.086	61	44	16.1*	−19.93
2	7	11	Cassiopeæ,	β	2½		1	12.20*	3.149	31	40	39.9*	19.89
3	11		Phœnicis,	ε	4		1	47.34	3.089	136	34	24.7	20.04
4	16	22	Andromedæ,		5		2	32.59	3.093	44	45	47.1	20.05
5	19		Octantis,	γ^3	5		3	6.19	2.820	173	3	32.9	19.89
6	26	88	Pegasi,	γ	2		5	30.99*	3.084	75	39	2.1*	20.04
7	52	24	Andromedæ,	θ	5		9	16.17	3.111	52	9	5.1	20.03
8	62	8	Ceti,	ι	4		11	47.03*	3.060	99	39	22.0*	19.98
9	64		Tucanæ,	ζ	5		12	11.93	3.159	155	45	26.9	21.14
10	70		Tucanæ,	π	4½		13	41.72	2.834	160	27	31.9	19.88
11	72		Sculptoris,	ι	5		13	58.54	3.025	119	48	42.0*	19.92
12	88		Hydri,	β	3		17	47.38	3.297	168	6	4.4	20.26
13	93		Phœnicis,	κ	4		18	49.23	2.994	134	30	50.8	19.79
14	94		Phœnicis,	α	2		18	51.42	2.983	133	7	11.7	19.69
15	103		Sculptoris,		5		20	29.10	2.989	123	50	5.0	20.02
16	121	14	Cassiopeæ,	λ	5		23	31.35	3.263	36	18	22.7	19.97
17	124		Phœnicis,	λ	5		24	9.91	2.906	139	38	2.8	19.94
18	126	15	Cassiopeæ,	κ	4		24	30.74*	3.344	27	53	48.2*	19.96
19	127		Tucanæ,	β^1	4		24	38.29	2.770	153	47	9.2	19.87
20	128		Tucanæ,	β^2	4		24	39.13	2.773	153	47	34.3	19.89
21	134		Tucanæ,		5		25	51.93	2.751	153	51	36.8	19.50
22	143		Phœnicis,		5		27	19.48	2.901	143	12	5.6	19.95
23	153	17	Cassiopeæ,	ζ	4		28	38.45*	3.299	36	55	45.8*	19.91
24	155	29	Andromedæ,	π	4½		28	52.91	3.183	57	6	25.8*	19.92
25	164	30	Andromedæ,	ε	4		30	38.35*	3.154	61	30	11.9*	19.67
26	166	31	Andromedæ,	δ	3		31	18.99	3.187	59	57	38.5*	19.76
27	169	18	Cassiopeæ,	α	3		32	1.58*	3.354	34	17	9.9*	19.83
28	176		Tucanæ,		5		33	21.60	2.805	150	18	1.6	20.19
29	183		Phœnicis,	μ	5		34	13.89	2.869	136	54	27.5	20.03
30	188		Phœnicis,	ξ	5		34	54.51	2.738	147	19	33.9	19.96
31	189	20	Cassiopeæ,	π	5		35	11.45	3.289	43	47	47.9	19.81
32	192		Sculptoris,	λ^1	5		35	29.35	2.901	129	17	8.4	19.89
33	196	16	Ceti,	β	2½		36	3.42*	3.016	108	48	39.1*	19.86
34	199		Phœnicis,	η	5		36	35.64	2.713	148	17	17.3	19.29
35	200	17	Ceti,	ϕ^1	5		36	37.45	3.031	101	25	37.1	19.70
36	202		Sculptoris,	λ^2	5		36	57.03	2.919	129	14	56.6	19.88
37	215	34	Andromedæ,	ζ	4		39	23.79*	3.169	66	32	59.4*	19.69
38	218	24	Cassiopeæ,	η	4		40	3.40*	3.564	32	58	53.5*	19.27
39	219	25	Cassiopeæ,	ν	5		40	21.66	3.359	39	51	7.9	19.68
40	222	63	Piscium,	δ	5		40	54.20*	3.107	83	13	56.5*	19.73
41	227	35	Andromedæ,	ν	4		41	33.42*	3.279	49	44	19.9*	19.72
42	242	20	Ceti,		5		45	20.57*	3.064	91	57	35.4*	19.67
43	245		Cassiopeæ,		5		46	35.01	3.369	42	8	7.8	19.64
44	253	27	Cassiopeæ,	γ	3		47	41.83*	3.548	30	5	48.1*	19.64
45	259	37	Andromedæ,	μ	4		48	26.48*	3.301	52	18	54.0*	19.68
46	262	2	Ursæ Minoris,		5		49	9.55*	6.716	4	33	2.4*	19.59
47	264	38	Andromedæ,	η	5		49	12.50	3.190	67	23	32.1	19.60
48	272		Sculptoris,	α	5		51	22.51*	2.899	120	10	8.7*	19.52
49	288	71	Piscium,	ε	4		55	9.75*	3.114	82	55	7.2*	19.50
50	317		Phœnicis,	β	3½	0	59	23.10	+2.692	137	31	21.3	−19.37

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
1	+8.8790	+6.3273	+0.4875	+8.5544	−9.5808	−9.6753	−1.3022	+7.4483
2	9.1036	6.8230	0.4888	+9.0336	9.3491	−9.9299	1.3022	7.7194
3	8.9867	6.8791	0.4857	−8.8478	9.4829	+9.8611	1.3022	7.8924
4	8.9762	7.0214	0.4893	+8.8275	9.4738	−9.8512	1.3022	8.0452
5	9.7416	7.8733	0.4657	−9.7385	8.8182	+9.9968	1.3022	8.1316
6	8.8375	7.2192	0.4884	+8.2317	9.6174	−9.3940	1.3021	8.3815
7	8.9261	7.5333	0.4931	+8.7140	9.5023	−9.7875	1.3019	8.6068
8	8.8295	7.5411	0.4856	−8.0542	9.6399	+9.2241	1.3017	8.7110
9	9.2099	7.9364	0.4643	−9.1698	9.3555	+9.9593	1.3016	8.7259
10	9.2988	8.0757	0.4542	−9.2730	9.3043	+9.9735	1.3014	8.7761
11	8.8848	7.6705	0.4806	−8.5812	9.6094	+9.6957	1.3014	8.7849
12	9.5083	8.3982	0.4116	−9.4989	9.2180	+9.9893	1.3009	8.8886
13	8.9693	7.8847	0.4717	−8.8151	9.5647	+9.8443	1.3008	8.9140
14	8.9592	7.8754	0.4724	−8.7939	9.5717	+9.8333	1.3008	8.9148
15	8.9028	7.8552	0.4758	−8.6485	9.6130	+9.7440	1.3005	8.9507
16	9.0492	8.0620	0.5128	+8.9555	9.2416	−9.9040	1.2999	9.0105
17	9.0101	8.0348	0.4632	−8.8921	9.5579	+9.8795	1.2998	9.0222
18	9.1513	8.1822	0.5238	+9.0976	9.0358	−9.9439	1.2997	9.0284
19	9.1763	8.2094	0.4439	−9.1291	9.4594	+9.9504	1.2997	9.0306
20	9.1763	8.2097	0.4439	−9.1292	9.4592	+9.9504	1.2997	9.0308
21	9.1771	8.2315	0.4415	−9.1303	9.4658	+9.9504	1.2995	9.0516
22	9.0434	8.1218	0.4561	−8.9469	9.5504	+9.9004	1.2991	9.0753
23	9.0418	8.1408	0.5175	+8.9445	9.2071	−9.8994	1.2988	9.0957
24	8.8963	7.9991	0.5023	+8.6312	9.4714	−9.7314	1.2988	9.0993
25	8.8761	8.0048	0.5007	+8.5547	9.5022	−9.6747	1.2983	9.1248
26	8.8825	8.0208	0.5019	+8.5820	9.4879	−9.6954	1.2982	9.1342
27	9.0689	8.2170	0.5242	+8.9860	9.1119	−9.9129	1.2980	9.1439
28	9.1243	8.2904	0.4363	−9.0631	9.5328	+9.9342	1.2976	9.1615
29	8.9845	8.1620	0.4561	−8.8480	9.6077	+9.8586	1.2974	9.1726
30	9.0866	8.2727	0.4400	−9.0118	9.5587	+9.9201	1.2972	9.1811
31	8.9786	8.1683	0.5164	+8.8370	9.2785	−9.8533	1.2971	9.1845
32	8.9299	8.1233	0.4627	−8.7315	9.6371	+9.7963	1.2970	9.1882
33	8.8424	8.0428	0.4770	−8.3508	9.6640	+9.5031	1.2968	9.1950
34	9.0977	8.3046	0.4356	−9.0274	9.5604	+9.9242	1.2967	9.2014
35	8.8270	8.0343	0.4811	−8.1240	9.6598	+9.2914	1.2967	9.2017
36	8.9293	8.1405	0.4617	−8.7305	9.6412	+9.7955	1.2966	9.2056
37	8.8549	8.0945	0.5010	+8.4547	9.5185	−9.5934	1.2958	9.2332
38	9.0814	8.3283	0.5351	+9.0050	8.9557	−9.9170	1.2956	9.2402
39	9.0104	8.2607	0.5252	+8.8956	9.1572	−9.8784	1.2955	9.2435
40	8.8200	8.0762	0.4912	+7.8914	9.6128	−9.0644	1.2953	9.2493
41	8.9341	8.1974	0.5152	+8.7446	9.3316	−9.8032	1.2950	9.2561
42	8.8156	8.1177	0.4860	−7.3496	9.6439	+8.5254	1.2937	9.2935
43	8.9882	8.3023	0.5275	+8.8584	9.1505	−9.8611	1.2932	9.3051
44	9.1142	8.4388	0.5499	+9.0513	8.5900	−9.9276	1.2928	9.3151
45	8.9157	8.2474	0.5168	+8.7020	−9.3326	−9.7765	1.2924	9.3218
46	9.9143	9.2523	0.8224	+9.9130	+9.2497	−9.9886	1.2922	9.3279
47	8.8485	8.1872	0.5037	+8.4334	−9.5034	−9.5747	1.2921	9.3285
48	8.8761	8.2341	0.4621	−8.5773	9.6874	+9.6902	1.2912	9.3469
49	8.8145	8.2045	0.4928	+7.9054	9.6035	−9.0782	1.2895	9.3773
50	+8.9797	+8.4030	+0.4308	−8.8475	−9.6831	+9.8530	−1.2875	+9.4086

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annu Variat
						h.	m.	s.	s.	°	′	″	′
51	318	41	Andromedæ,		5	0	59	25.30	+3.410	46	51	30.1	—19
52	328	80	Piscium,	e	5	1	0	38.79*	3.083	85	8	41.8*	19
53	330	42	Andromedæ,	φ	5		0	49.11*	3.437	43	33	34.1*	19
54	332	31	Ceti,	η	3½		1	2.67	3.019	100	58	42.5	19
55	333		Tucanæ,	ι	5		1	19.95	2.357	152	34	36.9	19
56	334	43	Andromedæ,	β	2		1	20.90*	3.336	55	10	33.2*	19
57	339	33	Cassiopeæ,	θ	4½		1	59.95*	3.593	35	38	58.3*	19
58	340		Phœnicis,	ζ	5		2	4.35	2.555	146	3	1.5	18
59	348	84	Piscium,	χ	5		3	24.08	3.210	69	45	52.6	19
60	360	1	Ursæ Minoris,	α	2		5	0.83*	17.546	1	29	24.7*	19
61	380		Phœnicis,	ν	4½		8	25.15	2.725	136	19	59.4	19
62	392		Tucanæ,	κ	5		10	41.92	2.089	159	40	29.8	19
63	398		Tucanæ,		5		11	50.37	2.117	157	11	31.3	18
64	404	46	Andromedæ,	ξ	4½		13	31.85	3.493	45	15	34.4	19
65	412	36	Cassiopeæ,	ψ	4½		15	24.16*	4.119	22	39	18.1*	19
66	416	37	Cassiopeæ,	δ	3		16	2.88*	3.853	30	32	47.3*	18
67	420	45	Ceti,	θ	3		16	31.58*	3.000	98	57	32.2*	18
68	422		Tucanæ,		5		16	47.59	2.041	157	10	19.1	18
69	426		Phœnicis,		5		18	2.32	2.666	132	16	26.8	18
70	427	93	Piscium,	ρ	5		18	10.60	3.216	71	36	34.5	18
71	429	46	Ceti,		5		18	14.70	2.952	105	22	50.6	18
72	431	94	Piscium,		5		18	36.06	3.225	71	32	16.7	18
73	432	48	Andromedæ,	ω	5		18	42.29*	3.550	45	22	10.6*	18
74	438	38	Cassiopeæ,	A	5		20	9.01	4.324	20	30	36.0	18
75	441	49	Andromedæ,	A	5		21	7.73	3.556	43	46	5.5	18
76	447		Phœnicis,	γ	3		21	51.16	2.634	134	5	14.8	18
77	448	98	Piscium,	μ	4½		22	19.73*	3.137	84	37	53.2*	18
78	453	99	Piscium,	η	4		23	27.80*	3.200	75	25	45.0*	18
79	461		Phœnicis,	δ	4		25	0.11	2.509	139	51	15.6	18
80	480	50	Andromedæ,		5		28	0.81*	3.489	49	20	48.7*	18
81	487	51	Andromedæ,		3½		28	48.69*	3.639	42	8	2.5*	18
82	488	102	Piscium,	π	5		29	9.21*	3.171	78	37	38.1*	18
83	502	53	Andromedæ,	τ	5		31	44.87	3.516	50	11	8.1	18
84	507		Eridani,	α	1		32	7.23	2.238	148	0	0.5	18
85	518	106	Piscium,	ν	5		33	37.69*	3.117	85	16	25.3*	18
86	522	54	Andromedæ,		4		34	17.28*	3.716	40	4	9.1*	18
87	536	52	Ceti,	τ	3½		37	6.09	2.788	106	43	43.5	19
88	537	110	Piscium,	o	5		37	28.67*	3.162	81	35	56.9*	18
89	541		Sculptoris,	ε	5		38	37.35	2.819	115	48	14.9*	18
90	550		Eridani,	q^2	5		40	23.30	2.287	144	16	37.6	17
91	559	53	Ceti,	χ	5		42	13.00*	2.946	101	25	49.9*	18
92	564	45	Cassiopeæ,	ε	3		43	39.68*	4.221	27	4	17.9*	18
93	565	55	Ceti,	ζ	3		44	3.54	2.960	101	4	42.9	17
94	569	2	Trianguli,	α	3½		44	32.57*	3.399	61	9	14.9*	17
95	572	5	Arietis,	γ^1	4½		45	18.43*	3.277	71	26	38.5*	17
96	573	5	Arietis,	γ^2	4½		45	18.43*	3.279	71	26	29.0*	17
97	577	6	Arietis,	β	3		46	21.69*	3.297	69	55	38.7*	17
98	582		Phœnicis,		5		47	37.92	2.412	137	2	17.7	17
99	585		Phœnicis,	φ	5		48	8.43	2.491	133	14	1.8	17
100	589		Hydri,	η^1	5	1	48	45.68	+1.458	158	40	58.3	—17

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a'	b'	c'	d'
51	+8.9460	+8.3697	+0.5304	+8.7810	−9.1501	−9.8202	−1.2875	+9.4089
52	8.8101	8.2430	0.4914	+7.7376	9.6131	−8.9121	1.2868	9.4175
53	8.9702	8.4044	0.5365	+8.8303	9.0374	−9.8447	1.2868	9.4187
54	8.8163	8.2523	0.4775	−8.0961	9.6777	+9.2642	1.2866	9.4203
55	9.1449	8.5829	0.3783	−9.0931	9.6381	+9.9325	1.2865	9.4223
56	8.8939	8.3320	0.5207	+8.6506	9.3122	−9.7409	1.2865	9.4224
57	9.0423	8.4852	0.5525	+8.9522	8.5539	−9.8938	1.2861	9.4269
58	9.0608	8.5043	0.4047	−8.9796	9.6668	+9.9027	1.2861	9.4274
59	8.8348	8.2880	0.5059	+8.3737	−9.4951	−9.5221	1.2854	9.4364
60	0.3911	9.8559	1.2420	+0.3909	+9.4289	−9.9821	1.2845	9.4470
61	8.9651	8.4533	0.4247	−8.8245	−9.7096	+9.8397	1.2826	9.4685
62	9.2622	8.7655	0.2955	−9.2342	9.6389	+9.9511	1.2812	9.4823
63	9.2138	8.7245	0.3202	−9.1784	9.6555	+9.9429	1.2805	9.4891
64	8.9497	8.4713	0.5426	+8.7972	−8.9355	−9.8248	1.2795	9.4988
65	9.2143	8.7476	0.6134	+9.1794	+9.1173	−9.9412	1.2783	9.5094
66	9.0935	8.6307	0.5808	+9.0286	+8.7774	−9.9108	1.2779	9.5129
67	8.8046	8.3448	0.4773	−7.9969	−9.6810	+9.1677	1.2775	9.5155
68	9.2103	8.7521	0.3067	−9.1749	9.6735	+9.9397	1.2774	9.5170
69	8.9290	8.4784	0.4257	−8.7569	9.7370	+9.8021	1.2765	9.5237
70	8.8209	8.3711	0.5078	+8.3199	9.4862	−9.4732	1.2765	9.5244
71	8.8139	8.3646	0.4695	−8.2376	9.7052	+9.3978	1.2764	9.5248
72	8.8208	8.3736	0.5080	+8.3214	9.4844	−9.4746	1.2762	9.5267
73	8.9455	8.4989	0.5459	+8.7922	−8.8591	−9.8205	1.2761	9.5273
74	9.2523	8.8142	0.6330	+9.2238	+9.2276	−9.9444	1.2751	9.5348
75	8.9562	8.5239	0.5508	+8.8148	−8.6981	−9.8308	1.2744	9.5399
76	8.9393	8.5112	0.4179	−8.7818	9.7442	+9.8142	1.2739	9.5436
77	8.7972	8.3718	0.4934	+7.7682	9.6012	−8.9424	1.2736	9.5460
78	8.8087	8.3898	0.5044	+8.2093	9.5190	−9.3712	1.2728	9.5517
79	8.9840	8.5738	0.3972	−8.8673	9.7458	+9.8528	1.2716	9.5592
80	8.9110	8.5175	0.5442	+8.7249	8.9299	−9.7811	1.2694	9.5736
81	8.9638	8.5746	0.5598	+8.8340	8.0374	−9.8367	1.2688	9.5774
82	8.7988	8.4115	0.5014	+8.0937	9.5448	−9.2612	1.2685	9.5790
83	8.9027	8.5292	0.5447	+8.7091	8.9232	−9.7706	1.2665	9.5908
84	9.0636	8.6921	0.3490	−8.9921	9.7496	+9.8924	1.2662	9.5924
85	8.7881	8.4245	0.4934	+7.7040	−9.6018	−8.8786	1.2649	9.5991
86	8.9774	8.6172	0.5689	+8.8612	+8.4249	−9.8460	1.2644	9.6020
87	8.8025	8.4567	0.4632	−8.2617	−9.7275	+9.4190	1.2620	9.6140
88	8.7881	8.4442	0.4986	+7.9527	9.5670	−9.1241	1.2617	9.6156
89	8.8280	8.4898	0.4473	−8.4668	9.7576	+9.5973	1.2607	9.6203
90	9.0145	8.6851	0.3583	−8.9240	9.7766	+9.8664	1.2592	9.6275
91	8.7879	8.4675	0.4704	−8.0850	−9.7083	+9.2523	1.2575	9.6349
92	9.1198	8.8063	0.6247	+9.0694	+9.2824	−9.9036	1.2562	9.6405
93	8.7857	8.4742	0.4707	−8.0693	−9.7077	+9.2373	1.2558	9.6421
94	8.8346	8.5254	0.5308	+8.5181	9.2243	−9.6366	1.2554	9.6440
95	8.7995	8.4940	0.5145	+8.3023	9.4319	−9.4552	1.2547	9.6469
96	8.7995	8.4940	0.5145	+8.3023	9.4317	−9.4552	1.2547	9.6469
97	8.8026	8.5020	0.5171	+8.3381	9.4048	−9.4870	1.2537	9.6509
98	8.9407	8.6461	0.3840	−8.8051	9.7972	+9.8146	1.2525	9.6557
99	8.9112	8.6190	0.3978	−8.7469	9.7977	+9.7854	1.2520	9.6576
100	+9.2125	+8.9232	+0.1775	−9.1817	−9.7660	+9.9184	−1.2514	+9.6599

TABLE XXX.

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			An Vari
						h.	m.	s.	s.	°	′	″	
101	595	48	Cassiopeæ,		5	1	49	43.96*	+4.778	19	49	26.5*	—1
102	596		Eridani,	χ	4		50	6.71	2.325	142	21	25.7	1
103	600	50	Cassiopeæ,		4		50	44.28*	4.940	18	18	28.9*	1
104	603		Hydri,	η^2	4½		51	7.99	1.486	158	23	12.9	1
105	618	59	Ceti,	υ	4½		52	56.22	2.827	111	48	22.7	1
106	623		Hydri,	α	3		54	2.62	1.889	152	18	4.7	1
107	625	113	Piscium,	α	3½		54	17.50	3.102	87	57	44.7	1
108	628	57	Andromedæ,	γ	3		54	42.71*	3.644	48	23	33.9*	1
109	634		Phœnicis,	χ	5		55	41.21	2.402	135	26	11.5	1
110	635		Hydri,		5		55	43.87	1.570	156	47	55.7	1
111	648	13	Arietis,	α	2		58	43.64*	3.364	67	14	57.9*	17
112	656	4	Trianguli,	β	4	2	0	38.02*	3.545	55	43	30.0*	17
113	684	65	Ceti,	ξ^1	5		5	3.28*	3.169	81	51	34.0*	17
114	688		Fornacis,	μ	5		6	17.93	2.642	121	25	47.3*	17
115	717		Eridani,	φ	4		11	9.34	2.156	142	12	26.7	16
116	720	68	Ceti,	o	Var.		11	46.53	3.021	93	39	41.8	16
117	721	9	Persei,	i	5		11	56.19*	4.114	34	50	39.6*	16
118	744		Cassiopeæ,		4		16	46.71*	4.812	23	16	34.3*	16
119	754	72	Ceti,	ρ	5		18	42.42	2.897	102	58	9.6	16
120	756		Hydri,	δ	4		19	5.79	1.037	159	20	36.3	16
121	760	73	Ceti,	ξ^2	4		20	11.34*	3.182	82	12	54.0*	16
122	763		Eridani,	κ	4½		21	28.98	2.186	138	22	48.1	15
123	781	76	Ceti,	σ	5		24	58.93	2.844	105	54	19.4	16
124	794	78	Ceti,	ν	4½		28	0.42*	3.135	85	3	50.7*	15
125	811	82	Ceti,	δ	4		31	48.13	3.074	90	19	18.5	15
126	815	83	Ceti,	ε	4½		32	18.69	2.899	102	30	41.0	15
127	827	13	Persei,	θ	4		33	58.81*	4.051	41	24	35.0*	15
128	828		Eridani,		5		34	4.65	2.276	133	32	15.1	15
129	831	35	Arietis,		4		34	39.69	3.499	62	56	2.1	15
130	832		Eridani,	ι	4		34	45.10	2.378	130	29	56.2	15
131	837	86	Ceti,	γ	3		35	31.93*	3.102	87	23	57.8*	15
132	845		Arietis,		4		36	50.37*	3.232	80	31	20.5*	15
133	847	89	Ceti,	π	4		36	59.13	2.851	104	29	49.1	15
134	849		Hydri,	ε	5		37	17.66	0.861	158	54	42.9	15
135	856	1	Eridani,	τ^1	4½		38	6.34	2.800	109	12	37.0	15
136	861	39	Arietis,		4		38	59.30*	3.549	61	22	44.5*	15
137	863	15	Persei,	η	4		39	47.50*	4.312	34	43	52.4*	15
138	870	42	Arietis,	π	5		40	55.74*	3.335	73	9	45.5*	15
139	871	16	Persei,		4½		41	7.91	3.759	52	18	9.3	15
140	872	41	Arietis,		3		41	9.98*	3.510	63	21	40.7*	15
141	879		Fornacis,	β	5		42	48.68	2.509	123	2	17.9*	15
142	882		Hydri,	ζ	5		43	14.78	0.874	158	14	54.0	15
143	885	18	Persei,	τ	5		43	39.05*	4.199	37	51	20.3*	15
144	887	2	Eridani,	τ^3	4½		44	14.15	2.720	111	37	28.9	15
145	910	3	Eridani,	η	3		49	6.07	2.928	99	29	53.0	14
146	912	22	Persei,	π	5		49	11.22*	3.806	50	56	29.0*	14
147	921	48	Arietis,	ε	5		50	38.60*	3.418	69	15	46.4*	14
148	931		Horologii,		5		51	43.49	1.226	153	43	28.3	14
149	937		Eridani,	θ	3½		52	34.47	2.269	130	54	30.5	14
150	947	23	Persei,	γ	3½	2	53	57.70*	+4.296	37	5	8.1*	—14

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a'	b'	c'	d'
101	+9.2417	+8.9570	+0.6794	+9.2152	+9.4567	−9.9217	−1.2504	+9.6634
102	8.9859	8.7029	0.3559	−8.8845	−9.8000	+9.8465	1.2500	9.6648
103	9.2740	8.9939	0.6944	+9.2515	+9.4840	−9.9247	1.2494	9.6671
104	9.2045	8.9262	0.1754	−9.1728	−9.7732	+9.9151	1.2490	9.6685
105	8.8011	8.5311	0.4499	−8.3711	−9.7624	+9.5149	1.2472	9.6749
106	9.1005	8.8354	0.2683	−9.0476	−9.7954	+9.8910	1.2461	9.6788
107	8.7678	8.5039	0.4904	+7.3187	−9.6198	−8.4945	1.2458	9.6797
108	8.8934	8.6313	0.5612	+8.7155	−7.7709	−9.7654	1.2454	9.6811
109	8.9199	8.6623	0.3828	−8.7727	−9.8123	+9.7949	1.2444	9.6845
110	9.1706	8.9131	0.1936	−9.1339	−9.7894	+9.9055	1.2443	9.6847
111	8.7980	8.5539	0.5248	+8.3854	−9.3195	−9.5263	1.2411	9.6948
112	8.8436	8.6078	0.5476	+8.5942	−8.8791	−9.6875	1.2391	9.7011
113	8.7602	8.5435	0.5011	+7.9112	−9.5516	−9.0829	1.2341	9.7152
114	8.8232	8.6118	0.4221	−8.5405	−9.8084	+9.6476	1.2326	9.7190
115	8.9612	8.7702	0.3298	−8.8590	−9.8413	+9.8224	1.2269	9.7336
116	8.7487	8.5603	0.4806	−7.5539	−9.6701	+8.7291	1.2261	9.7355
117	8.9907	8.8029	0.6145	+8.9049	+9.2986	−9.8379	1.2259	9.7359
118	9.1447	8.9769	0.6828	+9.1078	+9.5376	−9.8807	1.2198	9.7497
119	8.7502	8.5902	0.4617	−8.1013	−9.7408	+9.2662	1.2173	9.7551
120	9.1910	9.0326	0.0204	−9.1621	−9.8368	+9.8857	1.2168	9.7561
121	8.7410	8.5870	0.5018	+7.8728	−9.5468	−9.0449	1.2153	9.7591
122	8.9130	8.7642	0.3423	−8.7867	−9.8579	+9.7850	1.2136	9.7626
123	8.7475	8.6126	0.4541	−8.1853	−9.7630	+9.3444	1.2088	9.7718
124	8.7279	8.6050	0.4969	+7.6625	−9.5805	−8.8370	1.2046	9.7795
125	8.7207	8.6128	0.4866	−6.4704	−9.6410	+7.6465	1.1990	9.7889
126	8.7304	8.6244	0.4605	−8.0662	−9.7461	+9.2318	1.1983	9.7901
127	8.8970	8.7975	0.6036	+8.7721	+9.2548	−9.7686	1.1958	9.7941
128	8.8571	8.7580	0.3578	−8.6952	−9.8713	+9.7315	1.1957	9.7943
129	8.7668	8.6700	0.5438	+8.4249	−9.0090	−9.5506	1.1948	9.7957
130	8.8353	8.7388	0.3723	−8.6478	−9.8668	+9.7049	1.1946	9.7959
131	8.7156	8.6221	0.4926	+7.3724	−9.6075	−8.5481	1.1934	9.7978
132	8.7191	8.6307	0.5067	+7.9357	−9.5104	−9.1058	1.1914	9.8008
133	8.7269	8.6391	0.4551	−8.1255	−9.7623	+9.2875	1.1912	9.8012
134	9.1564	9.0697	9.9413	−9.1262	−9.8735	+9.8584	1.1907	9.8019
135	8.7360	8.6525	0.4431	−8.2533	−9.7918	+9.4045	1.1895	9.8038
136	8.7664	8.6863	0.5487	+8.4467	−8.8727	−9.5662	1.1881	9.8058
137	8.9528	8.8758	0.6344	+8.8676	+9.4479	−9.7994	1.1868	9.8076
138	8.7257	8.6531	0.5227	+8.1876	−9.3585	−9.3447	1.1850	9.8102
139	8.8080	8.7362	0.5727	+8.5944	+8.7160	−9.6688	1.1847	9.8106
140	8.7550	8.6833	0.5446	+8.4067	−8.9912	−9.5340	1.1846	9.8107
141	8.7802	8.7149	0.3986	−8.5168	−9.8570	+9.6163	1.1820	9.8143
142	9.1340	9.0703	9.9447	−9.1020	−9.8853	+9.8469	1.1812	9.8153
143	8.9143	8.8522	0.6230	+8.8117	+9.3993	−9.7757	1.1806	9.8162
144	8.7330	8.6731	0.4350	−8.2995	−9.8099	+9.4439	1.1796	9.8175
145	8.6991	8.6577	0.4654	−7.9166	−9.7316	+9.0867	1.1714	9.8278
146	8.8028	8.7618	0.5799	+8.6022	+8.9395	−9.6685	1.1713	9.8280
147	8.7195	8.6840	0.5332	+8.2686	−9.2199	−9.4156	1.1687	9.8310
148	9.0424	9.0110	0.0885	−8.9951	−9.9046	+9.8173	1.1668	9.8332
149	8.8086	8.7805	0.3576	−8.6248	−9.8898	+9.6793	1.1653	9.8350
150	+8.9042	+8.8813	+0.6323	+8.8061	+9.4583	−9.7625	−1.1629	+9.8377

CATALOGUE OF 1500 STARS.

No.	B.A.C.		Constellation.		Mag.	Right Ascension, Jan. 1, 1850.	Annual Variation.	North Polar Dist., Jan. 1, 1850.	Ar Var
						h. m. s.	s.	° ′ ″	
151	948		Persei,		5	2 54 18.40*	+4.440	33 53 16.6*	—1
152	949	92	Ceti,	α	2½	54 26.60*	3.129	86 30 7.4*	1
153	952	9	Eridani,	ρ^2	5	55 20.58	2.937	98 16 42.0	1
154	953	25	Persei,	ρ	4	55 34.81	3.815	51 44 40.5	1
155	954	11	Eridani,	τ^3	4	55 46.82	2.643	114 12 56.1	1
156	956		Horologii,		5	55 55.87	1.109	154 40 7.2	1
157	959	10	Eridani,	ρ^3	5	56 54.63	2.943	98 11 26.6	1
158	962		Persei,	ι	4	58 15.67*	4.280	40 57 51.6*	1
159	963	26	Persei,	β	2½	58 25.56*	3.871	49 37 34.0*	1
160	967	27	Persei,	κ	5	59 23.96*	4.009	45 42 54.2*	1
161	981	28	Persei,	ω	5	3 1 37.41	+3.845	50 57 44.3	1
162	982		Hydri,	θ	5	1 58.36	—0.008	162 29 27.7	1
163	986	57	Arietis,	δ	4	3 3.58*	+3.418	70 50 39.9*	1
164	997	12	Eridani,	α	3½	5 42.08*	2.550	119 34 53.0*	1
165	999	58	Arietis,	ζ	5	6 17.21*	3.434	69 30 53.9*	1
166	1001		Cassiopeæ,		5	6 51.15*	5.152	24 54 8.5*	1
167	1013	13	Eridani,	ζ	4	8 33.08*	2.910	99 22 48.8*	1
168	1028	96	Ceti,	κ^1	5	11 29.91	3.139	87 11 3.0	1
169	1034	61	Arietis,	τ^1	5	12 34.43*	3.449	69 23 50.2*	1
170	1037	16	Eridani,	τ^4	3½	12 50.67*	+2.665	112 18 24.8*	1
171	1038		Mensæ,		5	12 52.60	—2.334	169 32 55.2	1
172	1043	33	Persei,	α	2½	13 38.35*	+4.241	40 40 39.3*	1
173	1044		Eridani,		4½	13 55.05	2.365	133 38 48.6	1
174	1056		Hydri,		5	16 18.90	0.673	157 28 29.7	1
175	1057	1	Tauri,	ο	4½	16 44.83	3.222	81 30 10.7*	1
176	1058		Camelopardi,		4	16 57.92*	4.789	30 35 16.6*	1
177	1062		Camelopardi,		4	17 58.42*	4.725	31 38 49.0*	1
178	1065		Camelopardi,		5	18 35.65	4.523	35 4 25.0	1
179	1068	2	Tauri,	ξ	4	19 2.68*	+3.243	80 47 37.8*	1
180	1070		Hydri,		5	19 44.02	—1.794	167 55 57.7	1
181	1071	35	Persei,	σ	5	20 1.31*	+4.194	42 31 38.7*	1
182	1090	17	Eridani,		4½	23 10.75	2.973	95 35 36.6	1
183	1099	37	Persei,	ψ	5	25 51.02*	4.228	42 18 40.8*	1
184	1100	18	Eridani,	ε	3½	25 52.25	2.826	99 58 8.3*	1
185	1104	19	Eridani,	τ^5	4	27 9.79	2.643	112 8 20.3	1
186	1112	10	Tauri,		4½	29 13.43	3.058	90 4 40.9	1
187	1125		Eridani,		5	31 42.77	2.147	130 46 10.0	1
188	1129	39	Persei,	δ	3	32 15.85*	4.235	42 41 50.0*	1
189	1133		Camelopardi,		5	32 58.06*	5.161	27 8 6.4*	1
190	1137		Camelopardi,	γ	4½	34 34.89	6.139	19 8 14.5	1
191	1138	38	Persei,	ο	4	34 55.48*	3.733	58 11 27.3*	1
192	1139	41	Persei,	ν	4	35 1.13*	4.046	47 54 1.5*	1
193	1144		Camelopardi,		5	35 50.07	5.392	24 56 42.6	1
194	1147	17	Tauri,		4½	35 58.74*	3.548	66 21 43.9*	1
195	1148	23	Eridani,	δ	3½	36 4.05	2.871	100 16 28.0*	1
196	1150		Eridani,		5	36 17.03	2.378	122 25 13.5	1
197	1151	19	Tauri,		5	36 17.22*	3.557	66 0 28.0*	1
198	1154	20	Tauri,		5	36 54.50*	3.555	66 6 19.0*	1
199	1159		Eridani,	υ^1	5	37 17.05	2.248	127 47 19.6	1
200	1161	23	Tauri,		5	3 37 25.92*	+3.549	66 31 23.4*	—1

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a'	b'	c'	d'
151	+8.9376	+8.9160	+0.6477	+8.8568	+9.5188	−9.7792	−1.1622	+9.8384
152	8.6845	8.6634	0.4951	+7.4699	−9.5923	−8.6452	1.1620	9.8387
153	8.6866	8.6689	0.4678	−7.8449	−9.7235	+9.0164	1.1604	9.8405
154	8.7866	8.7698	0.5800	+8.5784	+8.9469	−9.6495	1.1599	9.8410
155	8.7213	8.7053	0.4238	−8.3342	−9.8328	+9.4703	1.1596	9.8413
156	9.0497	9.0343	0.0450	−9.0058	−9.9103	+9.8132	1.1593	9.8416
157	8.6836	8.6719	0.4678	−7.8374	−9.7234	+9.0090	1.1575	9.8436
158	8.8600	8.8534	0.6182	+8.7381	+9.3899	−9.7308	1.1550	9.8462
159	8.7945	8.7885	0.5876	+8.6059	+9.0945	−9.6639	1.1547	9.8465
160	8.8197	8.8174	0.6010	+8.6637	+9.2594	−9.6946	1.1528	9.8483
161	8.7800	8.7862	0.5846	+8.5793	+9.0469	−9.6456	1.1486	9.8525
162	9.1913	9.1988	8.6822	−9.1707	−9.9089	+9.8251	1.1479	9.8532
163	8.6923	8.7039	0.5319	+8.2083	−9.2425	−9.3597	1.1458	9.8552
164	8.7230	8.7446	0.4015	−8.4164	−9.8664	+9.5318	1.1406	9.8600
165	8.6895	8.7134	0.5357	+8.2335	−9.1827	−9.3813	1.1395	9.8611
166	9.0357	9.0616	0.7132	+8.9933	+9.6790	−9.7938	1.1384	9.8621
167	8.6624	8.6949	0.4637	−7.8746	−9.7385	+9.0448	1.1349	9.8651
168	8.6510	8.6946	0.4941	+7.3423	−9.5988	−8.5179	1.1288	9.8702
169	8.6769	8.7247	0.5372	+8.2233	−9.1587	−9.3707	1.1265	9.8721
170	8.6814	8.7302	+0.4251	−8.2607	−9.8354	+9.4030	1.1259	9.8725
171	9.3889	9.4378	−0.3681	−9.3817	−9.9096	+9.8164	1.1259	9.8726
172	8.8318	8.8837	+0.6268	+8.7117	+9.4538	−9.7019	1.1243	9.8739
173	8.7859	8.8387	0.3255	−8.6248	−9.9195	+9.6604	1.1237	9.8743
174	9.0569	9.1189	9.8026	−9.0224	−9.9366	+9.7818	1.1185	9.8783
175	8.6440	8.7077	0.5081	+7.8136	−9.5016	−8.9849	1.1175	9.8790
176	8.9321	8.9967	0.6798	+8.8671	+9.6348	−9.7498	1.1171	9.8794
177	8.9166	8.9850	0.6739	+8.8467	+9.6228	−9.7427	1.1148	9.8810
178	8.8757	8.9465	0.6553	+8.7887	+9.5732	−9.7242	1.1134	9.8820
179	8.6398	8.7123	+0.5100	+7.8438	−9.4862	−9.0143	1.1124	9.8827
180	9.3123	9.3875	−0.2344	−9.3026	−9.9238	+9.7990	1.1109	9.8838
181	8.8020	8.8783	+0.6219	+8.6695	+9.4335	−9.6755	1.1102	9.8843
182	8.6268	8.7152	0.4726	−7.6157	−9.7054	+8.7897	1.1030	9.8892
183	8.7903	8.8891	0.6253	+8.6592	+9.4568	−9.6634	1.0967	9.8933
184	8.6250	8.7239	0.4604	−7.8634	−9.7504	+9.0328	1.0967	9.8934
185	8.6485	8.7525	0.4221	−8.2247	−9.8439	+9.3675	1.0936	9.8953
186	8.6103	8.7223	0 4870	−5.7446	−9.6386	+6.9206	1.0886	9.8983
187	8.7248	8.8466	0.3326	−8.5397	−9.9293	+9.5951	1.0824	9.9019
188	8.7714	8.8953	0.6262	+8.6377	+9.4672	−9.6451	1.0810	9.9027
189	8.9419	9.0686	0.7127	+8.8912	+9.7119	−9.7264	1.0792	9.9037
190	9.0812	9.2143	0.7906	+9.0565	+9.7914	−9.7482	1.0751	9.9060
191	8.6666	8.8011	0.5728	+8.3885	+8.7497	−9.4939	1.0742	9.9065
192	8.7253	8.8601	0.6069	+8.5516	+9.3399	−9.5981	1.0740	9.9066
193	8.9685	9.1066	0.7320	+8.9260	+9.7405	−9.7271	1.0719	9.9077
194	8.6313	8.7699	0.5495	+8.2343	−8.8645	−9.3724	1.0715	9.9079
195	8.6000	8.7390	0.4586	−7.8513	−9.7569	+9.0204	1.0713	9.9081
196	8.6660	8.8059	0.3771	−8.3953	−9.9033	+9.4978	1.0707	9.9084
197	8.6316	8.7715	0.5505	+8.2408	−8.8274	−9.3777	1.0707	9.9084
198	8.6297	8.7720	0.5504	+8.2372	−8.8338	−9.3744	1.0691	9.9092
199	8.6920	8.8359	0.3480	−8.4793	−9.9247	+9.5531	1.0681	9.9097
200	+8.6269	+8.7714	+0.5493	+8.2272	−8.8704	−9.3658	−1.0677	+9.9099

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
201	1166	25	Tauri,	η	3	3	38	34.56*	+3.552	66	21	46.7*	−11.56
202	1168	26	Eridani,	π	5		39	3.23	2.830	102	34	32.2	11.61
203	1174	30	Tauri,	e	5		40	2.99	3.279	79	19	20.7*	11.45
204	1176	27	Tauri,		5		40	15.09*	3.552	66	24	35.0*	11.44
205	1181	27	Eridani,	τ^6	4½		40	23.78	2.582	113	41	43.0	10.97
206	1191	28	Eridani,	τ^7	5		41	12.59*	2.577	114	20	34.4*	11.45
207	1197		Reticuli,		4		42	19.71	0.703	155	16	46.5	11.47
208	1199		Eridani,		5		43	3.90	2.220	128	4	51.8	11.40
209	1201		Eridani,	υ^2	4		43	50.30	2.233	126	39	26.5	11.15
210	1203		Camelopardi,		5		44	13.81*	5.197	27	22	26.5*	11.25
211	1207	44	Persei,	ζ	3½		44	42.86*	3.755	58	33	59.4*	11.14
212	1211		Cassiopeæ,		5½		45	13.24*	+9.565	9	43	36.5*	11.18
213	1215		Hydri,		5		46	10.10	−0.445	162	23	54.8	11.06
214	1216	32	Eridani,		5		46	45.82	+3.011	93	24	6.8	11.01
215	1217	33	Eridani,	τ^8	5		47	20.24	2.556	115	3	40.4	10.88
216	1219	45	Persei,	ε	3½		47	48.05*	3.997	50	25	43.6*	10.91
217	1220		Eridani,	υ^3	5		47	55.89	2.277	125	10	44.4	10.88
218	1228	46	Persei,	ξ	5		49	14.57	+3.869	54	38	43.9	10.79
219	1230		Hydri,	γ	3		49	39.76	−1.038	164	41	53.0	10.87
220	1234	34	Eridani,	γ	2½		51	1.93*	+2.796	103	56	19.7*	10.60
221	1241	35	Tauri,	λ	4		52	22.49*	3.315	77	56	15.8*	10.61
222	1243	36	Eridani,	τ^9	5		53	31.92	2.555	114	26	42.0	10.52
223	1245	35	Eridani,		5		53	56.19	3.034	91	58	27.3	10.43
224	1251	38	Tauri,	ν	5		55	10.90	3.185	84	25	51.0	10.38
225	1254	47	Persei,	λ	4½		55	25.82*	4.431	40	3	41.5*	10.31
226	1257	37	Tauri,	A^1	5		55	50.03*	3.534	68	19	57.2*	10.28
227	1259		Reticuli,	δ	5		56	23.41	0.954	151	49	34.7	9.99
228	1266	48	Persei,	c	5		57	47.24*	4.326	42	41	36.9*	10.14
229	1270		Reticuli,	γ	5		58	44.36	0.831	152	34	42.5	10.24
230	1271		Reticuli,	ι	5		58	54.05	0.986	151	30	3.7	9.95
231	1287	51	Persei,	μ	4½	4	3	53.99*	4.372	41	58	39.0*	9.67
232	1290	38	Eridani,	o^1	4½		4	32.77	2.922	97	13	58.0	9.74
233	1291	52	Persei,	f	5		4	41.49	4.059	49	54	8.8	9.60
234	1299		Horologii,	δ	5		5	47.66	2.027	132	23	14.5	9.65
235	1301		Persei,	b^1	5		6	58.98	4.481	40	4	47.0	9.44
236	1303	39	Eridani,	A	5		7	15.73	2.851	100	37	57.8	9.29
237	1304	49	Tauri,	μ	5		7	23.51	3.250	81	29	14.2	9.44
238	1309	40	Eridani,	o^2	4½		8	22.16	2.763	97	53	23.5	5.94
239	1315		Horologii,	α	5		9	2.23	1.992	132	39	57.2	9.19
240	1326	52	Tauri,	φ	5		11	8.15	3.673	63	0	44.3	9.13
241	1328	54	Tauri,	γ	3½		11	15.75*	3.407	74	44	21.1*	9.14
242	1331		Doradûs,	γ	4		12	6.08	1.556	141	52	0.4	9.34
243	1333	41	Eridani,	υ^4	3½		12	12.96*	2.264	124	10	6.4*	9.07
244	1336		Reticuli,	α	3½		12	30.33	0.745	152	51	4.2	9.04
245	1344		Reticuli,	ε	5		13	54.12	1.008	149	39	53.9	8.58
246	1346	61	Tauri,	δ^1	4		14	17.36*	3.450	72	48	50.1*	8.91
247	1348		Horologii,		5		14	32.37	1.907	134	37	48.9	8.82
248	1356	64	Tauri,	δ^2	4½		15	27.26*	3.451	72	54	29.2*	8.83
249	1358		Reticuli,	θ	5		16	0.90	0.683	153	37	10.4	9.03
250	1365	68	Tauri,	δ^3	5	4	16	49.01*	+3.464	72	25	9.1	− 8.74

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a'*	*b'*	*c'*	*d'*
201	+8.6244	+8.7734	+0.5500	+8.2275	−8.8488	−9.3655	−1.0646	+9.9115
202	8.5956	8.7466	0.4513	−7.9335	−9.7791	+9.0991	1.0634	9.9121
203	8.5900	8.7449	0.5155	+7.8578	−9.4387	−9.0263	1.0607	9.9134
204	8.6197	8.7755	0.5501	+8.2220	−8.8426	−9.3602	1.0601	9.9137
205	8.6197	8.7761	0.4132	−8.2238	−9.8615	+9.3616	1.0597	9.9139
206	8.6197	8.7793	0.4105	−8.2348	−9.8659	+9.3704	1.0575	9.9150
207	8.9548	9.1190	9.8292	−8.9130	−9.9689	+9.7105	1.0544	9.9164
208	8.6780	8.8452	0.3433	−8.4682	−9.9302	+9.5403	1.0524	9.9174
209	8.6676	8.8380	0.3514	−8.4436	−9.9257	+9.5240	1.0502	9.9184
210	8.9083	9.0802	0.7171	+8.8567	+9.7301	−9.6954	1.0492	9.9189
211	8.6384	8.8123	0.5740	+8.3557	+8.7993	−9.4628	1.0478	9.9195
212	9.3404	9.5163	+0.9805	+9.3341	+9.8732	−9.7379	1.0464	9.9201
213	9.0848	9.2647	−9.6488	−9.0639	−9.9668	+9.7206	1.0436	9.9213
214	8.5644	8.7467	+0.4777	−7.3377	−9.6838	+8.5130	1.0419	9.9221
215	8.6049	8.7896	0.4061	−8.2318	−9.8737	+9.3650	1.0403	9.9228
216	8.6736	8.8603	0.6017	+8.4778	+9.2999	−9.5409	1.0389	9.9234
217	8.6478	8.8350	0.3580	−8.4083	−9.9231	+9.4968	1.0385	9.9235
218	8.6449	8.8376	+0.5876	+8.4073	+9.1242	−9.4949	1.0347	9.9251
219	9.1338	9.3280	−0.0197	−9.1181	−9.9671	+9.7156	1.0335	9.9256
220	8.5640	8.7642	+0.4456	−7.9458	−9.7958	+9.1089	1.0294	9.9273
221	8.5567	8.7625	0.5203	+7.8768	−9.3927	−9.0432	1.0253	9.9289
222	8.5842	8.7949	0.4071	−8.2010	−9.8736	+9.3363	1.0218	9.9302
223	8.5425	8.7549	0.4816	−7.0796	−9.6657	+8.2555	1.0205	9.9307
224	8.5404	8.7581	0.5027	+7.5274	−9.5427	−8.7014	1.0166	9.9321
225	8.7289	8.9477	0.6465	+8.6128	+9.5748	−9.5975	1.0159	9.9324
226	8.5681	8.7886	0.5472	+8.1354	−8.9390	−9.2797	1.0146	9.9329
227	8.8604	9.0833	9.9679	−8.8057	−9.9829	+9.6558	1.0128	9.9335
228	8.6988	8.9277	0.6353	+8.5651	+9.5298	−9.5724	1.0084	9.9351
229	8.8637	9.0968	9.9274	−8.8120	−9.9851	+9.6513	1.0053	9.9361
230	8.8478	9.0816	9.9749	−8.7917	−9.9850	+9.6465	1.0048	9.9363
231	8.6845	8.9403	0.6405	+8.5557	+9.5561	−9.5571	0.9881	9.9417
232	8.5111	8.7698	0.4657	−7.6111	−9.7330	+8.7837	0.9859	9.9424
233	8.6235	8.8828	0.6082	+8.4324	+9.3661	−9.4922	0.9854	9.9425
234	8.6349	8.8992	0.3008	−8.4636	−9.9603	+9.5081	0.9816	9.9437
235	8.6903	8.9600	0.6503	+8.5741	+9.5962	−9.5590	0.9774	9.9449
236	8.5057	8.7766	0.4548	−7.7717	−9.7702	+8.9403	0.9765	9.9452
237	8.5025	8.7740	0.5115	+7.6728	−9.4752	−8.8441	0.9760	9.9453
238	8.4983	8.7743	0.4634	−7.6359	−9.7413	+8.8079	0.9725	9.9463
239	8.6253	8.9044	0.2967	−8.4564	−9.9633	+9.4989	0.9701	9.9470
240	8.5342	8.8230	0.5654	+8.1911	+8.2279	−9.3171	0.9625	9.9490
241	8.4993	8.7887	0.5309	+7.9196	−9.2662	−9.0801	0.9620	9.9492
242	8.6900	8.9833	0.1911	−8.5857	−9.9863	+9.5524	0.9589	9.9500
243	8.5624	8.8563	0.3544	−8.3119	−9.9344	+9.4057	0.9584	9.9501
244	8.8198	9.1150	9.8721	−8.7691	−9.9964	+9.6045	0.9574	9.9504
245	8.7705	9.0723	0.0113	−8.7065	−9.9963	+9.5860	0.9521	9.9517
246	8.4922	8.7958	0.5367	+7.9627	−9.1761	−9.1190	0.9507	9.9521
247	8.6191	8.9239	0.2760	−8.4658	−9.9723	+9.4941	0.9497	9.9523
248	8.4875	8.7967	0.5365	+7.9557	−9.1787	−9.1122	0.9462	9.9532
249	8.8180	9.1299	9.8114	−8.7702	−9.9991	+9.5940	0.9440	9.9537
250	+8.4833	+8.7991	+0.5381	+7.9634	−9.1514	−9.1187	−0.9409	+9.9544

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.		Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation
					h.	m.	s.	s.	°	′	″	″
251	1367	69 Tauri,	υ^1	5	4	17	20.30*	+3.579	67	31	53.1*	— 8.6
252	1370	73 Tauri,	π	5		18	8.11	3.380	75	37	48.5	8.5
253	1372	43 Eridani,	υ^5	4		18	24.35	2.254	124	22	10.0*	8.5
254	1376	74 Tauri,	ε	3½		19	51.70*	3.494	71	9	25.1*	8.4
255	1380	77 Tauri,	θ^1	4½		20	0.55*	3.413	74	22	31.5*	8.4
256	1381	78 Tauri,	θ^2	4½		20	6.18*	3.420	74	28	0.0*	8.4
257	1383	Reticuli,	η	5		20	16.91	0.619	153	44	34.4	8.7
258	1409	86 Tauri,	ρ	5		25	20.41*	3.402	75	28	33.0*	8.0
259	1413	Cæli,	δ	5		26	14.77	1.841	135	16	41.7	8.0
260	1419	47 Eridani,		5		26	58.23*	2.888	98	32	57.7*	7.9
261	1420	87 Tauri,	α	1		27	19.12*	3.436	73	47	49.0*	7.7
262	1421	88 Tauri,	d	5		27	24.85	3.285	80	9	4.7	7.8
263	1422	50 Eridani,	υ^6	4½		27	37.56	2.351	120	4	20.5	7.6
264	1429	48 Eridani,	ν	4		28	49.69	2.994	93	39	46.8	7.7
265	1433	52 Eridani,	υ^7	3½		29	43.40	2.333	120	52	23.7*	7.6
266	1434	90 Tauri,	c^1	5		29	46.75	3.350	77	47	39.8	7.6
267	1438	Doradûs,	α	3		30	45.81	1.289	145	21	25.4	7.6
268	1441	53 Eridani,		4		31	18.87	2.746	104	36	3.0	7.4
269	1442	93 Tauri,	c^2	5		31	42.69*	3.336	78	6	1.2	7.5
270	1449	94 Tauri,	τ	5		33	14.84*	3.592	67	20	8.4*	7.4
271	1451	54 Eridani,		4		33	53.01	2.623	109	57	48.3	7.2
272	1456	4 Camelopardi,		5		35	31.65*	4.960	33	30	58.0*	7.0
273	1458	Cæli,	α	4½		35	43.95	1.930	132	9	9.0	7.1
274	1464	Cæli,	β	5		36	45.55	2.122	127	26	25.3	7.3
275	1469	57 Eridani,	μ	5		38	0.43	3.001	93	32	0.3	7.0
276	1473	Pictoris,	λ	5		38	55.97	1.531	140	45	55.1	7.0
277	1474	9 Camelopardi,	α	4		39	9.42*	5.906	23	55	13.7*	6.9
278	1486	1 Orionis,	π^1	4		41	42.15	3.258	83	18	18.7	6.7
279	1491	2 Orionis,	π^2	5		42	26.61	3.272	81	21	41.7	6.6
280	1495	3 Orionis,	π^3	4		43	13.30	3.194	84	39	22.8	6.5
281	1500	4 Orionis,	o^1	5		44	3.06*	3.388	76	0	14.8*	6.4
282	1504	7 Camelopardi,		5		45	16.57	4.785	36	29	41.4	6.4
283	1507	61 Eridani,	ω	5		45	31.69	2.946	95	42	28.7	6.4
284	1514	8 Orionis,	π^5	4½		46	26.51	3.122	87	48	32.4	6.3
285	1520	3 Aurigæ,	ι	4		47	13.87*	3.895	57	4	37.7*	6.2
286	1525	9 Orionis,	o^2	5		47	56.48*	3.370	76	43	38.0*	6.1
287	1530	4 Aurigæ,		5		49	4.77	+4.058	52	20	30.8	6.0
288	1532	Mensæ,		5		49	25.09	—2.322	166	34	29.9	5.8
289	1536	10 Camelopardi,	β	4½		50	5.72	+5.303	29	47	5.2*	6.0
290	1540	7 Aurigæ,	ε	4		51	12.91*	4.291	46	24	16.7*	5.9
291	1541	8 Aurigæ,	ζ	4		52	0.11*	4.181	49	8	56.9*	5.8
292	1544	63 Eridani,		5		52	44.67	2.840	100	29	12.7	5.6
293	1546	11 Camelopardi,		5		53	7.18	5.184	31	14	39.5	5.7
294	1551	102 Tauri,	ι	4½		54	8.02*	3.581	68	37	45.8*	5.6
295	1552	65 Eridani,	ψ	5		54	10 09	2.906	97	23	52.8	5.6
296	1554	9 Aurigæ,		5		54	56.42	4.678	38	36	27.7	5.4
297	1557	11 Orionis,		5		56	0.07*	3.424	74	48	34.9*	5.4
298	1558	10 Aurigæ,	η	4		56	0.30*	4.195	48	58	28.2*	5.4
299	1559	Leporis,		5		56	3.90	2.438	116	29	26.7	5.4
300	1565	Camelopardi,		5	4	57	55.03	+9.671	10	57	24.2	— 5.4

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
251	+8.4948	+8.8131	+0.5526	+8.0771	−8.7521	−9.2189	−0.9388	+9.9549
252	8.4712	8.7933	0.5289	+7.8659	−9.2929	−9.0282	0.9357	9.9556
253	8.5396	8.8631	0.3511	−8.2913	−9.9386	+9.3841	0.9346	9.9559
254	8.4743	8.8050	0.5421	+7.9835	−9.0723	−9.1357	0.9287	9.9572
255	8.4662	8.7976	0.5327	+7.8965	−9.2401	−9.0562	0.9281	9.9573
256	8.4656	8.7974	0.5324	+7.8934	−9.2438	−9.0534	0.9278	9.9574
257	8.8029	9.1356	9.7873	−8.7556	−0.0223	+9.5775	0.9270	9.9575
258	8.4416	8.7999	0.5299	+7.8409	−9.2797	−9.0029	0.9058	9.9619
259	8.5762	8.9391	0.2629	−8.4277	−9.9811	+9.4512	0.9018	9.9626
260	8.4252	8.7920	0.4603	−7.5974	−9.7526	+8.7686	0.8987	9.9632
261	8.4364	8.8050	0.5350	+7.8821	−9.2047	−9.0406	0.8971	9.9635
262	8.4248	8.7939	0.5164	+7.6580	−9.4319	−8.8276	0.8967	9.9636
263	8.4803	8.8504	0.3726	−8.1802	−9.9224	+9.2935	0.8958	9.9637
264	8.4130	8.7895	0.4759	−7.2184	−9.6920	+8.3936	0.8904	9.9647
265	8.4744	8.8557	0.3678	−8.1846	−9.9275	+9.2943	0.8863	9.9654
266	8.4177	8.7993	0.5235	+7.7428	−9.3597	−8.9090	0.8861	9.9654
267	8.6485	9.0354	0.1074	−8.5638	−0.0034	+9.4946	0.8816	9.9662
268	8.4150	8.8048	0.4390	−7.8165	−9.8151	+8.9783	0.8790	9.9669
269	8.4083	8.8003	0.5227	+7.7226	−9.3683	−8.8892	0.8772	9.9669
270	8.4265	8.8269	0.5550	+8.0123	−8.6274	−9.1535	0.8699	9.9681
271	8.4155	8.8194	0.4181	−7.9488	−9.8606	+9.0979	0.8669	9.9686
272	8.6385	9.0516	0.6949	+8.5595	+9.7309	−9.4777	0.8589	9.9698
273	8.5095	8.9238	0.2881	−8.3363	−9.9768	+9.3824	0.8579	9.9700
274	8.4747	8.8948	0.3250	−8.2585	−9.9604	+9.3344	0.8528	9.9707
275	8.3690	8.7963	0.4762	−7.1588	−9.6909	+8.3341	0.8465	9.9716
276	8.5624	8.9951	0.1860	−8.4515	−0.0005	+9.4286	0.8418	9.9722
277	8.7543	9.1884	0.7708	+8.7153	+9.8337	−9.4993	0.8405	9.9724
278	8.3520	8.8010	0.5077	+7.4186	−9.5068	−8.5917	0.8273	9.9742
279	8.3500	8.8035	0.5135	+7.5266	−9.4583	−8.6978	0.8233	9.9746
280	8.3427	8.8010	0.5036	+7.3118	−9.5367	−8.4860	0.8191	9.9752
281	8.3494	8.8127	0.5297	+7.7329	−9.2842	−8.8959	0.8146	9.9757
282	8.5551	9.0261	0.6797	+8.4603	+9.7020	−9.4108	0.8078	9.9765
283	8.3302	8.8027	0.4689	−7.3279	−9.7211	+8.5018	0.8064	9.9767
284	8.3232	8.8015	0.4940	+6.9056	−9.5991	−8.0814	0.8012	9.9772
285	8.3944	8.8777	0.5903	+8.1296	+9.1818	−9.2296	0.7967	9.9777
286	8.3260	8.8138	0.5277	+7.6870	−9.3098	−8.8513	0.7925	9.9782
287	8.4090	8.9042	+0.6078	+8.1950	+9.3771	−9.2696	0.7859	9.9789
288	8.9397	9.4372	−0.3555	−8.9277	−0.0119	+9.4696	0.7839	9.9791
289	8.6053	9.1073	+0.7242	+8.5438	+9.7869	−9.4160	0.7798	9.9795
290	8.4348	8.9442	0.6321	+8.2734	+9.5371	−9.3093	0.7730	9.9801
291	8.4111	8.9257	0.6208	+8.2267	+9.4731	−9.2816	0.7681	9.9806
292	8.2925	8.8123	0.4524	−7.5526	−9.7788	+8.7214	0.7635	9.9810
293	8.5679	9.0903	0.7144	+8.4999	+9.7736	−9.3909	0.7612	9.9812
294	8.3073	8.8367	0.5530	+7.8689	−8.7372	−9.0141	0.7547	9.9818
295	8.2798	8.8094	0.4630	−7.3896	−9.7434	+8.5621	0.7545	9.9818
296	8.4760	9.0110	0.6701	+8.3689	+9.6807	−9.3402	0.7495	9.9823
297	8.2797	8.8222	0.5340	+7.6980	−9.2225	−8.8587	0.7426	9.9828
298	8.3866	8.9292	0.6221	+8.2038	+9.4823	−9.2575	0.7425	9.9828
299	8.3120	8.8550	0.3856	−7.9614	−9.9123	+9.0893	0.7421	9.9829
300	+8.9725	+9.5289	+0.9879	+8.9645	+9.9363	−9.4195	−0.7297	+9.9839

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
301	1573		Cæli,	γ^1	5	4	59	0.80	+2.153	125	41	29.3	− 5.20
302	1575	2	Leporis,	ε	4		59	6.74*	+2.538	112	34	34.1*	5.22
303	1587		Mensæ,		4½	5	0	0.15	−1.533	165	10	3.1	4.91
304	1588	67	Eridani,	β	3		0	28.73	+2.948	95	17	4.7	5.07
305	1591	15	Orionis,		5		1	7.04*	3.430	74	35	56.4*	5.12
306	1597	69	Eridani,	λ	4		1	58.26	2.872	98	57	1.7	5.00
307	1600		Doradûs,	ζ	5		2	56.53	1.001	147	40	45.2	5.10
308	1602	11	Aurigæ,	μ	5		3	10.09	4.094	51	41	55.8	4.86
309	1608	3	Leporis,	ι	4½		5	18.17	2.798	102	3	12.7	4.72
310	1611	17	Orionis,	ρ	5		5	27.16	3.135	87	19	18.7	4.72
311	1612		Doradûs,	μ	5		5	35.19	0.514	152	0	9.7	4.04
312	1613	13	Aurigæ,	α	1		5	36.97*	4.422	44	9	39.4*	4.30
313	1614	14^2	Aurigæ,		5		5	38.73*	3.899	57	29	30.0*	4.73
314	1616	5	Leporis,	μ	5		6	11.59*	2.688	106	23	12.3*	4.66
315	1617	4	Leporis,	κ	5		6	18.39	2.770	103	7	21.3	4.64
316	1623	19	Orionis,	β	1		7	19.86*	2.884	98	22	45.2*	4.56
317	1631	15	Aurigæ,	λ	5		8	35.61*	4.210	50	2	26.4*	3.80
318	1638	20	Orionis,	τ	4		10	19.58	2.916	97	0	39.1	4.28
319	1650		Columbæ,	ο	5		12	4.55	2.162	125	2	46.1	3.66
320	1653	6	Leporis,	λ	4½		12	39.98*	+2.764	103	20	8.2	4.09
321	1659		Doradûs,	θ	5		13	53.59	−0.075	157	21	15.8	4.07
322	1665	23	Orionis,	m	5		14	57.12*	+3.152	86	36	16.3*	3.91
323	1672		Pictoris,	ζ	5		15	41.42	1.457	140	46	8.3	4.11
324	1681	112	Tauri,	β	2		16	48.80*	3.791	61	31	29.5*	3.57
325	1684	28	Orionis,	η	4½		16	56.22	3.014	92	32	22.6	3.73
326	1687	24	Orionis,	γ	2		17	5.31*	3.220	83	47	27.7*	3.72
327	1690	24	Aurigæ,	φ	5		17	42.51	3.974	55	39	27.9	3.63
328	1695	114	Tauri,	ο	5		18	37.61*	3.602	68	11	47.5*	3.63
329	1700	30	Orionis,	ψ^2	5		18	58.87	3.143	87	2	18.8	3.56
330	1704		Pictoris,	κ	5		19	36.52	1.112	146	16	50.9	3.12
331	1706		Camelopardi,		5		19	45.79	8.002	15	4	2.4*	3.50
332	1715	9	Leporis,	β	4		21	49.19*	2.572	110	52	56.3	3.25
333	1717	31	Orionis,		5		22	7.09	3.046	91	12	53.8	3.26
334	1722	32	Orionis,	A	5		22	45.58*	3.208	84	10	16.9*	3.22
335	1723	25	Aurigæ,	χ	5		22	57.71*	3.905	57	55	27.2*	3.27
336	1730	34	Orionis,	δ	2		24	20.72*	3.066	90	24	53.1*	3.07
337	1731	36	Orionis,	υ	5		24	40.70	2.905	97	24	58.8	3.07
338	1739		Columbæ,	ε	4		25	53.31	2.131	125	35	2.6*	2.85
339	1741	11	Leporis,	α	3½		26	6.96*	2.648	107	56	0.7*	2.96
340	1748	37	Orionis,	ϕ^1	4½		26	35.26	3.291	80	37	0.9	2.89
341	1749	39	Orionis,	λ	4		26	52.73	3.302	80	10	18.4	2.85
342	1756		Columbæ,		5		27	47.53	2.013	128	37	21.5	2.81
343	1759	42	Orionis,	c	5		27	59.34	2.958	94	56	30.2	2.78
344	1762	44	Orionis,	ι	3½		28	5.91	2.936	96	0	45.2	2.77
345	1765	46	Orionis,	ε	2½		28	36.22*	3.044	91	18	8.2*	2.73
346	1766	40	Orionis,	ϕ^2	4½		28	40.14*	3.293	80	47	44.6*	2.42
347	1767	123	Tauri,	ζ	3½		28	40.97*	3.585	68	57	14.9*	2.71
348	1768	26	Aurigæ,		5		29	0.35*	3.846	59	36	8.1*	2.69
349	1780	48	Orionis,	σ	4		31	13.08	3.010	92	41	28.2	2.50
350	1785	49	Orionis,	d	5	5	31	37.71*	+2.902	97	18	4.1*	− 2.42

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
301	+8.3342	+8.8987	+0.3312	−8.1002	−9.9616	+9.1859	−0.7221	+9.9844
302	8.2778	8.8430	+0.4038	−7.8620	−9.8871	+9.0034	0.7214	9.9845
303	8.8287	9.4006	−0.2567	−8.8139	−0.0190	+9.3983	0.7152	9.9849
304	8.2354	8.8110	+0.4700	−7.1996	−9.7169	+8.3739	0.7118	9.9852
305	8.2448	8.8253	0.5349	+7.6690	−9.2082	−8.8292	0.7072	9.9855
306	8.2280	8.8152	0.4574	−7.4200	−9.7629	+8.5908	0.7010	9.9859
307	8.4875	9.0822	0.0097	−8.4144	−0.0216	+9.3185	0.6939	9.9864
308	8.3191	8.9157	0.6122	+8.1114	+9.4153	−9.1822	0.6922	9.9865
309	8.2072	8.8211	0.4461	−7.5270	−9.7975	+8.6934	0.6759	9.9875
310	8.1969	8.8120	0.4957	+6.8664	−9.5889	−8.0420	0.6747	9.9876
311	8.5238	9.1400	9.7968	−8.4697	−0.0261	+9.3174	0.6737	9.9876
312	8.3521	8.9685	0.6443	+8.2079	+9.5966	−9.2270	0.6734	9.9877
313	8.2690	8.8856	0.5909	+7.9993	+9.1942	−9.1014	0.6733	9.9877
314	8.2086	8.8298	0.4295	−7.6590	−9.8393	+8.8171	0.6689	9.9879
315	8.2012	8.8234	0.4421	−7.5573	−9.8085	+8.7219	0.6680	9.9880
316	8.1861	8.8170	0.4592	−7.3496	−9.7568	+8.5211	0.6597	9.9884
317	8.2866	8.9284	0.6194	+8.0943	+9.4678	−9.1549	0.6494	9.9890
318	8.1597	8.8169	0.4639	−7.2463	−9.7403	+8.4191	0.6348	9.9897
319	8.2280	8.9012	0.3331	−7.9871	−9.9624	+9.0763	0.6194	9.9904
320	8.1477	8.8265	+0.4410	−7.5106	−9.8116	+8.6749	0.6141	9.9907
321	8.5391	9.2296	−8.8420	−8.5042	−0.0300	+9.2658	0.6029	9.9912
322	8.1154	8.8162	+0.4981	+6.8879	−9.5742	−8.0632	0.5929	9.9916
323	8.3065	9.0147	0.1654	−8.1956	−0.0148	+9.1727	0.5858	9.9918
324	8.1525	8.8722	0.5778	+7.8308	+8.9385	−8.9509	0.5748	9.9922
325	8.0957	8.8166	0.4789	−6.7422	−9.6785	+7.9179	0.5736	9.9923
326	8.0963	8.8188	0.5070	+7.1304	−9.5120	−8.3039	0.5721	9.9923
327	8.1707	8.8997	0.5986	+7.9221	+9.2929	−9.0150	0.5658	9.9926
328	8.1103	8.8490	0.5559	+7.6802	−8.5740	−8.8240	0.5564	9.9929
329	8.0750	8.8175	0.4968	+6.7882	−9.5827	−7.9637	0.5527	9.9930
330	8.3234	9.0727	0.0408	−8.2434	−0.0252	+9.1639	0.5461	9.9932
331	8.6512	9.4023	0.9010	+8.6360	+9.9232	−9.2270	0.5444	9.9935
332	8.0731	8.8474	0.4095	−7.6251	−9.8791	+8.7716	0.5219	9.9940
333	8.0403	8.8181	0.4832	−6.3667	−9.6578	+7.5427	0.5185	9.9940
334	8.0351	8.8204	0.5059	+7.0418	−9.5205	−8.2157	0.5111	9.9942
335	8.1024	8.8902	0.5908	+7.8275	+9.1942	−8.9316	0.5087	9.9943
336	8.0141	8.8186	0.4859	−5.8740	−9.6446	+7.0501	0.4924	9.9947
337	8.0137	8.8224	0.4622	−7.1246	−9.7465	+8.2970	0.4884	9.9948
338	8.0848	8.9089	0.3273	−7.8496	−9.9677	+8.9360	0.4733	9.9952
339	8.0138	8.8408	0.4220	−7.5022	−9.8558	+8.6566	0.4704	9.9952
340	7.9919	8.8251	0.5171	+7.2042	−9.4264	−8.3745	0.4644	9.9954
341	7.9887	8.8258	0.5185	+7.2210	−9.4128	−8.3906	0.4606	9.9955
342	8.0775	8.9268	0.3038	−7.8728	−9.9809	+8.9417	0.4486	9.9957
343	7.9692	8.8213	0.4707	−6.9045	−9.7140	+8.0789	0.4459	9.9958
344	7.9685	8.8221	0.4670	−6.9887	−9.7286	+8.1624	0.4444	9.9958
345	7.9594	8.8199	0.4830	−6.3159	−9.6594	+7.4919	0.4376	9.9959
346	7.9640	8.8255	0.5166	+7.1680	−9.4312	−8.3385	0.4367	9.9959
347	7.9881	8.8498	0.5539	+7.5434	−8.6920	−8.6895	0.4365	9.9959
348	8.0180	8.8841	0.5852	+7.7221	+9.1042	−8.8340	0.4320	9.9960
349	7.9222	8.8210	0.4783	−6.5938	−9.6813	+7.7695	0.4000	9.9966
350	+7.9190	+8.8241	+0.4625	−7.0231	−9.7455	+8.1957	−0.3938	+9.9967

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
351	1791		Doradûs,	β	4	5	32	19.75	+0.506	152	35	20.0	− 2.45
352	1794	50	Orionis,	ζ	2		33	11.51*	3.030	92	1	37.2*	2.33
353	1802		Columbæ,	α	2		34	13.15*	2.177	124	9	28.4*	2.25
354	1823	13	Leporis,	γ	4		38	12.61*	2.500	112	30	2.8*	1.55
355	1830	29	Aurigæ,	τ	5		38	46.97	4.153	50	52	35.8	1.79
356	1837	132	Tauri,		5		39	48.77*	3.683	65	29	17.9*	1.75
357	1840	14	Leporis,	ζ	4½		40	9.70	2.720	104	52	55.7	1.72
358	1843	53	Orionis,	κ	3		40	38.64	2.846	99	43	38.6	1.66
359	1845	32	Aurigæ,	ν	5		41	5.72	4.156	50	54	6.0	1.68
360	1849	31	Camelopardi,		5		41	32.08*	5.365	30	9	13.1*	1.57
361	1854	30	Aurigæ,	ξ	5		42	16.53	5.019	34	20	9.1	1.52
362	1855		Pictoris,		5		42	19.03	1.667	136	39	17.7	1.47
363	1861		Pictoris,	β	4½		43	43.82	1.416	141	7	22.3	1.54
364	1863	136	Tauri,		4½		43	54.06*	3.771	62	25	44.6*	1.35
365	1868		Doradûs,	δ	4½		44	30.50	0.077	155	47	35.3	1.07
366	1871	15	Leporis,	δ	5		44	52.20	2.578	110	53	43.3	0.66
367	1876	54	Orionis,	χ^1	5		45	30.03*	3.552	69	45	24.7*	1.17
368	1878		Columbæ,	β	3		45	40.61*	2.109	125	49	43.0*	1.53
369	1883	58	Orionis,	α	1		47	3.14*	3.249	82	37	33.0*	1.13
370	1884		Pictoris,	γ	4½		47	6.37	1.073	146	12	21.3	1.06
371	1885	33	Aurigæ,	δ	3½		47	10.44*	4.937	35	44	3.4*	1.01
372	1890		Pictoris,		5		47	29.53	1.338	142	8	41.1	1.08
373	1891		Columbæ,	λ	5		47	39.73	2.167	123	50	11.8	1.17
374	1895	34	Aurigæ,	β	2		48	31.46*	4.404	45	4	27.7*	0.97
375	1897	35	Aurigæ,	π	5		48	48.11*	4.452	44	4	59.6	0.96
376	1900	37	Aurigæ,	θ	4		49	29.59*	4.092	52	48	12.4*	0.81
377	1901	16	Leporis,	η	4		49	34.47*	+2.735	104	11	56.3*	1.06
378	1905		Doradûs,	ε	5		50	2.41	−0.120	156	56	16.1	1.08
379	1922		Columbæ,	γ	4		52	13.03*	+2.127	125	18	10.8*	0.61
380	1928	61	Orionis,	μ	5		54	7.97	3.302	80	21	28.0	0.50
381	1933		Puppis,		5		54	33.31	1.833	132	49	32.2	0.50
382	1934	64	Orionis,	χ^3	5		54	34.74	3.559	70	18	46.1	0.43
383	1938	1	Geminorum,		5		55	0.14*	3.648	66	44	2.6*	0.33
384	1939	62	Orionis,	χ^4	5		55	0.76*	3.565	69	51	48.1*	0.40
385	1943	37	Camelopardi,		5		56	44.97*	5.298	31	3	10.4*	0.29
386	1958	67	Orionis,	ν	4½		59	0.49*	3.428	75	13	7.4*	0.06
387	1959	18	Leporis,	θ	4½		59	22.12	2.718	104	55	34.4	− 0.05
388	1979	40	Camelopardi,		5	6	2	11.99*	5.393	29	58	3.4*	+ 0.22
389	1980		Camelopardi,		5		2	18.53*	6.623	20	38	13.2*	0.29
390	1982		Columbæ,	θ	5		2	23.37	2.069	127	14	5.9	0.21
391	1990	70	Orionis,	ξ	5		3	24.69	3.415	75	45	44.8	0.34
392	1992	1	Lyncis,		5		4	4.90*	5.538	28	26	40.4*	0.37
393	1994		Monocerotis,		5		4	32.98	2.918	96	31	12.5	0.40
394	2001	44	Aurigæ,	κ	4		5	49.14*	3.828	60	27	9.0*	0.80
395	2002	7	Geminorum,	η	4		5	49.35*	3.624	67	27	18.2*	0.52
396	2007	2	Lyncis,		4½		6	23.21*	5.310	30	56	33.3*	0.56
397	2015	5	Monocerotis,		4½		7	32.45	2.928	96	14	0.9	0.79
398	2034		Columbæ,	κ	4½		11	12.95	2.133	125	5	37.2*	1.01
399	2044	46	Aurigæ,		5		13	20.48*	4.629	40	38	33.2*	1.23
400	2047	13	Geminorum,	μ	3	6	13	53.13*	+3.636	67	24	53.3*	+ 1.34

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
351	+8.2416	+9.1576	+9.7086	−8.1899	−0.0339	+9.0291	−0.3830	+9.9968
352	7.8913	8.8212	0.4805	−6.4398	−9.6710	+7.6156	0.3693	9.9970
353	7.9563	8.9034	0.3363	−7.7056	−9.9624	+8.7995	0.3524	9.9973
354	7.8357	8.8563	0.4013	−7.4185	−9.8932	+8.5602	0.2796	9.9980
355	7.9000	8.9323	0.6184	+7.7001	+9.4648	−8.7659	0.2681	9.9981
356	7.8093	8.8632	0.5656	+7.4272	+8.2672	−8.5623	0.2465	9.9983
357	7.7755	8.8371	0.4340	−7.1852	−9.8295	+8.3464	0.2390	9.9984
358	7.7563	8.8287	0.4537	−6.9841	−9.7753	+8.1539	0.2283	9.9985
359	7.8499	8.9325	0.6184	+7.6497	+9.4649	−8.7157	0.2181	9.9985
360	8.0287	9.1215	0.7295	+7.9656	+9.8088	−8.8426	0.2080	9.9986
361	7.9606	9.0713	0.7009	+7.8774	+9.7623	−8.8048	0.1902	9.9987
362	7.8743	8.9860	0.2197	−7.7359	−0.0099	+8.7486	0.1891	9.9987
363	7.8770	9.0251	0.1512	−7.7683	−0.0207	+8.7421	0.1531	9.9989
364	7.7225	8.8752	0.5760	+7.3880	+8.8865	−8.5117	0.1485	9.9989
365	8.0407	9.2101	9.0145	−8.0007	−0.0366	+8.7896	0.1318	9.9990
366	7.6728	8.8525	0.4085	−7.2251	−9.8816	+8.3716	0.1216	9.9991
367	7.6524	8.8507	0.5518	+7.1915	−8.7917	−8.3399	0.1031	9.9991
368	7.7107	8.9142	0.3237	−7.4781	−9.9713	+8.5631	0.0979	9.9992
369	7.5793	8.8268	0.5110	+6.6877	−9.4803	−7.8601	0.0540	9.9993
370	7.8286	9.0780	0.0319	−7.7482	−0.0298	+8.6696	0.0522	9.9993
371	7.8050	9.0568	0.6925	+7.7145	+9.7459	−8.6570	0.0498	9.9993
372	7.7728	9.0353	0.1313	−7.6702	−0.0231	+8.6342	0.0390	9.9994
373	7.6353	8.9039	0.3376	−7.3811	−9.9621	+8.4766	0.0331	9.9994
374	7.6732	8.9733	0.6437	+7.5221	+9.5999	−8.5483	0.0016	9.9995
375	7.6703	8.9810	0.6483	+7.5266	+9.6187	−8.5451	9.9910	9.9995
376	7.5838	8.9222	0.6111	+7.3653	+9.4120	−8.4426	9.9634	9.9995
377	7.4951	8.8369	+0.4366	−6.8848	−9.8233	+8.0474	9.9599	9.9996
378	7.8689	9.2305	−8.8248	−7.8327	−0.0371	+8.6018	9.9402	9.9996
379	7.4429	8.9119	+0.3272	−7.2047	−9.9693	+8.2926	9.8329	9.9998
380	7.2383	8.8300	0.5182	+6.4623	−9.4156	−7.6322	9.7104	9.9999
381	7.3343	8.9584	0.2629	−7.1666	−9.9991	+8.2081	9.6780	9.9999
382	7.2243	8.8500	0.5501	+6.7518	−8.8561	−7.9017	9.6764	9.9999
383	7.1991	8.8607	0.5617	+6.7957	−7.5798	−7.9350	9.6406	9.9999
384	7.1889	8.8512	0.5515	+6.7258	−8.8007	−7.8745	9.6398	9.9999
385	7.2638	9.1114	0.7235	+7.1967	+9.8012	−8.0853	9.4547	0.0000
386	6.4745	8.8385	0.5345	+5.8812	−9.2162	−7.0427	8.9382	0.0000
387	+6.2787	8.8388	0.4336	−5.6896	−9.8305	+6.8508	−8.7421	0.0000
388	−7.1065	9.1254	0.7315	−7.0442	+9.8126	+7.9188	+9.2834	0.0000
389	7.2796	9.2768	0.8209	−7.2508	+9.8937	+7.9740	9.3050	0.0000
390	6.9411	8.9229	0.3127	+6.7229	−9.9780	−7.8000	9.3204	0.0000
391	7.0102	8.8374	0.5327	−6.4010	−9.2423	+7.5635	9.4749	0.0000
392	7.3960	9.1460	0.7434	−7.3401	+9.8276	+8.1941	9.5522	9.9999
393	7.1245	8.8266	0.4651	+6.1797	−9.7362	−7.3530	9.6000	9.9999
394	7.2892	8.8843	0.5830	−6.9822	+9.0626	+8.0978	9.7070	9.9999
395	7.2632	8.8583	0.5594	−6.8469	−8.2430	+7.9885	9.7070	9.9999
396	7.5579	9.1126	0.7243	−7.4912	+9.8022	+8.3784	9.7473	9.9998
397	7.3437	8.8263	0.4661	+6.3794	−9.7323	−7.5529	9.8194	9.9998
398	7.6005	8.9105	0.3289	+7.3601	−9.9682	−8.4491	9.9917	9.9995
399	7.7748	9.0094	0.6652	−7.6550	+9.6765	+8.6449	0.0670	9.9993
400	−7.6407	+8.8578	+0.5594	−7.2251	−8.2406	+8.3665	+0.0843	+9.9992

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.	Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
				h.	m.	s.	s.	°	′	″	″
451	2345	25 Canis Majoris, δ	3½	7	2	17.61*	+2.441	116	9	31.1*	+ 5.36
452	2349	18 Lyncis,	5		2	47.52*	5.277	30	6	10.4*	5.76
453	2355	Puppis, A	5		3	48.85	2.007	129	25	3.7	5.56
454	2358	22 Monocerotis,	4½		4	12.27	3.069	90	14	56.7	5.56
455	2362	51 Geminorum,	5		4	45.34*	3.454	73	35	28.9*	5.58
456	2379	Lyncis,	5		7	6.75	4.581	40	16	30.5	5.79
457	2380	Puppis, E	5		7	17.85	1.986	130	14	50.4	5.81
458	2381	64 Aurigæ,	5		7	35.89*	4.191	48	51	21.5*	5.82
459	2388	27 Canis Majoris,	4½		8	8.47	2.446	116	5	49.5	5.85
460	2389	Puppis, I	5		8	16.64	1.676	136	30	46.0	6.20
461	2392	Puppis, L^1	5		8	44.30	1.808	134	55	32.0	6.17
462	2398	54 Geminorum, λ	4½		9	28.22*	+3.458	73	11	37.7*	5.99
463	2400	Volantis, γ	5		10	0.28	−0.472	160	15	17.0	5.86
464	2407	19 Lyncis,	5		10	36.41	+4.926	34	26	33.8	6.13
465	2410	55 Geminorum, δ	3		11	9.67*	3.597	67	44	47.5*	6.14
466	2414	Argûs, π	3		11	51.05	2.141	126	49	51.4	6.17
467	2416	65 Aurigæ,	5		12	0.87	4.028	52	57	44.4	6.20
468	2418	30 Canis Majoris,	5		12	29.41	2.491	114	41	4.5	6.24
469	2427	Puppis, F	5		13	26.54	2.045	128	56	19.9	6.31
470	2429	66 Aurigæ,	5		13	44.69	4.175	49	2	39.6	6.35
471	2439	Camelopardi,	5		15	12.91	6.318	21	14	12.1*	6.53
472	2442	60 Geminorum, ι	4		16	24.36*	+3.742	61	54	32.3*	6.65
473	2447	Volantis, δ	5		16	53.34	−0.012	157	40	57.3	6.60
474	2458	31 Canis Majoris, η	2		18	9.72*	+2.372	119	0	51.2*	6.72
475	2462	3 Canis Minoris, β	3		19	0.92	3.261	81	24	45.3	6.83
476	2464	62 Geminorum, ρ	5		19	27.39*	3.871	57	55	21.1*	6.61
477	2478	Puppis,	5		23	17.73	2.317	121	9	1.0	7.18
478	2482	Argûs, σ	4		24	28.71	1.921	133	0	2.2	7.11
479	2484	Puppis,	5		24	52.56	2.321	120	39	4.6	7.38
480	2485	66 Geminorum, α^2	1½		25	1.33*	3.848	57	47	17.0*	7.34
481	2486	68 Geminorum,	5		25	2.64*	3.433	73	51	19.0*	7.2
482	2493	69 Geminorum, υ	5		26	40.43*	3.714	62	46	33.1*	7.4
483	2497	Puppis, n^1	4½		27	58.35	2.540	113	9	3.9	7.61
484	2500	Puppis, g	5		28	18.20	2.475	115	47	33.7	7.72
485	2522	10 Canis Minoris, α	1		31	26.78*	3.145	84	23	39.7*	8.77
486	2530	Puppis, k^1	4½		32	40.95	2.460	116	27	48.3	7.85
487	2531	Puppis, k^2	5		32	41.31	2.459	116	27	52.8	7.77
488	2540	75 Geminorum, σ	5		33	55.80	3.764	60	45	30.8	8.23
489	2542	26 Monocerotis, γ	4½		34	4.89	2.868	99	12	17.5	8.02
490	2551	77 Geminorum, κ	4		35	23.15*	3.634	65	14	49.5*	8.16
491	2555	78 Geminorum, β	2		36	7.80*	3.682	61	36	59.0*	8.23
492	2562	3 Puppis,	5		37	47.27*	2.409	118	35	55.2*	8.27
493	2570	Puppis, W	4½		38	35.45	2.039	130	34	13.5	8.68
494	2580	Puppis, c	5		39	54.63	2.132	127	36	23.7	8.40
495	2590	Camelopardi,	5½		40	56.71	9.819	10	7	24.3	8.65
496	2594	Puppis, o	5		41	51.22	2.502	115	34	5.	8.6
497	2602	Argûs, ξ	3½		42	59.28*	+2.527				
498	2607	Volantis, ζ	5		43	38.25	−0.6				
499	2617	83 Geminorum, φ	5		44	18.61*	+3.6				
500	2620	Puppis, P	4½	7	44	40.19	+1.8				

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
451	−8.2998	+8.8546	+0.3870	+7.9440	−9.9107	−9.0732	+0.7311	+9.9838
452	8.5558	9.1071	0.7235	−8.4929	+9.7890	+9.3693	0.7345	9.9835
453	8.3751	8.9189	0.3041	+8.1779	−9.9758	−9.2419	0.7413	9.9829
454	8.2656	8.8066	0.4864	+5.9040	−9.6417	−7.0801	0.7439	9.9827
455	8.2873	8.8244	0.5377	−7.7383	−9.1614	+8.8963	0.7475	9.9824
456	8.4738	8.9945	0.6610	−8.3563	+9.6532	+9.3429	0.7626	9.9811
457	8.4028	8.9222	0.2983	+8.2131	−9.9775	−9.2719	0.7638	9.9810
458	8.4105	8.9279	0.6220	−8.2287	+9.4812	+9.2816	0.7657	9.9808
459	8.3374	8.8511	0.3882	+7.9808	−9.9087	−9.1102	0.7690	9.9805
460	8.4539	8.9666	0.2363	+8.3145	−9.9966	−9.3283	0.7699	9.9804
461	8.4444	8.9540	0.2545	+8.2933	−9.9921	−9.3194	0.7727	9.9802
462	8.3178	8.8226	+0.5386	−7.7790	−9.1443	+8.9361	0.7772	9.9797
463	8.7734	9.2746	−9.6829	+8.7471	−0.0188	−9.4519	0.7804	9.9794
464	8.5532	9.0505	+0.6927	−8.4695	+9.7328	+9.3981	0.7840	9.9791
465	8.3426	8.8363	0.5553	−7.9209	−8.6128	+9.0634	0.7873	9.9787
466	8.4097	8.8989	0.3260	+8.1875	−9.9626	−9.2669	0.7913	9.9783
467	8.4119	8.9000	0.6053	−8.1917	+9.3551	+9.2699	0.7923	9.9782
468	8.3584	8.8434	0.3956	+7.9792	−9.8983	−9.1136	0.7951	9.9779
469	8.4313	8.9103	0.3108	+8.2296	−9.9704	−9.2966	0.8005	9.9773
470	8.4459	8.9230	0.6203	−8.2624	+9.4685	+9.3166	0.8023	9.9771
471	8.7732	9.2411	0.8012	−8.7427	+9.8604	+9.4778	0.8105	9.9762
472	8.3932	8.8537	+0.5734	−8.0661	+8.7938	+9.1878	0.8171	9.9754
473	8.7619	9.2195	−7.6812	+8.7281	−0.0163	−9.4837	0.8197	9.9751
474	8.4065	8.8564	+0.3751	+8.0923	−9.9225	−9.2101	0.8266	9.9742
475	8.3577	8.8025	0.5133	−7.5318	−9.4603	+8.7030	0.8311	9.9737
476	8.4271	8.8692	0.5864	−8.1522	+9.1199	+9.2564	0.8334	9.9734
477	8.4424	8.8622	0.3646	+8.1561	−9.9319	−9.2646	0.8531	9.9707
478	8.5165	8.9296	0.2805	+8.3503	−9.9794	−9.3905	0.8589	9.9698
479	8.4479	8.8588	0.3677	+8.1553	−9.9287	−9.2660	0.8609	9.9695
480	8.4558	8.8659	0.5861	−8.1826	+9.1126	+9.2861	0.8616	9.9694
481	8.4009	8.8108	0.5355	−7.8450	−9.1976	+9.0036	0.8617	9.9694
482	8.4422	8.8431	0.5694	−8.1026	+8.5944	+9.2277	0.8695	9.9682
483	8.4338	8.8276	0.4049	+8.0284	−9.8829	−9.1680	0.8757	9.9672
484	8.4445	8.8364	0.3930	+8.0831	−9.8998	−9.2136	0.8772	9.9669
485	8.4154	8.7905	0.5040	−7.4052	−9.5339	+8.5792	0.8916	9.9645
486	8.4669	8.8355	0.3907	+8.1159	−9.9021	−9.2439	0.8971	9.9635
487	8.4669	8.8355	0.3907	+8.1159	−9.9021	−9.2439	0.8972	9.9635
488	8.4835	8.8456	0.5749	−8.1724	[illegible]	[illegible]	0.9026	9.9625
489	8.4306	8.7919	0.4582	+7.6346	[illegible]	[illegible]	0.9033	9.9623
490	8.4725	8 8270	0.5604	−8.0944	[illegible]	[illegible]	0.9089	9.9613
491	8.4894	8.8402	0 5718	−8.1665	[illegible]	[illegible]	[illegible]	9.9606
492	8.4973	8.8396	0.3815	+8.1773	[illegible]	[illegible]	[illegible]	9.9592
493	8.5635	8.9018	0.3075	+8.3767	[illegible]	[illegible]	[illegible]	9.9585
494	8.5506	8.8824	0.3298	+8.3361	[illegible]	[illegible]	[illegible]	9.9574
495	9.2087	9.5354	0.9932	[illegible]	[illegible]	[illegible]	[illegible]	9.9564
496	8.5021	8.8243	0 3967	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
497	8.5027	8.8194	+0.4018	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
498	8.9803	9.2939	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
499	8.5177	8.8280	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
500	−8.6266	+8.9352	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Catalogue of 1500 Stars.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
501	2622	9	Puppis,		5	7	44	49.57	+2.780	103	30	10.8	+ 9.19
502	2629		Puppis,		5		46	40.65	2.263	124	19	58.0	8.91
503	2634		Puppis,	a	5		47	3.67	2.063	130	11	32.2	9.12
504	2635		Puppis,	b	5		47	20.41	2.132	128	28	38.4	9.12
505	2642		Velorum,		5		48	49.76	1.678	139	13	31.2	9.35
506	2644		Puppis,	R	4		48	53.31	1.737	137	42	45.1	8.86
507	2665		Argûs,	χ	4		52	57.65	1.518	142	34	54.4	9.50
508	2666		Puppis,		5		53	8.56	2.688	107	59	25.7	9.50
509	2670		Velorum,		5		53	56.71	1.755	138	50	20.2	9.48
510	2673		Canis Minoris,		5		54	27.69	3.129	87	15	25.9	9.49
511	2697	27	Lyncis,		5		57	9.06*	4.556	38	4	0.2*	9.82
512	2707	55	Camelopardi,		5		57	49.11*	6.096	21	5	31.0*	9.86
513	2710		Argûs,	ζ	2½		58	18.98	2.114	129	35	0.9	9.96
514	2714	10	Cancri,	μ^2	5		58	55.89	3.545	67	59	7.9	9.97
515	2728		Argûs,	ρ	3½	8	1	9.42*	2.558	113	52	30.3*	10.04
516	2730	14	Cancri,	ψ^2	4		1	24.66	3.630	64	2	28.9	10.47
517	2736	16	Puppis,		5		2	19.91	2.682	108	48	30.7	10.17
518	2754		Velorum,		5		4	52.10	1.849	136	54	17.1	10.39
519	2755		Argûs,	γ	2		4	54.50	1.840	136	53	47.9	10.46
520	2769	20	Puppis,		5		6	26.41	2.761	105	20	24.2	10.54
521	2773		Volantis,	ε	5		7	25.99	0.251	158	10	37.9	10.62
522	2774		Puppis,	r	5		7	50.06	2.268	125	26	53.5*	10.71
523	2776	30	Lyncis,		5		8	17.15*	4.907	31	47	42.5*	10.63
524	2778	17	Cancri,	β	4		8	22.72	3.263	80	21	22.2	10.72
525	2792		Lyncis,		5		12	25.34*	4.592	36	18	8.6*	10.99
526	2793	31	Lyncis,		5		12	32.94*	4.141	46	20	8.2*	11.06
527	2795		Puppis,	q	5		12	56.69	2.244	126	11	51.1*	10.99
528	2802		Puppis,	w	5		15	29.23	2.377	122	34	50.1	11.26
529	2819	1	Ursæ Majoris,	o	4		17	45.46*	5.066	28	47	10.8*	11.46
530	2823		Velorum,	B	5		17	55.20	1.838	138	0	40.5	11.30
531	2832		Argûs,	ε	2		19	25.89	1.241	149	1	40.3	11.35
532	2842	2	Ursæ Majoris,	A	5		21	6.20	+5.476	24	20	59.0	11.66
533	2849		Chamæleontis,	α	4½		22	18.70	−1.426	166	26	38.7	11.59
534	2856		Volantis,	η	5		23	22.60	−0.480	162	54	53.2	11.80
535	2863		Volantis,	β	5		24	5.41	+0.662	155	38	13.6	12.00
536	2870		Chamæleontis,	θ	5		25	2.81	−1.662	166	59	53.1	11.81
537	2884	4	Ursæ Majoris,	π	5		27	2.92*	+5.358	25	9	15.6*	11.99
538	2901	4	Hydræ,	δ	4		29	42.64*	3.184	83	46	36.1*	12.17
539	2911	5	Hydræ,	σ	5		30	55.07	3.147	86	8	6.9	12.29
540	2926		Velorum,	e	5		32	22.28	2.107	132	27	57.7	12.35
541	2935		Mali,	b	5		34	14.13*	2.363	124	46	50.2*	12.60
542	2937	43	Cancri,	γ	4½		34	35.91*	3.488	67	59	44.7*	12.50
543	2945	7	Hydræ,	η	5		35	23.02	3.143	86	3	58.1	12.59
544	2947		Velorum,	b	5		35	39.07	1.989	136	7	3.3	12.67
545	2950		Argûs,	o	4		35	59.80	1.717	142	23	29.8	12.69
546	2953	47	Cancri,	δ	4½		36	9.26*	3.425	71	17	53.0*	12.87
547	2962		Carinæ,	d	5		37	18.24	1.344	149	13	35.1	12.55
548	2964		Mali,	a	4½		37	34.13*	2.410	122	38	55.4*	12.64
549	2965	48	Cancri,	ι	5		37	36.76	3.654	60	41	44.0	12.80
550	2971	11	Hydræ,	e	4	8	38	49.78*	+3.189	83	2	3.5*	+12.84

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
501	−8.4811	+8.7890	+0.4444	+7.8494	−9.8004	−9.0133	+0.9473	+9.9529
502	8.5591	8.8582	0.3531	+8.3104	−9.9357	−9.4034	0.9543	9.9512
503	8.5944	8.8917	0.3143	+8.4042	−9.9579	−9.4633	0.9558	9.9508
504	8.5848	8.8807	0.3268	+8.3787	−9.9518	−9.4485	0.9568	9.9505
505	8.6691	8.9580	0.2284	+8.5483	−9.9809	−9.5394	0.9623	9.9491
506	8.6563	8.9450	0.2464	+8.5254	−9.9776	−9.5294	0.9626	9.9490
507	8.7153	8.9852	0.1850	+8.6152	−9.9838	−9.5750	0.9772	9.9450
508	8.5213	8.7905	0.4295	+8.0111	−9.8355	−9.1654	0.9779	9.9448
509	8.6840	8.9495	0.2371	+8.5608	−9.9765	−9.5552	0.9807	9.9440
510	8.5047	8.7678	0.4951	−7.1846	−9.5927	+8.3602	0.9825	9.9434
511	8.7234	8.9745	0.6589	−8.6195	+9.6223	+9.5856	0.9917	9.9406
512	8.9595	9.2077	0.7846	−8.9294	+9.8170	+9.6616	0.9939	9.9399
513	8.6304	8.8764	0.3241	+8.4347	−9.9487	−9.4977	0.9956	9.9394
514	8.5522	8.7955	0.5490	−8.1261	−8.8854	+9.2693	0.9977	9.9387
515	8.5655	8.7990	0.4082	+8.1727	−9.8729	−9.3099	1.0050	9.9363
516	8.5737	8.8061	0.5602	−8.2149	−8.0969	+9.3448	1.0058	9.9360
517	8.5543	8.7827	0.4279	+8.0627	−9.8378	−9.2150	1.0088	9.9350
518	8.7039	8.9214	0.2668	+8.5674	−9.9643	−9.5780	1.0168	9.9321
519	8.7040	8.9213	0.2669	+8.5674	−9.9642	−9.5781	1.0169	9.9320
520	8.5591	8.7699	0.4406	+7.9816	−9.8088	−9.1419	1.0217	9.9303
521	8.9762	9.1828	9.3705	+8.9439	−9.9776	−9.6902	1.0247	9.9291
522	8.6367	8.8416	0.3547	+8.4001	−9.9272	−9.4871	1.0259	9.9286
523	8.8273	9.0303	0.6900	−8.7567	+9.6916	+9.6545	1.0273	9.9281
524	8.5555	8.7581	0.5136	−7.7795	−9.4562	+8.9494	1.0276	9.9280
525	8.7889	8.9746	0.6623	−8.6952	+9.6214	+9.6436	1.0396	9.9231
526	8.7023	8.8875	0.6169	−8.5414	+9.4219	+9.5768	1.0399	9.9229
527	8.6559	8.8395	0.3526	+8.4272	−9.9265	−9.5101	1.0411	9.9224
528	8.6444	8.8175	0.3731	+8.3756	−9.9100	−9.4773	1.0483	9.9192
529	8.8938	9.0576	0.7057	−8.8365	+9.7108	+9.6952	1.0547	9.9163
530	8.7514	8.9146	0.2663	+8.6226	−9.9557	−9.6241	1.0551	9.9161
531	8.8695	9.0265	0.0944	+8.8027	−9.9670	−9.6902	1.0593	9.9141
532	8.9703	9.1206	+0.7390	−8.9298	+9.7520	+9.7211	1.0638	9.9119
533	9.2188	9.3642	−0.1582	+9.2065	−9.9506	−9.7525	1.0670	9.9103
534	9.1235	9.2647	−9.6586	+9.1039	−9.9554	−9.7480	1.0698	9.9088
535	8.9779	9.1163	+9.8342	+8.9374	−9.9618	−9.7290	1.0717	9.9078
536	9.2437	9.3783	−0.2037	+9.2324	−9.9461	−9.7607	1.0742	9.9065
537	8.9725	9.0992	+0.7285	−8.9293	+9.7330	+9.7338	1.0793	9.9037
538	8.6102	8.7264	0.5033	−7.6453	−9.5382	+8.8188	1.0860	9.8999
539	8.6116	8.7230	0.4972	−7.4403	−9.5794	+8.6154	1.0889	9.8981
540	8.7463	8.8520	0.3238	+8.5757	−9.9302	−9.6196	1.0925	9.8960
541	8.7041	8.8026	0.3701	+8.4603	−9.9045	−9.5509	1.0969	9.8932
542	8.6523	8.7494	0.5432	−8.2260	−9.0402	+9.3692	1.0978	9.8926
543	8.6224	8.7164	0.4972	−7.4587	−9.5792	+8.6338	1.0996	9.8915
544	8.7811	8.8741	0.2986	+8.6389	−9.9353	−9.6558	1.1003	9.8910
545	8.8373	8.9289	0.2360	+8.7361	−9.9447	−9.6977	1.1011	9.8905
546	8.6467	8.7377	0.5343	−8.1527	−9.2093	+9.3053	1.1015	9.8903
547	8.9169	9.0034	0.1251	+8.8509	−9.9477	−9.7360	1.1041	9.8885
548	8.7011	8.7867	0.3818	+8.4331	−9.8934	−9.5345	1.1047	9.8881
549	8.6860	8.7714	0.5624	−8.3757	−6.3010	+9.4923	1.1048	9.8880
550	−8.6325	+8.7132	+0.5047	−7.7163	−9.5283	+8.8892	+1.1076	+9.8861

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation
						h.	m.	s.	s.	°	′	″	″
551	2978	13	Hydræ,	ρ	5	8	40	29.11	+3.186	83	36	39.5	+12.9
552	2979		Argûs,	δ	3		40	33.66	1.652	144	9	38.6	13.0
553	2981		Velorum,	a	5		40	56.75	2.034	135	29	43.6	12.9
554	2998		Carinæ,	f	5		42	49.93	+1.562	146	13	9.1	12.8
555	3023		Chamæleontis,	η	5		46	17.84	—1.816	168	24	53.5	12.8
556	3032	16	Hydræ,	ζ	4		47	27.90	+3.184	83	29	11.2	13.3
557	3048	9	Ursæ Majoris,	ι	3½		48	54.53*	4.125	41	22	24.4*	13.7
558	3049	8	Ursæ Majoris,	ρ	5		48	56.30	5.553	21	47	29.9	13.4
559	3055	65	Cancri,	a	4		50	16.71*	3.293	77	33	53.8*	13.6
560	3059		Lyncis,		4		50	53.07*	3.931	47	37	37.7*	13.8
561	3073		Carinæ,	b^1	4		53	18.01	1.458	148	39	4.8	13.6
562	3075	12	Ursæ Majoris,	κ	4		53	21.54*	4.140	42	15	15.6*	13.8
563	3087	11	Ursæ Majoris,	σ^1	5		55	9.18*	5.399	22	31	48.0*	13.9
564	3089		Carinæ,	b^2	4		55	43.11	1.461	148	30	37.5	13.4
565	3097		Lyncis,		5		56	58.42*	3.855	50	57	7.0*	14.0
566	3099	13	Ursæ Majoris,	σ^2	5		57	7.36*	5.407	22	15	44.0*	14.1
567	3106	15	Ursæ Majoris,	f	5		58	15.49	4.287	37	47	37.9	14.1
568	3108	14	Ursæ Majoris,	τ	5		58	29.38	5.044	25	52	52.7	14.1
569	3110		Velorum,	c	5		58	59.47	2.084	136	30	11.6	14.2
570	3111	76	Cancri,	κ	5		59	37.12*	3.262	78	43	52.8*	14.1
571	3114		Volantis,	a	4½	9	0	4.02	0.962	155	47	54.3	14.3
572	3125	16	Ursæ Majoris,	c	5		2	26.16*	4.844	27	57	51.0*	14.3
573	3126		Argûs,	λ	3		2	29.05	2.202	132	49	45.7	14.4
574	3136		Carinæ,	G	5		4	43.28	0.200	162	0	6.6	14.9
575	3140	18	Ursæ Majoris,	e	5		5	21.45*	4.382	35	21	46.7*	14.4
576	3146	22	Hydræ,	θ	4½		6	33.53	3.131	87	3	19.9	14.9
577	3149		Carinæ,	a	5		7	1.17	1.562	148	21	19.5	14.9
578	3152		Carinæ,	i	5		7	52.21	1.354	151	42	9.9	14.6
579	3162	38	Lyncis,		4		9	29.81	3.763	52	33	55.6	14.8
580	3163		Velorum,	l	5		9	42.55	2.371	127	56	44.4	14.5
581	3177		Argûs,	β	1		11	32.18	0.690	159	6	0.6	14.7
582	3178	40	Lyncis,	a	4		11	54.29	3.682	54	58	37.2	14.9
583	3186		Argûs,	ι	2		13	4.63	1.602	148	38	49.6	14.9
584	3187		Velorum,	K	5		13	6.71	2.006	140	25	23.2	15.0
585	3195		Mali,	h	5		14	51.55	2.663	115	19	45.6*	14.9
586	3199		Draconis,		5		15	15.42*	9.258	8	1	6.5*	15.1
587	3204	1	Leonis,	κ	5		15	54.60	3.514	63	10	28.2	15.1
588	3213		Argûs,	κ	3		17	28.39	1.851	144	22	20.2	15.2
589	3221	23	Ursæ Majoris,	h	4		19	38.53*	4.833	26	17	12.0*	15.3
590	3223	30	Hydræ,	a	2		20	12.94*	2.951	98	0	40.2*	15.3
591	3226		Hydræ,		5		20	20.77	2.989	95	25	7.7	15.3
592	3232	24	Ursæ Majoris,	d	5		21	7.77*	5.482	19	30	54.9*	15.4
593	3242	25	Ursæ Majoris,	θ	3		22	47.56*	4.049	37	38	33.1*	16.1
594	3246	4	Leonis,	λ	4½		23	9.26*	3.441	66	22	24.9*	15.5
595	3249		Carinæ,	n	5		23	29.12	1.318	154	16	51.9	15.5
596	3250	5	Leonis,	ξ	5		23	51.37*	3.245	78	2	19.3*	15.6
597	3257		Argûs,	ψ	4		24	48.15	2.367	129	48	43.9	15.5
598	3261	10	Leonis Minoris,		5		25	1.27	3.707	52	56	20.7	15.6
599	3269		Velorum,	N	5		26	40.23	1.825	146	22	25.4	15.5
600	3289		Carinæ,	h	5	9	28	5.38	+1.710	148	33	43.2	+15.9

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
551	−8.6358	+8.7101	+0.5031	−7.6822	−9.5393	+8.8556	+1.1114	+9.8835
552	8.8657	8.9397	0.2189	+8.7745	−9.9414	−9.7182	1.1116	9.8834
553	8.7884	8.8610	0.3080	+8.6416	−9.9287	−9.6634	1.1124	9.8827
554	8.8932	8.9585	+0.1919	+8.8129	−9.9402	−9.7341	1.1166	9.8797
555	9.3430	9.3951	−0.2573	+9.3341	−9.9138	−9.8130	1.1241	9.8740
556	8.6511	8.6987	+0.5030	−7.7059	−9.5400	+8.8791	1.1266	9.8720
557	8.8312	8.8732	0.6227	−8.7065	+9.4296	+9.7027	1.1297	9.8695
558	9.0817	9.1237	0.7440	−9.0495	+9.7213	+9.7593	1.1297	9.8695
559	8.6645	8.7014	0.5169	−7.9976	−9.4236	+9.1634	1.1325	9.8671
560	8.7869	8.8215	0.5984	−8.6155	+9.2418	+9.6601	1.1337	9.8661
561	8.9441	8.9695	0.1685	+8.8756	−9.9279	−9.7679	1.1386	9.8618
562	8.8328	8.8580	0.6171	−8.7021	+9.3918	+9.7059	1.1388	9.8617
563	9.0806	9.0990	0.7322	−9.0462	+9.6994	+9.8056	1.1423	9.8585
564	8.9472	8.9634	0.1757	+8.8780	−9.9247	−9.7720	1.1434	9.8575
565	8.7774	8.7888	0.5852	−8.5767	+9.0581	+9.6430	1.1459	9.8551
566	9.0894	9.1003	0.7331	−9.0558	+9.6974	+9.8103	1.1462	9.8549
567	8.8827	8.8893	0.6334	−8.7805	+9.4729	+9.7439	1.1484	9.8528
568	9.0305	9.0362	0.7017	−8.9846	+9.6542	+9.8007	1.1488	9.8523
569	8.8337	8.8375	0.3160	+8.6943	−9.9106	−9.7082	1.1498	9.8514
570	8.6811	8.6826	0.5131	−7.9721	−9.4582	+9.1398	1.1510	9.8502
571	9.0608	9.0605	9.9857	+9.0208	−9.9152	−9.8097	1.1518	9.8494
572	9.0069	8.9976	0.6842	−8.9530	+9.6174	+9.8001	1.1563	9.8448
573	8.8127	8.8033	0.3432	+8.6451	−9.8997	−9.6865	1.1564	9.8447
574	9.1922	9.1743	9.3448	+9.1704	−9.8991	−9.8365	1.1605	9.8404
575	8.9208	8.9005	0.6406	−8.8322	+9.4939	+9.7708	1.1616	9.8391
576	8.6860	8.6612	0.4939	−7.3967	−9.6000	+8.5723	1.1638	9.8367
577	8.9664	8.9398	0.1998	+8.8965	−9.9090	−9.7925	1.1646	9.8358
578	9.0120	8.9821	0.1386	+8.9567	−9.9067	−9.8086	1.1661	9.8341
579	8.7908	8.7548	0.5756	−8.5746	+8.8261	+9.6506	1.1690	9.8307
580	8.7942	8.7573	0.3739	+8.5830	−9.8798	−9.6559	1.1693	9.8303
581	9.1419	9.0981	9.8588	+9.1123	−9.8928	−9.8407	1.1725	9.8265
582	8.7816	8.7364	0.5680	−8.5404	+8.4594	+9.6298	1.1731	9.8257
583	8.9806	8.9309	0.2068	+8.9120	−9.9000	−9.8044	1.1751	9.8232
584	8.8927	8.8429	0.2998	+8.7796	−9.8974	−9.7599	1.1752	9.8232
585	8.7437	8.6872	0.4238	+8.3750	−9.8290	−9.5071	1.1781	9.8194
586	9.5559	9.4979	0.9695	−9.5517	+9.7720	+9.8723	1.1788	9.8186
587	8.7510	8.6905	0.5459	−8.4054	−8.9595	+9.5321	1.1799	9.8171
588	8.9388	8.8723	0.2685	+8.8488	−9.8933	−9.7902	1.1824	9.8137
589	9.0613	8.9865	0.6825	−9.0139	+9.5857	+9.8363	1.1859	9.8089
590	8.7128	8.6358	0.4698	+7.8569	−9.7153	−9.0287	1.1868	9.8076
591	8.7107	8.6331	0.4756	+7.6858	−9.6925	−8.8599	1.1870	9.8073
592	9.1861	9.1056	0.7389	−9.1604	+9.6602	+9.8604	1.1883	9.8055
593	8.9267	8.8397	0.6200	−8.8253	+9.3740	+9.7873	1.1909	9.8017
594	8.7511	8.6628	0.5366	−8.3540	−9.1590	+9.4921	1.1914	9.8006
595	9.0762	8.9865	0.1202	+9.0309	−9.8790	−9.8444	1.1919	9.8001
596	8.7237	8.6326	0.5118	−8.0403	−9.4684	+9.2068	1.1925	9.7992
597	8.8302	8.7355	0.3752	+8.6366	−9.8660	−9.6981	1.1940	9.7970
598	8.8140	8.7184	0.5685	−8.5940	+8.4900	+9.6721	1.1943	9.7965
599	8.9751	8.8731	0.2610	+8.8956	−9.8789	−9.8150	1.1968	9.7926
600	−9.0062	+8.8908	+0.2405	+8.9372	−9.8724	−9.8307	+1.2018	+9.7842

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.	Mag.	Right Ascension, Jan. 1, 1850.	Annual Variation.	North Polar Dist., Jan. 1, 1850.	Annual Variation.
				h. m. s.	s.	° ′ ″	″
601	3300	Velorum, M	4	9 31 27.89	+2.157	138 41 4.2	+16.08
602	3303	35 Hydræ, ι	5	32 11.75	3.072	90 27 52.9	16.14
603	3311	38 Hydræ, κ	5	33 7.26	2.881	103 39 13.0	16.07
604	3312	14 Leonis, ο	4	33 8.53*	3.228	79 25 40.7*	16.13
605	3315	28 Ursæ Majoris,	5	34 19.08*	4.725	25 39 37.8*	16.15
606	3320	Carinæ, m	5	35 11.56	1.651	150 39 2.7	16.25
607	3331	17 Leonis, ε	3	37 19.69*	3.425	65 32 16.2*	16.33
608	3346	29 Ursæ Majoris, υ	4	40 16.98*	4.353	30 15 33.0*	16.63
609	3353	Carinæ, l	5	41 7.51	1.637	151 49 4.4	16.57
610	3358	30 Ursæ Majoris, φ	5	41 51.86*	4.150	35 14 17.6*	16.57
611	3365	Argûs, υ	3	43 21.18	1.502	154 22 38.1	16.58
612	3371	24 Leonis, μ	3	44 13.41*	3.428	63 17 21.3*	16.70
613	3372	39 Hydræ, $υ^1$	5	44 15.88	2.885	104 8 41.1	16.66
614	3410	Argûs, φ	4	51 36.23	2.092	143 51 19.0	16.95
615	3415	29 Leonis, π	4½	52 17.01*	3.182	81 14 18.4*	17.05
616	3446	21 Leonis Minoris,	5	58 34.14*	3.571	54 1 35.1*	17.31
617	3453	30 Leonis, η	3½	59 8.85*	3.285	72 30 29.5*	17.32
618	3457	31 Leonis, A	5	59 56.27*	3.194	79 16 9.5*	17.42
619	3458	15 Sextantis,	5	10 0 15.59	3.077	89 38 26.0	17.39
620	3459	32 Leonis, α	1	0 22.74*	3.206	77 18 6.8*	17.39
621	3473	41 Hydræ, λ	4½	3 16.75	2.927	101 36 51.8	17.60
622	3495	Ursæ Minoris,	5½	6 58.81*	10.207	4 59 29.4*	17.73
623	3496	32 Ursæ Majoris,	5	7 4.60	4.468	24 8 46.2*	17.71
624	3505	33 Ursæ Majoris, λ	3½	8 1.86*	3.659	46 20 20.0*	17.77
625	3508	36 Leonis, ζ	4½	8 20.43	3.355	65 50 13.6	17.70
626	3509	Velorum, q	4	8 27.11	2.513	131 22 52.7	17.91
627	3516	Argûs, ω	4	10 10.11	1.426	159 17 38.0	17.77
628	3523	41 Leonis, γ	2	11 41.78*	3.322	69 24 7.0*	18.01
629	3526	Carinæ, q	5	12 4.94	1.986	150 35 2.2	17.87
630	3528	Draconis,	5½	12 15.65	8.132	6 40 57.7*	17.95
631	3531	Ursæ Majoris,	5	13 15.20	4.445	23 40 38.4	17.98
632	3533	34 Ursæ Majoris, μ	3	13 22.36*	3.615	47 44 53.3*	17.89
633	3536	Velorum, V	5	13 59.05	2.239	144 16 39.4	18.09
634	3546	Velorum, T	5	15 20.25	2.196	145 17 27.3	18.34
635	3552	Velorum, r	5	15 54.07	2.558	130 53 50.1	18.08
636	3560	30 Leonis Minoris,	4½	17 18.33	3.470	55 26 31.1	18.18
637	3568	42 Hydræ, μ	4	18 50.40	2.900	106 4 19.4	18.25
638	3572	31 Leonis Minoris, β	4½	19 11.64*	3.502	52 31 33.1	18.26
639	3578	Antliæ, α	4½	20 17.69	2.736	120 18 23.4*	18.24
640	3580	36 Ursæ Majoris,	5	20 59.82*	3.911	33 15 8.1*	18.19
641	3585	Carinæ I	4½	21 24.76	1.212	163 16 7.0	18.24
642	3586	Carinæ,	5	21 41.59	1.885	154 52 38.1	18.24
643	3589	Velorum, P	5	21 50.35	2.227	146 52 24.0	17.92
644	3594	Carinæ, s	5	22 22.58	2.161	147 58 25.3	18.23
645	3607	Ursæ Majoris,	5	24 27.94	3.544	48 48 15.3	18.34
646	3609	47 Leonis, ρ	4	24 54.58*	3.171	79 55 24.0*	18.39
647	3610	34 Leonis Minoris,	5	24 55.62	3.459	54 14 25.5	18.37
648	3612	37 Ursæ Majoris,	5	25 27.85*	3.939	32 8 49.3*	18.39
649	3619	Carinæ, p	4	26 42.13	2.111	150 54 52.4	18.38
650	3640	37 Leonis Minoris,	4½	10 30 16.07	+3.403	57 14 46.3	+18.54

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
601	−8.9058	+8.7850	+0.3329	+8.7815	−9.8692	−9.7773	+1.2038	+9.7808
602	8.7266	8.6029	0.4863	+6.6356	−9.6424	−7.8117	1.2049	9.7790
603	8.7403	8.6130	0.4589	+8.1133	−9.7504	−9.2770	1.2062	9.7766
604	8.7353	8.6079	0.5078	−7.9989	−9.5009	+9.1675	1.2062	9.7766
605	9.0930	8.9610	0.6742	−9.0479	+9.5406	+9.8605	1.2079	9.7736
606	9.0404	8.9049	0.2217	+8.9808	−9.8618	−9.8472	1.2091	9.7713
607	8.7745	8.6305	0.5347	−8.3916	−9.1864	+9.5269	1.2120	9.7657
608	9.0353	8.8794	0.6418	−8.9717	+9.4408	+9.8501	1.2160	9.7578
609	9.0646	8.9052	0.2173	+9.0098	−9.8494	−9.8601	1.2171	9.7555
610	8.9786	8.8163	0.6174	−8.8907	+9.3276	+9.8279	1.2180	9.7535
611	9.1057	8.9374	0.1776	+9.0608	−9.8414	−9.8728	1.2200	9.7494
612	8.7918	8.6199	0.5374	−8.4445	−9.1358	+9.5716	1.2211	9.7469
613	8.7562	8.5841	0.4598	+8.1442	−9.7460	−9.3070	1.2211	9.7468
614	8.9811	8.7786	0.3218	+8.8883	−9.8358	−9.8351	1.2302	9.7254
615	8.7578	8.5524	0.5024	−7.9406	−9.5420	+9.1115	1.2310	9.7234
616	8.8518	8.6194	0.5516	−8.6207	−8.7348	+9.7049	1.2382	9.7036
617	8.7811	8.5462	0.5162	−8.2590	−9.4193	+9.4146	1.2388	9.7017
618	8.7690	8.5307	0.5048	−8.0390	−9.5228	+9.2075	1.2397	9.6992
619	8.7617	8.5220	0.4878	−6.5592	−9.6344	+7.7353	1.2400	9.6981
620	8.7726	8.5324	0.5080	−8.1147	−9.4967	+9.2800	1.2402	9.6977
621	8.7740	8.5209	0.4678	+8.0779	−9.7188	−9.2450	1.2433	9.6880
622	9.8292	9.5598	1.0137	−9.8275	+9.6375	+9.9432	1.2471	9.6754
623	9.1571	8.8870	0.6514	−9.1174	+9.4050	+9.9052	1.2472	9.6749
624	8.9105	8.6360	0.5646	−8.7496	+8.0043	+9.7851	1.2482	9.6715
625	8.8100	8.5341	0.5252	−8.4221	−9.3103	+9.5584	1.2485	9.6704
626	8.8951	8.6187	0.4013	+8.7153	−9.8026	−9.7666	1.2486	9.6700
627	9.2235	8.9392	0.1582	+9.1945	−9.7669	−9.9191	1.2503	9.6638
628	8.8022	8.5108	0.5184	−8.3485	−9.3911	+9.4959	1.2518	9.6582
629	9.0827	8.7894	0.2999	+9.0227	−9.7841	−9.8900	1.2522	9.6567
630	9.7081	9.4140	0.9158	−9.7051	+9.6014	+9.9472	1.2524	9.6561
631	9.1712	8.8725	0.6474	−9.1330	+9.3748	+9.9129	1.2533	9.6524
632	8.9058	8.6065	0.5582	−8.7334	−8.2900	+9.7788	1.2534	9.6519
633	9.0094	8.7072	0.3504	+8.9189	−9.7894	−9.8613	1.2540	9.6496
634	9.0215	8.7129	0.3462	+8.9364	−9.7850	−9.8680	1.2553	9.6444
635	8.8990	8.5877	0.4087	+8.7151	−9.7891	−9.7696	1.2558	9.6422
636	8.8631	8.5450	0.5403	−8.6169	−9.0477	+9.7086	1.2571	9.6367
637	8.7975	8.4719	0.4633	+8.2397	−9.7286	−9.3985	1.2585	9.6306
638	8.8809	8.5535	0.5449	−8.6650	−8.9310	+9.7408	1.2588	9.6292
639	8.8453	8.5125	0.4379	+8.5482	−9.7689	−9.6605	1.2598	9.6248
640	9.0430	8.7068	0.5938	−8.9654	+9.0500	+9.8805	1.2604	9.6219
641	9.3232	8.9849	0.0849	+9.3044	−9.7196	−9.9397	1.2607	9.6202
642	9.1547	8.8150	0.2752	+9.1116	−9.7488	−9.9156	1.2610	9.6190
643	9.0452	8.7047	0.3463	+8.9682	−9.7672	−9.8819	1.2611	9.6184
644	9.0587	8.7155	0.3399	+8.9870	−9.7637	−9.8877	1.2616	9.6162
645	8.9085	8.5547	0.5495	−8.7272	−8.7767	+9.7798	1.2633	9.6073
646	8.7922	8.4361	0.5006	−8.0351	−9.5524	+9.2044	1.2637	9.6054
647	8.8761	8.5200	0.5389	−8.6428	−9.0671	+9.7282	1.2637	9.6054
648	9.0599	8.7010	0.5937	−8.9876	+9.0354	+9.8897	1.2642	9.6030
649	9.1001	8.7348	0.3261	+9.0416	−9.7459	−9.9044	1.2652	9.5976
650	−8.8649	+8.4807	+0.5313	−8.5982	−9.2000	+9.6991	+1.2681	+9.5816

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
651	3644		Velorum,	p	5	10	30	59.76	+2.436	137	26	48.3	+18.38
652	3646		Hydræ,	ϕ^2	5		31	16.67	2.920	106	5	56.5	18.53
653	3647	38	Ursæ Majoris,		5		31	39.19	4.214	23	30	0.1*	18.66
654	3652		Ursæ Majoris,		5		32	15.29*	4.435	20	8	28.7*	18.65
655	3655		Carinæ,	t^2	5		33	2.82	2.244	148	24	12.3	18.67
656	3660		Chamæleontis,	γ	5		33	39.04	0.732	167	49	48.3	18.56
657	3685	42	Leonis Minoris,		4½		37	30.86	3.361	58	31	42.0	18.79
658	3686		Argûs,	θ	3		37	37.19	2.118	153	36	33.5	18.76
659	3695		Argûs,	η	2		39	15.44	2.305	148	53	49.5	18.73
660	3702		Argûs,	μ	3		40	19.80	2.560	138	37	41.2	18.92
661	3715		Hydræ,	ν	4		42	13.55	2.954	105	24	37.1	18.74
662	3724		Chamæleontis,	δ^2	5		44	19.32	0.631	169	44	57.9	18.97
663	3728	46	Leonis Minoris,		4½		44	54.49	3.379	54	58	39.8*	19.25
664	3729	45	Ursæ Majoris,	ω	5		45	19.44*	3.491	46	0	46.4*	19.03
665	3740		Carinæ,	u	5		47	24.97	2.396	148	3	26.9	19.09
666	3742	54	Leonis,		4½		47	29.24	3.275	64	27	4.4	19.07
667	3766	7	Crateris,	α	4		52	28.25	2.919	107	30	1.8	19.05
668	3767	48	Ursæ Majoris,	β	2		52	45.41*	3.685	32	48	53.6*	19.17
669	3768	58	Leonis,	d	5		52	48.71*	3.104	85	34	42.3*	19.25
670	3776	60	Leonis,	b	5		54	19.05	3.219	69	0	56.9	19.20
671	3777	50	Ursæ Majoris,	α	1½		54	25.72*	3.784	27	26	26.0*	19.33
672	3788	63	Leonis,	χ	4½		57	16.61*	3.103	81	51	15.6*	19.39
673	3793		Hydræ,	χ^1	5		58	6.98	2.884	116	29	5.1	19.36
674	3794		Hydræ,	χ^2	5		58	41.71	2.900	116	28	40.7	19.37
675	3812	52	Ursæ Majoris,	ψ	3½	11	1	12.67*	3.411	44	41	19.6*	19.49
676	3815		Hydræ,		5		1	29.16	2.897	117	16	5.4	19.43
677	3822		Hydræ,		5		2	40.85	2.856	121	33	16.3	19.55
678	3826	11	Crateris,	β	4		4	17.27	2.947	112	0	26.4	19.57
679	3834	68	Leonis,	δ	2½		6	7.46*	3.209	68	39	19.3*	19.64
680	3838	70	Leonis,	θ	3		6	21.99	3.161	73	45	0.7	19.54
681	3842	72	Leonis,		5		7	13.16	3.207	66	5	15.4	19.54
682	3848	74	Leonis,	φ	5		9	2.12*	3.053	92	49	57.3*	19.61
683	3851	53	Ursæ Majoris,	ξ	4		10	10.46	3.223	57	37	36.4	20.15
684	3852	54	Ursæ Majoris,	ν	4		10	22.05	3.268	56	5	16.2	19.55
685	3856	55	Ursæ Majoris,		5		10	56.51*	3.302	50	59	32.2*	19.68
686	3859	12	Crateris,	δ	3½		11	50.65*	2.997	103	58	3.4*	19.44
687	3862	77	Leonis,	σ	4		13	23.98*	3.099	83	8	58.1*	19.66
688	3866		Centauri,	π	4		14	10.55	2.758	143	40	14.5	19.98
689	3877	78	Leonis,	ι	4		16	6.12*	3.137	78	38	41.5*	19.75
690	3881	14	Crateris,	ε	5		17	2.28	3.028	100	2	15.4	19.69
691	3883	15	Crateris,	γ	4		17	23.56	2.990	106	51	36.1	19.67
692	3885		Ursæ Majoris,		5		17	27.47*	3.439	33	19	41.0*	19.67
693	3900	84	Leonis,	τ	4		20	13.31*	3.091	86	19	5.9*	19.78
694	3914	1	Draconis,	λ	3½		22	26.42*	3.671	19	50	30.4*	19.87
695	3916	87	Leonis,	e	4½		22	39.03*	3.066	92	10	35.9*	19.84
696	3922		Hydræ,		5		24	50.91	2.964	118	26	22.1*	19.67
697	3928		Hydræ,		4		25	38.25	2.941	121	1	39.2	19.86
698	3941		Centauri,	λ	4		28	53.66	2.724	152	11	24.0	19.87
699	3943	21	Crateris,	θ	4		29	4.66	3.043	98	58	21.5	19.86
700	3946	91	Leonis,	υ	4½	11	29	16.07*	+3.074	89	59	46.6*	+19.87

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
651	−8.9602	+8.5721	+0.4013	+8.8274	−9.7577	−9.8337	+1.2686	+9.5783
652	8.8079	8.4182	0.4661	+8.2508	−9.7173	−9.4096	1.2688	9.5770
653	9.1901	8.7984	0.6260	−9.1525	+9.2350	+9.9293	1.2691	9.5752
654	9.2543	8.8593	0.6466	−9.2269	+9.3038	+9.9400	1.2696	9.5724
655	9.0726	8.6733	0.3552	+9.0029	−9.7347	−9.8983	1.2702	9.5686
656	9.4684	9.0658	9.8975	+9.4586	−9.6544	−9.9586	1.2706	9.5658
657	8.8643	8.4398	0.5262	−8.5820	−9.2700	+9.6890	1.2735	9.5468
658	9.1474	8.7223	0.3269	+9.0996	−9.7059	−9.9235	1.2735	9.5463
659	9.0832	8.6487	0.3628	+9.0158	−9.7158	−9.9051	1.2747	9.5379
660	8.9770	8.5361	0.4071	+8.8523	−9.7343	−9.8485	1.2754	9.5323
661	8.8143	8.3621	0.4695	+8.2388	−9.7049	−9.3989	1.2767	9.5223
662	9.5495	9.0845	9.8283	+9.5425	−9.5980	−9.9689	1.2781	9.5109
663	8.8869	8.4183	0.5279	−8.6457	−9.2322	+9.7351	1.2785	9.5076
664	8.9434	8.4722	0.5420	−8.7851	−8.9538	+9.8182	1.2788	9.5053
665	9.0783	8.5938	0.3807	+9.0069	−9.6936	−9.9065	1.2801	9.4934
666	8.8465	8.3616	0.5145	−8.4813	−9.4104	+9.6127	1.2801	9.4930
667	8.8254	8.3075	0.4696	+8.3035	−9.7000	−9.4590	1.2831	9.4630
668	9.0710	8.5512	0.5647	−8.9955	+7.8976	+9.9055	1.2833	9.4612
669	8.8063	8.2861	0.4914	−7.6933	−9.6132	+8.8681	1.2833	9.4609
670	8.8356	8.3050	0.5073	−8.3896	−9.4829	+9.5359	1.2841	9.4513
671	9.1424	8.6110	0.5795	−9.0905	+8.7033	+9.9301	1.2842	9.4506
672	8.8118	8.2601	0.4945	−7.9632	−9.5925	+9.1349	1.2858	9.4318
673	8.8560	8.2982	0.4613	+8.5053	−9.7054	−9.6333	1.2862	9.4261
674	8.8563	8.2942	0.4616	+8.5055	−9.7043	−9.6335	1.2865	9.4221
675	8.9624	8.3811	0.5332	−8.8142	−9.0962	+9.8374	1.2878	9.4043
676	8.8608	8.2774	0.4619	+8.5218	−9.7002	−9.6467	1.2879	9.4024
677	8.8797	8.2870	0.4575	+8.5984	−9.6984	−9.7050	1.2885	9.3936
678	8.8438	8.2383	0.4684	+8.4175	−9.6924	−9.5608	1.2893	9.3815
679	8.8427	8.2220	0.5041	−8.4037	−9.5043	+9.5490	1.2901	9.3672
680	8.8296	8.2069	0.4998	−8.2765	−9.5462	+9.4349	1.2902	9.3653
681	8.8513	8.2213	0.5060	−8.4591	−9.4832	+9.5962	1.2906	9.3585
682	8.8136	8.1679	0.4852	+7.5075	−9.6477	−8.6830	1.2914	9.3435
683	8.8869	8.2311	0.5123	−8.6156	−9.3997	+9.7184	1.2919	9.3338
684	8.8946	8.2371	0.5137	−8.6412	−9.3806	+9.7363	1.2920	9.3322
685	8.9234	8.2606	0.5186	−8.7224	−9.3086	+9.7889	1.2922	9.3272
686	8.8273	8.1562	0.4773	+8.2100	−9.6734	−9.3730	1.2926	9.3193
687	8.8180	8.1322	0.4918	−7.8946	−9.6094	+9.0676	1.2932	9.3052
688	9.0425	8.3493	0.4329	+8.9487	−9.6202	−9.8974	1.2935	9.2980
689	8.8245	8.1121	0.4944	−8.1187	−9.5888	+9.2862	1.2942	9.2796
690	8.8229	8.1010	0.4809	+8.0642	−9.6626	−9.2336	1.2945	9.2704
691	8.8354	8.1098	0.4765	+8.2979	−9.6711	−9.4549	1.2947	9.2668
692	9.0765	8.3501	0.5373	−8.9985	−8.9253	+9.9145	1.2947	9.2661
693	8.8182	8.0621	0.4893	−7.6259	−9.6253	+8.8011	1.2957	9.2372
694	9.2873	8.5057	0.5653	−9.2607	+7.7924	+9.9676	1.2964	9.2126
695	8.8184	8.0344	0.4861	+7.3979	−9.6434	−8.5737	1.2964	9.2102
696	8.8746	8.0638	0.4713	+8.5525	−9.6574	−9.6727	1.2971	9.1841
697	8.8861	8.0652	0.4699	+8.5983	−9.6522	−9.7073	1.2973	9.1743
698	9.1510	8.2863	0.4358	+9.0977	−9.5081	−9.9427	1.2982	9.1313
699	8.8253	7.9580	0.4832	+8.0183	−9.6529	−9.1891	1.2983	9.1288
700	−8.8200	+7.9500	+0.4872	−4.6517	−9.6375	+5.8278	+1.2983	+9.1261

Table XXX.

Catalogue of 1500 Stars.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
701	3978	27	Crateris,	ζ	4	11	37	10.04	+3.033	107	30	58.5	+19.97
702	3979	2	Virginis,	ξ	5		37	33.03	3.097	80	54	30.1	20.00
703	3981	63	Ursæ Majoris,	χ	4		38	6.53*	3.209	41	23	20.5*	19.94
704	3982	3	Virginis,	ν	4½		38	8.97*	3.093	82	37	49.4*	20.16
705	3984		Muscæ,		4½		38	34.44	2.792	155	53	52.1	20.03
706	3990	93	Leonis,		4		40	14.66	3.107	68	56	49.3	19.98
707	3995	94	Leonis,	β	2½		41	24.27*	3.066	74	35	22.6*	20.09
708	4002	5	Virginis,	β	3½		42	52.90*	3.128	87	23	25.2*	20.28
709	4015		Hydræ,		4		45	20.47*	3.014	123	4	26.5*	20.04
710	4017	64	Ursæ Majoris,	γ	2		45	55.06*	3.202	35	28	16.8*	20.04
711	4048		Chamæleontis,	ε	5		52	14.93	2.830	167	23	13.0	20.10
712	4052	8	Virginis,	π	5		53	11.16*	3.079	82	32	57.7*	20.10
713	4072	9	Virginis,	ο	4½		57	34.07*	3.064	80	26	1.4*	20.05
714	4078		Crucis,	η	4½		59	5.65	3.041	153	46	38.6	20.24
715	4087		Centauri,	δ	3	12	0	36.54	3.070	139	53	14.1	20.15
716	4090	1	Corvi,	α	4½		0	41.27	3.082	113	53	28.0	20.10
717	4097	2	Corvi,	ε	4		2	25.25	3.077	111	47	4.5	20.03
718	4103		Centauri,	ρ	4		3	50.15	3.090	141	31	57.2	20.03
719	4112		Draconis,		5		5	6.43*	2.936	11	33	0.3*	20.03
720	4120		Crucis,	δ	3		7	12.68	3.128	147	54	49.3	20.00
721	4123	69	Ursæ Majoris,	δ	3		7	58.70*	3.016	32	8	1.7*	20.10
722	4124	4	Corvi,	γ	3		8	5.97	3.077	106	42	30.4	20.02
723	4125	6	Comæ,		5		8	22.93	3.054	74	15	53.2	20.05
724	4126	2	Canum Venat.,		5		8	35.84*	3.034	48	30	15.9*	20.07
725	4127	7	Comæ,		5		8	44.96	3.050	65	13	14.1	20.07
726	4128		Canum Venat.,		5		8	57.48	3.041	56	6	2.8	20.22
727	4131		Chamæleontis,	β	5		9	40.27	3.304	168	28	46.3	20.05
728	4133		Crucis,	ζ	5		10	20.59	3.153	153	10	20.8	20.46
729	4145	15	Virginis,	η	3½		12	13.89*	3.067	89	49	58.5*	20.07
730	4151	16	Virginis,	c	5		12	43.93*	3.049	85	51	6.5*	20.09
731	4156	11	Comæ,		5		13	8.13	3.036	71	22	39.8	19.95
732	4158		Crucis,	ε	4		13	18.27	3.177	149	34	19.6	19.88
733	4169	12	Comæ,		5		14	57.61	3.029	63	19	13.3	20.00
734	4181	13	Comæ,		5		16	46.83	3.022	63	4	5.6	20.02
735	4186		Crucis,		4½		18	13.18	3.283	152	17	23.6	20.01
736	4187		Crucis,	α	1		18	18.00	3.251	152	15	59.7	19.94
737	4191	14	Comæ,		5		18	53.77	3.015	61	53	59.6	20.01
738	4195	15	Comæ,	γ	4½		19	27.45	3.004	60	53	49.1	20.09
739	4196	16	Comæ,		5		19	29.08	3.018	62	20	33.2	19.98
740	4197		Centauri,	σ	4½		19	57.03	3.185	139	23	55.8	19.91
741	4202		Centauri,		5		20	24.87	3.155	128	12	37.2	20.23
742	4211	7	Corvi,	δ	3		22	6.81	3.106	105	40	46.5	20.12
743	4215		Crucis,	γ	2		22	52.84	3.275	146	16	17.7	20.13
744	4224		Muscæ,	γ	4		23	35.05	3.460	161	18	13.2	19.98
745	4226	8	Corvi,	η	4½		24	20.93	3.083	105	21	52.4	20.01
746	4234	9	Corvi,	β	2½		26	30.94*	3.131	112	34	0.1*	19.99
747	4235	8	Canum Venat.,	β	4		26	36.40*	2.865	47	49	35.5*	19.64
748	4239	5	Draconis,	κ	3½		27	3.10*	2.610	19	23	4.4*	19.96
749	4240	23	Comæ,		4½		27	22.75	3.015	66	32	37.3	19.91
750	4245		Muscæ,	α	4	12	28	18.20	+3.481	158	18	28.7	+19.90

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
701	−8.8424	+7.8422	+0.4812	+8.3209	−9.6471	−9.4764	+1.3001	+8.9977
702	8.8273	7.8197	0.4902	−8.0260	9.6160	+9.1966	1.3001	8.9903
703	9.0016	7.9829	0.5072	−8.8768	9.3333	+9.8732	1.3002	8.9793
704	8.8255	7.8061	0.4896	−7.9337	9.6214	+9.1062	1.3002	8.9786
705	9.2110	8.1830	0.4459	+9.1713	9.4190	−9.9585	1.3003	8.9701
706	8.8523	7.7889	0.4935	−8.4077	9.5730	+9.5538	1.3006	8.9350
707	8.8384	7.7485	0.4914	−8.2628	9.5987	+9.4230	1.3008	8.9087
708	8.8232	7.6972	0.4879	−7.4816	9.6336	+8.6572	1.3010	8.8728
709	8.8998	7.7061	0.4793	+8.6368	9.6005	−9.7361	1.3013	8.8055
710	9.0594	7.8484	0.5032	−8.9703	9.3049	+9.9100	1.3014	8.7882
711	9.4845	8.0138	0.4577	+9.4739	9.1065	−9.9891	1.3020	8.5291
712	8.8274	7.3006	0.4880	−7.9403	9.6298	+9.1127	1.3020	8.4730
713	8.8300	6.8554	0.4876	−8.0506	9.6295	+9.2206	1.3022	8.0254
714	9.1786	+6.7754	0.4857	+9.1315	9.2905	−9.9528	1.3022	+7.5967
715	9.0148	−6.4392	0.4878	+8.8984	9.4434	−9.8835	1.3022	−7.4244
716	8.8628	6.3401	0.4875	+8.4703	9.5973	−9.6075	1.3022	7.4773
717	8.8561	6.8799	0.4880	+8.4256	9.6011	−9.5695	1.3022	8.0238
718	9.0300	7.2537	0.4912	+8.9238	9.4098	−9.8937	1.3022	8.2236
719	9.5223	7.8702	0.4661	−9.5134	9.0362	+9.9910	1.3021	8.3478
720	9.0984	7.5964	0.4966	+9.0264	9.3092	−9.9278	1.3020	8.4978
721	9.0978	7.6398	0.4766	−9.0256	9.4155	+9.9275	1.3020	8.5417
722	8.8424	7.3909	0.4892	+8.3010	9.6079	−9.4584	1.3020	8.5482
723	8.8402	7.4036	0.4853	−8.2735	9.6310	+9.4330	1.3019	8.5631
724	8.9491	7.5236	0.4809	−8.7704	9.5439	+9.8209	1.3019	8.5742
725	8.8655	7.4475	0.4839	−8.4879	9.6128	+9.6220	1.3019	8.5817
726	8.9045	7.4967	0.4822	−8.6509	9.5821	+9.7461	1.3019	8.5919
727	9.5231	8.1482	0.5247	+9.5142	8.6571	−9.9908	1.3018	8.6247
728	9.1690	7.8238	0.5038	+9.1195	9.1920	−9.9501	1.3018	8.6543
729	8.8233	7.5510	0.4872	−6.2885	9.6377	+7.4646	1.3016	8.7271
730	8.8244	7.5696	0.4865	−7.6838	9.6403	+8.8587	1.3016	8.7445
731	8.8466	7.6053	0.4836	−8.3508	9.6330	+9.5035	1.3015	8.7580
732	9.1186	7.8829	0.5055	+9.0543	9.2299	−9.9349	1.3015	8.7636
733	8.8719	7.6872	0.4810	−8.5241	9.6202	+9.6513	1.3013	8.8144
734	8.8726	7.7380	0.4801	−8.5286	9.6233	+9.6549	1.3011	8.8642
735	9.1551	8.0564	0.5149	+9.1022	9.1189	−9.9457	1.3009	8.8999
736	9.1547	8.0578	0.5150	+9.1018	9.1183	−9.9456	1.3008	8.9016
737	8.8769	7.7941	0.4788	−8.5499	9.6249	+9.6716	1.3007	8.9157
738	8.8810	7.8109	0.4782	−8.5679	9.6238	+9.6854	1.3007	8.9284
739	8.8750	7.8056	0.4787	−8.5417	9.6272	+9.6651	1.3007	8.9290
740	9.0088	7.9497	0.5060	+8.8892	9.3353	−9.8787	1.3006	8.9392
741	8.9269	7.8778	0.5003	+8.7183	9.4564	−9.7896	1.3005	8.9492
742	8.8384	7.8242	0.4923	+8.2701	9.5931	−9.4298	1.3002	8.9838
743	9.0773	8.0780	0.5146	+8.9972	9.1989	−9.9178	1.3001	8.9986
744	9.3157	8.3297	0.5411	+9.2922	8.6212	−9.9742	1.2999	9.0117
745	8.8373	7.8652	0.4927	+8.2604	9.5914	−9.4207	1.2998	9.0255
746	8.8556	7.9209	0.4962	+8.4396	9.5519	−9.5811	1.2993	9.0624
747	8.9511	8.0179	0.4669	−8.7781	9.6014	+9.8240	1.2993	9.0639
748	9.2999	8.3739	0.4188	−9.2745	9.4067	+9.9716	1.2992	9.0710
749	8.8583	7.9376	0.4773	−8.4582	9.6489	+9.5968	1.2991	9.0762
750	−9.2528	−8.3467	+0.5422	+9.2209	−8.6618	−9.9648	+1.2989	−9.0906

Catalogue of 1500 Stars.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°		″	″
751	4251		Centauri,	τ	5	12	29	31.49	+3.241	137	42	49.2	+19.84
752	4257	26	Virginis,	χ	5		31	30.74	3.094	97	10	7.4	19.91
753	4262		Centauri,		5		31	46.59	3.220	129	9	37.5	19.92
754	4264		Centauri,	γ	3		33	16.31	3.266	138	8	6.9	19.89
755	4268	29	Virginis,	γ	4		34	3.69*	3.040	90	37	33.7*	19.85
756	4271	30	Virginis,	ρ	5		34	17.41	3.037	78	56	9.4	19.92
757	4280		Muscæ,	β	4		37	8.73	3.577	157	17	8.2	19.80
758	4289		Crucis,	β	2		38	59.94	3.443	148	52	0.4	19.71
759	4290	27	Comæ,		5		39	9.07	3.003	72	36	4.7	19.75
760	4293		Octantis,	ι	5		39	45.57	5.396	174	18	14.1	19.42
761	4321		Centauri,		5		45	8.75	3.287	129	21	42.1	19.77
762	4325		Centauri,		5		45	49.29	3.473	146	21	40.3	19.57
763	4328	35	Comæ,		5		45	54.41*	2.960	67	56	19.3*	19.69
764	4330	40	Virginis,	ψ	5		46	33.41*	3.116	98	43	23.4*	19.66
765	4335	77	Ursæ Majoris,	ε	3		47	24.86*	2.668	33	13	30.6*	19.69
766	4339		Ursæ Minoris,		5½		47	57.88*	0.304	5	45	58.9*	19.63
767	4340	43	Virginis,	δ	3		48	2.89*	3.023	85	47	11.6*	19.71
768	4342		Ursæ Minoris,		5½		48	5.49*	0.288	5	46	17.0*	19.60
769	4346	12	Canum Venat.,	α	2½		49	0.17*	2.823	50	52	13.9*	19.56
770	4351	36	Comæ,		4½		51	30.25	2.974	71	46	48.7	19.49
771	4353		Muscæ,	δ	4		52	2.64	3.995	160	44	16.9	19.55
772	4360	37	Comæ,		5		53	5.57	2.885	58	24	13.4	19.50
773	4366	78	Ursæ Majoris,		5		54	16.67*	2.601	32	49	26.6*	19.48
774	4367	47	Virginis,	ε	3		54	42.83	2.993	78	13	58.3	19.46
775	4379		Centauri,	ξ²	5		58	11.03	3.464	139	6	5.4	19.62
776	4384	14	Canum Venat.,		5		58	43.20*	2.825	53	23	50.6*	19.39
777	4387	39	Comæ,		5		59	2.53	2.932	68	2	22.6	19.42
778	4390	41	Comæ,		4		59	58.69*	2.888	61	34	8.4*	19.46
779	4391	49	Virginis,	g	5	13	0	2.60*	3.137	99	56	13.7*	19.40
780	4395	45	Hydræ,	ψ	4½		0	59.31	3.219	112	18	50.9	19.41
781	4401	51	Virginis,	θ	4½		2	11.26*	3.101	94	44	12.9*	19.37
782	4406	42	Comæ,	α	4½		2	41.49	2.924	71	40	33.6*	19.17
783	4409		Centauri,		5		2	50.02	3.398	132	34	4.1	19.38
784	4418	53	Virginis,		5		4	4.93*	3.181	105	23	17.1*	19.57
785	4421	43	Comæ,	β	4½		4	52.01*	2.811	61	21	37.3*	18.35
786	4426		Muscæ,	η	5		5	9.35	3.943	157	5	45.4	19.09
787	4433		Canum Venat.,		5		6	54.62*	2.734	49	3	6.7*	19.27
788	4449	61	Virginis,		4½		10	33.88*	3.129	107	28	31.6*	20.14
789	4450	46	Hydræ,	γ	4		10	46.68	3.247	112	22	39.5	19.11
790	4451	20	Canum Venat.,		5		10	48.56*	2.703	48	38	10.5*	19.08
791	4456	21	Canum Venat.,		5		11	51.25	2.571	39	31	41.2	19.11
792	4458		Centauri,	ι	3		12	11.09	3.348	125	55	12.2*	19.21
793	4480	67	Virginis,	α	1		17	17.79*	3.152	100	22	36.5*	18.98
794	4483		Octantis,	κ	5		17	38.76	8.075	175	0	59.7	19.54
795	4484	79	Ursæ Majoris,	ζ	3		17	52.63*	2.437	34	17	24.0*	18.95
796	4492	68	Virginis,	i	5		18	48.23	3.161	101	55	30.5	18.92
797	4493	80	Ursæ Majoris,	g	5		19	12.48*	2.424	34	13	44.0*	18.92
798	4507		Centauri,		4½		22	21.97	3.445	128	37	48.8	18.82
799	4532	79	Virginis,	ζ	4		27	3.24*	3.055	89	49	38.2*	18.57
800	4538	24	Canum Venat.,		5	13	28	18.98*	+2.466	40	12	56.5*	+18.60

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
751	−8.9924	−8.1048	+0.5132	+8.8615	−9.2945	−9.8655	+1.2986	−9.1088
752	8.8232	7.9643	0.4905	+7.9194	−9.6165	−9.0921	1.2981	9.1369
753	8.9302	8.0749	0.5080	+8.7306	−9.3966	−9.7962	1.2980	9.1405
754	8.9950	8.1599	0.5167	+8.8669	−9.2598	−9.8674	1.2976	9.1604
755	8.8191	7.9944	0.4875	+8.8577	−9.6358	−8.0337	1.2974	9.1705
756	8.8272	8.0054	0.4817	−8.1103	−9.6575	+9.2782	1.2973	9.1734
757	9.2314	8.4450	0.5546	+9.1964	−8.2718	−9.9592	1.2965	9.2078
758	9.1041	8.3392	0.5372	+9.0365	−8.8998	−9.9261	1.2959	9.2288
759	8.8379	8.0747	0.4770	−8.3136	−9.6674	+9.4693	1.2959	9.2305
760	9.8206	9.0643	0.7311	+9.8185	+9.1096	−9.9913	1.2957	9.2371
761	8.9272	8.2273	0.5166	+8.7294	−9.3251	−9.7938	1.2937	9.2916
762	9.0717	8.3784	0.5403	+8.9921	−8.8756	−9.9117	1.2935	9.2980
763	8.8482	8.1557	0.4717	−8.4229	−9.6784	+9.5660	1.2935	9.2988
764	8.8199	8.1338	0.4930	+8.0008	−9.6002	−9.1718	1.2932	9.3048
765	9.0758	8.3978	0.4235	−8.9983	−9.6124	+9.9131	1.2929	9.3126
766	9.8123	9.1393	9.5061	−9.8101	−9.3985	+9.9882	1.2926	9.3175
767	8.8155	8.1434	0.4843	−7.6816	−9.6514	+8.8565	1.2926	9.3183
768	9.8119	9.1401	9.4994	−9.8097	−9.3993	+9.9882	1.2926	9.3186
769	8.9242	8.2610	0.4533	−8.7243	−9.6726	+9.7901	1.2922	9.3268
770	8.8352	8.1943	0.4731	−8.3303	−9.6830	+9.4840	1.2912	9.3480
771	9.2942	8.6580	0.5946	+9.2692	+8.8407	−9.9637	1.2909	9.3525
772	8.8818	8.2546	0.4597	−8.6011	−9.6901	+9.7075	1.2905	9.3610
773	9.0776	8.4603	0.4123	−9.0020	−9.6359	+9.9122	1.2899	9.3704
774	8.8206	8.2069	0.4778	−8.1301	−9.6749	+9.2970	1.2897	9.3738
775	8.9937	8.4078	0.5389	+8.8722	−8.9745	−9.8643	1.2881	9.4000
776	8.9049	8.3232	0.4501	−8.6803	−9.6985	+9.7610	1.2878	9.4039
777	8.8421	8.2628	0.4673	−8.4149	−9.6971	+9.5583	1.2877	9.4062
778	8.8647	8.2926	0.4599	−8.5424	−9.7031	+9.6627	1.2872	9.4129
779	8.8154	8.2438	0.4957	+8.0524	−9.5830	−9.2219	1.2871	9.4133
780	8.8422	8.2776	0.5072	+8.4216	−9.4794	−9.5639	1.2867	9.4199
781	8.8092	8.2536	0.4914	+7.7261	−9.6132	−8.9007	1.2860	9.4282
782	8.8301	8.2781	0.4700	−8.3275	−9.6963	+9.4810	1.2858	9.4316
783	8.9402	8.3893	0.5319	+8.7705	−9.1351	−9.8137	1.2857	9.4326
784	8.8226	8.2807	0.5014	+8.2464	−9.5381	−9.4066	1.2850	9.4409
785	8.8630	8.3266	0.4574	−8.5436	−9.7116	+9.6630	1.2846	9.4461
786	9.2160	8.6817	0.5975	+9.1803	+8.9518	−9.9465	1.2844	9.4479
787	8.9270	8.4049	0.4372	−8.7435	−9.7130	+9.7977	1.2835	9.4591
788	8.8235	8.3260	0.5049	+8.3010	−9.5091	−9.4566	1.2813	9.4816
789	8.8369	8.3407	0.5103	+8.4175	−9.4558	−9.5596	1.2812	9.4828
790	8.9275	8.4315	0.4334	−8.7476	−9.7215	+9.7990	1.2812	9.4830
791	8.9984	8.5093	0.4101	−8.8857	−9.7111	+9.8655	1.2805	9.4891
792	8.8936	8.4066	0.5277	+8.6620	−9.2297	−9.7465	1.2803	9.4911
793	8.8059	8.3508	0.4985	+8.0615	−9.5650	−9.2304	1.2770	9.5197
794	8.8593	9.4063	0.9118	+9.8576	+9.4675	−9.9729	1.2768	9.5216
795	9.0475	8.5960	0.3833	−8.9646	−9.7159	+9.8915	1.2767	9.5228
796	8.8072	8.3612	0.5005	+8.1224	−9.5501	−9.2890	1.2760	9.5278
797	9.0473	8.6037	0.3811	−8.9647	−9.7195	+9.8909	1.2758	9.5299
798	8.9025	8.4773	0.5374	+8.6979	−9.0770	−9.7667	1.2736	9.5462
799	8.7918	8.3930	0.4870	−6.2719	−9.6386	+7.4679	1.2701	9.5691
800	−8.9808	−8.5890	+0.3938	−8.8637	−9.7537	+9.8498	+1.2691	−9.5751

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	'	"	"
801	4549		Centauri,	ε	3	13	30	25.18	+3.735	142	42	3.5	+18.59
802	4552	25	Canum Venat.,		5		30	46.67	2.681	52	56	25.8*	18.50
803	4568	83	Ursæ Majoris,		5		35	2.54*	2.294	34	33	28.4*	18.41
804	4579	1	Centauri,	i	5		37	10.64*	3.389	122	16	59.5*	18.47
805	4580		Centauri,		5		37	11.59	3.745	140	40	36.7	18.12
806	4597	4	Bootis,	τ	5		40	8.04*	2.856	71	47	36.6*	18.12
807	4601		Centauri,	ν	3½		40	31.88	3.562	130	56	15.5	18.25
808	4602		Centauri,	μ	3½		40	36.11	3.570	131	43	26.6	18.25
809	4603	2	Centauri,	g	5		40	46.47*	3.452	123	42	0.1*	18.30
810	4607	85	Ursæ Majoris,	η	2½		41	37.27*	2.352	39	56	11.1*	18.15
811	4615	5	Bootis,	υ	4		42	14.59	2.895	73	27	20.4	18.04
812	4623	3	Centauri,	k	4½		43	11.00*	3.443	122	14	54.6*	18.17
813	4629	4	Centauri,	h	5		44	35.42*	3.431	121	11	5.6*	18.06
814	4638		Centauri,	ζ	3		46	12.67	3.693	136	32	49.6	18.05
815	4646	10	Draconis,	i	4½		47	3.14*	1.755	24	32	5.4*	17.97
816	4648	8	Bootis,	η	3		47	32.54*	2.862	70	50	54.2*	18.24
817	4653		Centauri,	φ	4½		49	10.53	3.608	131	21	58.7	18.01
818	4654		Centauri,	υ^1	5		49	26.29	3.654	134	4	9.7	18.06
819	4656	9	Bootis,		5		49	43.15	2.746	61	46	13.2	17.87
820	4660		Apodis,	θ	5		50	54.43	5.527	166	4	14.9	18.05
821	4668		Centauri,	υ^2	5		52	23.67	3.695	134	52	28.7	17.81
822	4669		Centauri,	β	1		53	17.41	4.145	149	38	44.6	17.72
823	4672	93	Virginis,	τ	4½		54	1.07	3.050	87	43	37.5	17.69
824	4681		Centauri,	χ	5		56	54.81	3.636	130	27	31.9	17.70
825	4685	49	Hydræ,	π	4½		57	50.59	3.398	115	57	23.9	17.60
826	4686	5	Centauri,	θ	2½		57	51.34*	3.505	125	37	49.4*	18.09
827	4692		Apodis,	η	5		59	45.17	6.914	170	17	43.0	17.13
828	4696	11	Draconis,	α	3½	14	0	19.87*	1.618	24	54	21.4*	17.37
829	4705		Octantis,	δ	5		3	29.22	8.524	172	58	23.8	17.24
830	4708	50	Hydræ,		5		4	11.19	3.417	116	33	7.3	17.22
831	4712		Apodis,	ε	5		4	39.94	6.829	169	24	20.9	16.62
832	4716	98	Virginis,	κ	4		4	54.02*	3.195	99	34	23.5*	17.14
833	4726	17	Bootis,	κ	5		8	6.41*	2.159	37	30	24.0*	17.03
834	4727	99	Virginis,	ι	4		8	9.33*	3.142	95	16	55.1*	17.41
835	4729	16	Bootis,	α	1		8	49.24*	2.734	70	2	3.9*	18.93
836	4732		Ursæ Minoris,		5		9	19.15	+1.110	19	51	44.6*	17.01
837	4733	4	Ursæ Minoris,		Var.		9	30.89	−0.387	11	44	51.6*	16.95
838	4734		Lupi,	ι	4½		9	49.72	+3.806	135	21	48.9	17.12
839	4741	19	Bootis,	λ	4		10	40.70*	2.288	43	13	15.8*	16.75
840	4742	21	Bootis,	ι	4		10	51.01*	2.130	37	56	21.3*	16.80
841	4743	100	Virginis,	λ	4		11	0.01*	3.237	102	40	40.8*	16.84
842	4745		Centauri,	ψ	5		11	27.41	3.619	127	11	33.9	16.93
843	4759		Centauri,		5		13	48.86	3.659	128	49	21.3	16.73
844	4768		Lupi,	τ^1	5		16	32.09	3.812	134	32	24.0	16.78
845	4770		Lupi,	τ^2	5		16	33.81	3.822	134	41	53.5	16.71
846	4789	23	Bootis,	θ	4		20	5.32*	2.045	37	27	15.0*	16.85
847	4792	105	Virginis,	φ	5		20	28.88	3.090	91	33	9.4	16.42
848	4801		Lupi,	σ	5		22	32.78	3.987	139	47	17.2	16.24
849	4808	25	Bootis,	ρ	4		25	21.85*	2.590	58	58	4.0*	16.02
850	4811		Centauri,	η	3	14	26	0.24	+3.770	131	29	44.6	+16.24

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
801	−9.0067	−8.6262	+0.5735	+8.9074	+8.6314	−9.8659	+1.2675	−9.5848
802	8.8869	8.5083	0.4283	−8.6670	−9.7625	+9.7451	1.2672	9.5864
803	9.0317	8.6754	0.3596	−8.9474	−9.7619	+9.8772	1.2638	9.6052
804	8.8566	8.5112	0.5338	+8.5842	−9.1676	−9.6873	1.2619	9.6143
805	8.9818	8.6364	0.5731	+8.8703	+8.6365	−9.8482	1.2619	9.6144
806	8.8034	8.4727	0.4601	−8.2981	−9.7359	+9.4519	1.2594	9.6265
807	8.9025	8.5738	0.5518	+8.7189	−8.6946	−9.7732	1.2590	9.6281
808	8.9077	8.5794	0.5536	+8.7309	−8.6128	−9.7799	1.2590	9.6284
809	8.8604	8.5329	0.5378	+8.6046	−9.0962	−9.7008	1.2588	9.6291
810	8.9723	8.6489	0.3776	−8.8569	−9.7836	+9.8405	1.2581	9.6325
811	8.7975	8.4772	0.4623	−8.2520	−9.7315	+9.4098	1.2575	9.6350
812	8.8511	8.5353	0.5363	+8.5783	−9.1297	−9.6816	1.2566	9.6387
813	8.8448	8.5358	0.5349	+8.5590	−9.1556	−9.6673	1.2553	9.6441
814	8.9381	8.6368	0.5684	+8.7990	+8.4150	−9.8125	1.2538	9.6503
815	9.1564	8.8591	0.2435	−9.1153	−9.7707	+9.9097	1.2530	9.6535
816	8.7990	8.5040	0.4565	−8.3149	−9.7469	+9.4663	1.2525	9.6554
817	8.8973	8.6100	0.5576	+8.7174	−8.3560	−9.7689	1.2510	9.6614
818	8.9160	8.6298	0.5641	+8.7583	+7.8921	−9.7908	1.2507	9.6624
819	8.8271	8.5423	0.4377	−8.5020	−9.7783	+9.6231	1.2504	9.6634
820	9.3894	9.1101	0.7465	+9.3765	+9.5405	−9.9341	1.2493	9.6677
821	8.9190	8.6465	0.5679	+8.7676	+8.3945	−9.7941	1.2478	9.6730
822	9.0650	8.7965	0.6184	+9.0009	+9.2790	−9.8806	1.2469	9.6762
823	8.7681	8.5030	0.4836	−7.3665	−9.6556	+8.5423	1.2461	9.6787
824	8.8835	8.6313	0.5596	+8.6956	−8.1239	−9.7530	1.2431	9.6887
825	8.8100	8.5619	0.5303	+8.4511	−9.2428	−9.5810	1.2421	9.6918
826	8.8538	8.6059	0.5493	+8.6191	−8.8202	−9.7052	1.2421	9.6919
827	9.5349	9.2953	0.8434	+9.5287	+9.6219	−9.9315	1.2400	9.6982
828	9.1367	8.8995	0.2115	−9.0943	−9.8044	+9.8948	1.2394	9.7001
829	9.6700	9.4466	0.9363	+9.6668	+9.6592	−9.9304	1.2359	9.7102
830	8.8052	8.5847	0.5334	+8.4555	−9.1978	−9.5832	1.2351	9.7124
831	9.4918	9.2734	0.8306	+9.4843	+9.6324	−9.9248	1.2345	9.7139
832	8.7620	8.5447	0.5035	+7.9830	−9.5336	−9.1530	1.2343	9.7147
833	8.9677	8.7639	0.3318	−8.8671	−9.8356	+9.8277	1.2305	9.7245
834	8.7540	8.5504	0.4964	+7.7181	−9.5835	−8.8923	1.2305	9.7247
835	8.7783	8.5775	0.4490	−8.3116	−9.7708	+9.4608	1.2297	9.7267
836	9.2196	9.0209	+0.0379	−9.1930	−9.8132	+9.9002	1.2291	9.7282
837	9.4417	9.2439	−9.5707	−9.4325	−9.7873	+9.9174	1.2288	9.7288
838	8.9034	8.7069	+0.5795	+8.7557	+8.8865	−9.7785	1.2285	9.7297
839	8.9136	8.7206	0.3622	−8.7761	−9.8392	+9.7878	1.2274	9.7322
840	8.9602	8.7679	0.3312	−8.8571	−9.8407	+9.8219	1.2272	9.7327
841	8.7594	8.5678	0.5097	+8.1008	−9.4839	−9.2662	1.2270	9.7332
842	8.8469	8.6572	0.5589	+8.6283	−8.2480	−9.7057	1.2265	9.7345
843	8.8537	8.6737	0.5639	+8.6509	+7.8751	−9.7186	1.2236	9.7414
844	8.8889	8.7200	0.5808	+8.7348	+8.9258	−9.7639	1.2201	9.7491
845	8.8900	8.7213	0.5813	+8.7372	+8.9360	−9.7650	1.2201	9.7491
846	8.9532	8.7987	0.3158	−8.8529	−9.8570	+9.8130	1.2155	9.7588
847	8.7368	8.5840	0.4902	+7.1697	−9.6214	−8.3456	1.2149	9.7599
848	8.9239	8.7794	0.6012	+8.8068	+9.2167	−9.7929	1.2122	9.7654
849	8.7971	8.6638	0.4140	−8.5093	−9.8309	+9.6183	1.2083	9.7728
850	−8.8546	−8.7238	+0.5768	+8.6758	+8.8395	−9.7264	+1.2074	−9.7744

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
851	4812	27	Bootis,	γ	3½	14	26	2.20*	+2.429	51	2	0.1*	+15.98
852	4821		Lupi,	ρ	5		27	49.93	+3.998	138	46	7.9	16.25
853	4822	5	Ursæ Minoris,		4		27	54.43*	—0.244	13	38	14.3*	16.05
854	4823	28	Bootis,	σ	5		28	9.02*	+2.615	59	36	3.7	15.89
855	4831		Centauri,	α^1	4		29	26.46	4.018	150	12	53.8	15.11
856	4832		Centauri,	α^2	1		29	28.00	4.018	150	12	37.4	15.11
857	4833		Apodis,	α	4½		29	29.31	7.009	168	24	1.9	15.89
858	4835		Circini,	α	4		30	27.77	4.739	154	19	0.9	16.15
859	4839		Lupi,	α	3		31	58.96	3.941	136	44	26.2	15.88
860	4842		Centauri,		5		32	39.59	3.697	127	8	45.7	15.83
861	4847	29	Bootis,	π	3½		33	40.55	2.816	72	56	7.1	15.69
862	4849	30	Bootis,	ζ	3½		33	59.35*	2.861	75	37	31.6*	15.70
863	4850	31	Bootis,		5		34	16.99	2.947	81	11	36.9	15.70
864	4852		Centauri,		5		34	30.02	3.637	124	31	26.3	15.93
865	4855	107	Virginis,	μ	4½		35	9.62*	3.155	95	0	12.2*	15.96
866	4864	34	Bootis,		4½		36	49.91	2.642	62	49	53.6	15.53
867	4873	35	Bootis,	ο	4½		38	14.67	2.800	72	23	49.6	15.48
868	4876	36	Bootis,	ε	3		38	26.14*	2.622	62	17	27.3*	15.45
869	4878	109	Virginis,		4		38	40.24	3.029	87	28	17.4	15.46
870	4880	56	Hydræ,		5		39	0.00	3.478	115	27	20.5*	15.51
871	4882	57	Hydræ,		5		39	11.59*	3.483	116	0	50.9*	15.48
872	4890	7	Libræ,	μ	5		41	6.22*	3.279	103	31	14.0*	15.31
873	4891	58	Hydræ,		5		41	29.44*	3.503	117	19	55.8*	15.32
874	4892		Lupi,	ο	5		41	52.47	3.885	132	57	3.2	15.41
875	4895	9	Libræ,	α	3		42	35.31*	3.309	105	24	54.7*	15.29
876	4905	37	Bootis,	ξ	3½		44	28.28	2.767	70	16	28.4	15.25
877	4922	15	Libræ,	ξ^2	5		48	38.13*	3.245	100	48	4.4*	14.90
878	4924		Lupi,	β	3		48	43.91	3.891	132	31	32.0	14.98
879	4928		Centauri,	κ	3		49	25.53	+3.865	131	29	52.6	14.81
880	4936	7	Ursæ Minoris,	β	3		51	12.03*	—0.273	15	13	53.8*	14.78
881	4939	19	Libræ,	δ	4½		52	57.84*	+3.197	97	55	13.0*	14.62
882	4948		Lupi,	π	5		54	55.81	4.039	136	27	36.4	14.63
883	4949		Draconis,		5		55	12.83	0.935	23	28	9.4	14.42
884	4950	20	Libræ,		3½		55	18.10*	3.496	114	41	19.7*	14.50
885	4951	110	Virginis,		5		55	19.57	3.027	87	18	57.7	14.46
886	4958	42	Bootis,	β	3		56	17.80*	2.264	49	0	55.2*	14.46
887	4969	43	Bootis,	ψ	5		58	1.19	2.572	62	27	53.6*	14.30
888	4970	21	Libræ,	ν^1	5		58	16.19	3.335	105	40	16.8	14.32
889	4973		Lupi,	λ	5		58	45.79	3.996	134	41	57.5	14.48
890	4974	44	Bootis,	ι^2	5		58	50.82*	1.978	41	45	35.0*	14.22
891	4981	45	Bootis,	c	5	15	0	42.75	+2.632	64	32	36.0	14.30
892	4982		Ursæ Minoris,		5		0	44.05	—4.797	6	52	24.6	14.14
893	4986		Lupi,	κ	5		1	31.97	+4.115	138	9	44.8	14.19
894	4987		Lupi,	ζ	4		1	32.30	4.250	141	31	27.9	14.31
895	5005		Triang. Aust.,	γ	3		4	59.47	5.456	158	7	9.5	13.92
896	5011		Circini,	β	5		5	48.85	4.613	148	14	4.8	13.85
897	5028		Lupi,	μ	5		8	7.65	4.127	137	19	6.9	13.85
898	5031	48	Bootis,	χ	5		8	12.96	2.508	60	16	35.8	13.65
899	5032	2	Lupi,		4½		8	42.96*	3.631	119	35	33.9*	13.69
900	5034	27	Libræ,	β	2½	15	8	56.44*	+3.220	98	49	33.1*	+13.62

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
851	−8.8383	−8.7077	+0.3851	−8.6369	−9.8522	+9.7037	+1.2073	−9.7745
852	8.9076	8.7840	+0.6007	+8.7838	+9.2206	−9.7788	1.2048	9.7790
853	9.3539	9.2306	−9.3876	−9.3415	−9.8368	+9.8901	1.2047	9.7792
854	8.7903	8.6680	+0.4147	−8.4945	−9.8319	+9.6063	1.2044	9.7798
855	9.0281	8.9108	0.6521	+8.9665	+9.4929	−9.8387	1.2025	9.7831
856	9.0280	8.9109	0.6521	+8.9664	+9.4929	−9.8387	1.2025	9.7831
857	9.4208	9.3038	0.8466	+9.4118	+9.7053	−9.8913	1.2024	9.7832
858	9.0858	8.9726	0.6783	+9.0406	+9.5587	−9.8536	1.2010	9.7856
859	8.8846	8.7773	0.5961	+8.7469	+9.1787	−9.7589	1.1988	9.7893
860	8.8180	8.7133	0.5678	+8.5989	+8.4346	−9.6765	1.1978	9.7909
861	8.7375	8.6368	0.4496	−8.2050	−9.7762	+9.3616	1.1963	9.7934
862	8.7313	8.6318	0.4560	−8.1262	−9.7596	+9.2885	1.1958	9.7941
863	8.7222	8.6239	0.4686	−7.9071	−9.7196	+9.0781	1.1953	9.7948
864	8.8008	8.7034	0.5617	+8.5542	−7.5682	−9.6462	1.1950	9.7953
865	8.7174	8.6225	0.4975	+7.6579	−9.5772	−9.8323	1.1940	9.7969
866	8.7639	8.6755	0.4211	−8.4234	−9.8291	+9.5488	1.1914	9.8008
867	8.7318	8.6488	0.4472	−8.2124	−9.7826	+9.3676	1.1892	9.8041
868	8.7635	8.6813	0.4188	−8.4310	−9.8330	+9.5542	1.1889	9.8045
869	8.7107	8.6294	0.4819	−7.3553	−9.6644	+8.5310	1.1886	9.8051
870	8.7541	8.6741	0.5413	+8.3874	−9.0682	−9.5191	1.1881	9.8058
871	8.7558	8.6765	0.5426	+8.3979	−9.0382	−9.5276	1.1878	9.8062
872	8.7186	8.6467	0.5157	+8.0874	−9.4327	−9.2513	1.1847	9.8106
873	8.7572	8.6867	0.5463	+8.4191	−8.9450	−9.5438	1.1841	9.8114
874	8.8407	8.7717	0.5887	+8.6741	+9.0955	−9.7146	1.1835	9.8123
875	8.7199	8.6537	0.5199	+8.1445	−9.3897	−9.3047	1.1823	9.8139
876	8.7272	8.6681	0.4402	−8.2555	−9.7997	+9.4053	1.1792	9.8180
877	8.7017	8.6585	0.5108	+7.9744	−9.4778	−9.1428	1.1722	9.8269
878	8.8263	8.7835	0.5904	+8.6562	+9.1274	−9.6997	1.1720	9.8271
879	8.8181	8.7779	+0.5874	+8.6393	+9.0842	−9.6899	1.1709	9.8285
880	9.2699	9.2366	−9.4242	−9.2544	−9.8863	+9.8500	1.1678	9.8322
881	8.6905	8.6638	+0.5049	+7.8297	−9.5256	−9.0016	1.1646	9.8357
882	8.8447	8.8254	0.6067	+8.7049	+9.3058	−9.7192	1.1611	9.8397
883	9.0821	9.0640	9.9727	−9.0446	−9.9073	+9.8209	1.1606	9.8402
884	8.7237	8.7059	0.5436	+8.3446	−9.0216	−9.4791	1.1604	9.8404
885	8.6826	8.6648	0.4810	−7.3530	−9.6683	+8.5286	1.1604	9.8405
886	8.8024	8.7884	0.3546	−8.6192	−9.8942	+9.6732	1.1586	9.8424
887	8.7293	8.7218	0.4120	−8.3942	−9.8506	+9.5181	1.1554	9.8457
888	8.6931	8.6865	0.5229	+8.1247	−9.3583	−9.2843	1.1550	9.8462
889	8.8240	8.8193	0.6022	+8.6712	+9.2700	−9.6990	1.1540	9.8471
890	8.8521	8.8477	0.3047	−8.7248	−9.9105	+9.7244	1.1539	9.8473
891	8.7164	8.7191	+0.4182	−8.3497	−9.8431	+9.4814	1.1504	9.8508
892	9.5940	9.5968	−0.6810	−9.5909	−9.8786	+9.8450	1.1503	9.8509
893	8.8463	8.8521	+0.6163	+8.7185	+9.3817	−9.7187	1.1488	9.8524
894	8.8765	8.8824	0.6302	+8.7702	+9.4583	−9.7402	1.1488	9.8524
895	9.0924	9.1113	0.7384	+9.0599	+9.7064	−9.8074	1.1420	9.8587
896	8.9407	8.9628	0.6661	+8.8703	+9.5886	−9.7677	1.1404	9.8602
897	8.8263	8.8571	0.6161	+8.6926	+9.3869	−9.6999	1.1357	9.8644
898	8.7185	8.7497	0.4000	−8.4138	−9.8691	+9.5287	1.1356	9.8645
899	8.7169	8.7500	0.5596	+8.4105	−8.1818	−9.5259	1.1345	9.8654
900	−8.6609	−8.6949	+0.5083	+7.8468	−9.5000	−9.0178	+1.1341	−9.8658

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°		″	″
901	5036	49	Bootis,	δ	3½	15	9	27.44*	+2.421	56	7	21.8*	+13.67
902	5046		Lupi,	δ	4		11	32.50	3.898	130	6	4.1	13.65
903	5049		Lupi,	ν^1	5		11	42.44	4.138	137	22	36.1	13.67
904	5054		Lupi,	ϕ^1	5		12	18.66	3.791	125	42	44.9	13.46
905	5056		Lupi,	ε	4½		12	30.65	4.028	134	8	44.4	13.57
906	5060		Lupi,	ϕ^2	5		13	35.42	+3.802	126	18	58.8	13.39
907	5079	11	Ursæ Minoris,		5		17	15.32*	−0.086	17	37	53.5*	13.06
908	5084	51	Bootis,	μ	4		18	49.42*	+2.267	52	5	39.0*	12.89
909	5089	32	Libræ,	ζ^1	4		19	48.27*	+3.374	106	11	22.9*	12.94
910	5094	13	Ursæ Minoris,	γ	3½		21	0.38*	−0.146	17	37	56.0*	12.77
911	5097	12	Draconis,	ι	3		21	36.00*	+1.332	30	30	24.8*	12.76
912	5098	3	Coronæ Bor.,	β	4		21	38.99	2.480	60	22	27.3*	12.71
913	5103		Triang. Aust.,	ε	5		23	3.22	5.356	155	48	22.5	12.93
914	5118		Lupi,	γ	3		25	9.82	3.963	130	39	29.7	12.71
915	5125	37	Libræ,		4		25	59.19*	3.268	99	32	46.7*	12.72
916	5131	4	Coronæ Bor.,	θ	4½		26	52.95	2.415	58	7	52.5	12.44
917	5134	38	Libræ,	γ	4½		27	8.48*	3.344	104	17	6.7*	12.39
918	5135	13	Serpentis,	δ	3		27	38.64	2.866	78	57	20.3	12.32
919	5138	39	Libræ,		4		27	55.82*	3.624	117	38	1.5*	12.32
920	5139		Lupi,	ι	5		27	58.18	4.018	132	4	16.9	12.49
921	5143	5	Coronæ Bor.,	α	2½		28	20.25*	2.539	62	46	39.0*	12.39
922	5151	40	Libræ,		4½		29	27.28*	3.662	119	16	47.7*	12.29
923	5155	6	Coronæ Bor.,	μ	5		29	44.71*	2.200	50	29	20.8*	12.23
924	5165		Lupi,		5		30	53.87	4.090	134	9	32.9	12.49
925	5176	43	Libræ,	κ	5		33	18.75*	3.445	109	11	18.6*	12.06
926	5178	7	Coronæ Bor.,	ζ	5		33	44.09	2.271	52	52	28.2	12.05
927	5187	21	Serpentis,	ι	5		34	51.89	2.670	69	50	34.8	11.89
928	5190	44	Libræ,	η	4½		35	38.48*	+3.371	105	11	27.1*	11.86
929	5191	15	Ursæ Minoris,	θ	5		35	58.15*	−1.976	12	9	11.9*	11.79
930	5192	8	Coronæ Bor.,	γ	5		36	26.63	+2.519	63	13	31.5	11.68
931	5196	24	Serpentis,	α	2½		36	52.92*	2.953	83	5	56.0*	11.67
932	5214	27	Serpentis,	λ	4½		39	10.15*	2.912	82	10	23.9*	11.60
933	5216	28	Serpentis,	β	3½		39	15.99	2.767	74	6	16.2	11.56
934	5224		Triang. Aust.,	κ	5		40	46.23	5.836	158	8	52.2	11.46
935	5227	5	Lupi,	χ	4		41	26.51*	3.789	123	9	56.5*	11.46
936	5230	32	Serpentis,	μ	3½		41	47.94	3.128	92	58	0.4	11.38
937	5232	1	Scorpii,	b	5		41	58.05*	3.592	115	17	26.2*	11.40
938	5233		Triang. Aust.,	β	3		41	59.10	5.205	152	57	36.8	11.73
939	5234	35	Serpentis,	κ	4		41	59.54	2.700	71	23	28.4	11.41
940	5244	10	Coronæ Bor.,	δ	4½		43	18.29	2.512	63	28	6.7	11.30
941	5245	37	Serpentis,	ε	3		43	20.57	2.989	85	4	3.3	11.21
942	5250	2	Scorpii,	A	5		44	37.01*	3.587	114	52	29.1*	11.19
943	5251	45	Libræ,	λ	4		44	38.15*	3.471	109	42	50.7*	11.19
944	5252	38	Serpentis,	ρ	4½		44	40.80	2.635	68	34	1.4	11.11
945	5257	46	Libræ,	θ	4½		45	17.47*	3.410	106	17	5.3*	10.99
946	5259	11	Coronæ Bor.,	κ	5		45	34.78	2.256	53	52	25.6	11.43
947	5268		Lupi,	ξ	4½		47	18.95	3.817	123	31	16.4	10.98
948	5272	5	Scorpii,	ρ	4		47	37.93*	3.689	118	46	17.6*	10.97
949	5279		Draconis,		5		48	47.63*	1.401	33	43	42.6*	10.86
950	5284	41	Serpentis,	γ	3	15	49	31.72*	+2.769	73	50	40.4	+12.05

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
901	−8.7355	−8.7714	+0.3821	−8.4817	−9.8867	+9.5770	+1.1330	−9.8667
902	8.7668	8.8106	0.5917	+8.5758	+9.1641	−9.6355	1.1287	9.8703
903	8.8193	8.8638	0.6179	+8.6861	+9.4019	−9.6929	1.1284	9.8706
904	8.7392	8.7860	0.5782	+8.5054	+8.9133	−9.5911	1.1271	9.8716
905	8.7925	8.8400	0.6061	+8.6354	+9.3164	−9.6673	1.1267	9.8719
906	8.7398	8.7915	+0.5804	+8.5123	+8.9694	−9.5946	1.1244	9.8738
907	9.1568	9.2225	−9.0770	−9.1359	−9.9315	+9.7933	1.1164	9.8799
908	8.7375	8.8092	+0.3573	−8.5260	−9.9090	+9.5991	1.1129	9.8824
909	8.6500	8.7254	+0.5273	+8.0953	−9.3084	−9.2538	1.1107	9.8839
910	9.1484	9.2285	−9.2138	−9.1275	−9.9366	+9.7849	1.1080	9.8859
911	8.9228	9.0051	+0.1213	−8.8581	−9.9466	+9.7397	1.1066	9.8868
912	8.6891	8.7716	0.3953	−8.3831	−9.8798	+9.4983	1.1065	9.8869
913	9.0124	9.1004	0.7304	+8.9725	+9.7232	−9.7612	1.1033	9.8890
914	8.7400	8.8362	0.5984	+8.5540	+9.2533	−9.6101	1.0984	9.8923
915	8.6242	8.7235	0.5114	+7.8439	−9.4745	−9.0139	1.0964	9.8935
916	8.6869	8.7897	0.3835	−8.4095	−9.8938	+9.5147	1.0943	9.8949
917	8.6290	8.7328	0.5235	+8.0212	−9.3553	−9.1837	1.0936	9.8953
918	8.6222	8.7280	0.4572	−7.9046	−9.7610	+9.0725	1.0924	9.8960
919	8.6660	8.7729	0.5590	+8.3324	−8.2810	−9.4559	1.0917	9.8964
920	8.7427	8.8498	0.6044	+8.5688	+9.3137	−9.6155	1.0916	9.8965
921	8.6634	8.7719	0.4028	−8.3238	−9.8732	+9.4489	1.0907	9.8970
922	8.6691	8.7820	0.5640	+8.3584	+7.9445	−9.4752	1.0880	9.8987
923	8.7217	8.8357	0.3418	−8.5253	−9.9238	+9.5887	1.0873	9.8991
924	8.7505	8.8688	0.6132	+8.5935	+9.3856	−9.6254	1.0846	9.9007
925	8.6249	8.7529	0.5370	+8.1416	−9.1652	−9.2929	1.0784	9.9042
926	8.6974	8.8271	0.3537	−8.4781	−9.9195	+9.5558	1.0773	9.9048
927	8.6235	8.7578	0.4273	−8.1608	−9.8358	+9.3095	1.0744	9.9064
928	8.6095	8.7468	+0.5269	+8.0278	−9.3162	−9.1885	1.0724	9.9075
929	9.2699	9.4085	−0.2904	−9.2600	−9.9456	+9.7594	1.0715	9.9079
930	8.6412	8.7817	+0.4021	−8.2949	−9.8765	+9.4217	1.0703	9.9086
931	8.5940	8.7362	0.4683	−7.6737	−9.7229	+8.8467	1.0691	9.9092
932	8.5888	8.7402	0.4654	−7.7229	−9.7334	+8.8950	1.0631	9.9123
933	8.6014	8.7532	0.4408	−8.0390	−9.8071	+9.1981	1.0628	9.9124
934	9.0096	9.1675	0.7640	+8.9772	+9.7786	−9.7241	1.0587	9.9144
935	8.6558	8.8164	0.5786	+8.3939	+8.9385	−9.4927	1.0569	9.9153
936	8.5782	8.7402	0.4952	+7.2921	−9.5920	−8.4676	1.0559	9.9157
937	8.6209	8.7836	0.5552	+8.2515	−8.6064	−9.3839	1.0554	9.9160
938	8.9194	9.0823	0.7185	+8.8692	+9.7297	−9.7029	1.0554	9.9160
939	8.6004	8.7632	0.4313	−8.1043	−9.8289	+9.2571	1.0554	9.9160
940	8.6218	8.7899	0.4011	−8.2718	−9.8796	+9.3995	1.0517	9.9177
941	8.5749	8.7433	0.4735	−7.5093	−9.7020	+8.6838	1.0516	9.9177
942	8.6120	8.7856	0.5546	+8.2359	−8.6464	−9.3697	1.0480	9.9194
943	8.5959	8.7695	0.5401	+8.1240	−9.1082	−9.2738	1.0480	9.9194
944	8.6007	8.7745	0.4207	−8.1635	−9.8498	+9.3084	1.0479	9.9194
945	8.5856	8.7619	0.5309	+8.0334	−9.2629	−9.1917	1.0461	9.9202
946	8.6597	8.8372	0.3537	−8.4303	−9.9251	+9.5136	1.0453	9.9206
947	8.6410	8.8257	0.5811	+8.3831	+9.0022	−9.4802	1.0403	9.9228
948	8.6183	8.8043	0.5665	+8.3007	+8.3579	−9.4196	1.0394	9.9231
949	8.8132	9.0040	0.1422	−8.7331	−9.9732	+9.6537	1.0360	9.9246
950	−8.5730	−8.7669	+0.4384	−8.0175	−9.8137	+9.1761	+1.0338	−9.9255

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
951	5285	16	Ursæ Minoris,	ζ	4	15	49	31.76*	−2.316	11	44	48.0*	+10.81
952	5289	6	Scorpii,	π	3½		49	47.12*	+3.615	115	40	39.2*	10.85
953	5290	48	Libræ,		4½		49	47.74*	3.349	103	50	32.5*	10.78
954	5292		Lupi,	η	4½		50	11.69	3.946	127	57	44.5	10.87
955	5302	13	Coronæ Bor.,	ε	4½		51	22.73*	2.487	62	41	4.7*	10.70
956	5303	7	Scorpii,	δ	3		51	28.31*	3.535	112	11	24.8*	10.68
957	5322	44	Serpentis,	π	4½		55	50.14	2.581	66	46	31.7	10.26
958	5323		Normæ,	δ	5		55	54.22	4.198	134	45	41.1	10.49
959	5324	51	Libræ,		4½		56	7.67*	3.291	100	57	18.0	10.34
960	5329	8	Scorpii,	β^1	2		56	43.31*	3.478	109	23	25.3*	10.30
961	5331		Lupi,	θ	4½		56	45.38*	3.921	126	23	21.3*	10.35
962	5337	9	Scorpii,	ω^1	4½		58	2.51	3.502	110	15	27.2	10.17
963	5338	6	Herculis,	υ	5		58	7.43*	1.861	43	32	40.5*	10.24
964	5342	10	Scorpii,	ω^2	4½		58	36.94*	3.508	110	27	31.0*	10.15
965	5347		Scorpii,		5		58	59.55	3.642	115	55	13.3	10.17
966	5348	13	Draconis,	θ	3		59	5.22*	1.124	31	1	58.6*	9.78
967	5375		Triang. Aust.,	δ	4½	16	1	49.66	5.364	153	17	41.7	9.88
968	5381	13	Scorpii,	c^2	5		3	4.50*	3.689	117	31	55.1*	9.80
969	5382	14	Scorpii,	ν	4		3	17.04*	3.478	109	3	58.9*	9.75
970	5386	15	Scorpii,	ψ	5		3	48.53*	3.271	99	40	16.8	9.78
971	5388	11	Herculis,	φ	5		4	2.15	1.869	44	40	10.8	9.70
972	5406		Draconis,		5		5	55.91*	0.127	21	47	39.6*	9.50
973	5414	1	Ophiuchi,	δ	3		6	29.31*	3.138	93	18	14.5*	9.64
974	5420	18	Scorpii,		5		7	28.30*	3.250	97	58	3.9	9.96
975	5425		Normæ,	γ^2	5		8	38.46	4.455	139	46	55.2	9.52
976	5437	2	Ophiuchi,	ε	3		10	23.41*	3.168	94	19	22.2*	9.19
977	5439		Apodis,	γ	5		10	36.26	8.782	168	33	7.8	10.34
978	5447	20	Scorpii,	σ	4		12	4.73*	3.634	115	13	39.8*	9.10
979	5456	50	Serpentis,	σ	5		14	28.83	3.035	88	36	49.4	8.84
980	5459		Draconis,		5		14	45.56*	+0.983	29	51	47.5*	8.89
981	5462	19	Ursæ Minoris,		5		15	9.79*	−1.863	13	44	49.0*	8.86
982	5463	22	Herculis,	τ	4		15	13.93*	+1.799	43	19	37.6*	8.83
983	5466	20	Herculis,	γ	3½		15	18.29	2.645	70	29	28.4	8.80
984	5467	4	Ophiuchi,	ψ	5		15	19.86*	3.501	109	40	53.4*	8.90
985	5473	19	Coronæ Bor.,	ξ	5		16	15.27	2.340	58	45	22.3	8.63
986	5477	5	Ophiuchi,	ρ	5		16	35.90	3.586	113	5	44.8	8.72
987	5479	20	Coronæ Bor.,	ν^1	5		16	42.67*	2.261	55	50	41.2*	8.82
988	5480	21	Coronæ Bor.,	ν^2	5		16	50.23*	2.269	55	56	40.9*	8.75
989	5489	7	Ophiuchi,	χ	5		18	20.16*	3.468	108	6	40.1*	8.60
990	5490	24	Herculis,	ω	5		18	29.44	2.752	75	37	2.5	8.62
991	5494		Ophiuchi,		5		19	37.94	3.225	97	14	51.0	8.51
992	5495	3	Ophiuchi,	υ	5		19	41.78*	3.254	98	1	55.3*	8.50
993	5496	25	Herculis,		5		20	3.57	2.134	52	15	40.8	8.47
994	5498	21	Scorpii,	α	1		20	13.07*	3.668	116	5	38.9*	8.49
995	5502		Draconis,		5		21	8.75*	1.300	34	27	6.9*	8.39
996	5508		Scorpii,		4		21	35.40*	3.899	124	22	24.8*	8.43
997	5510		Apodis,	β	5		21	49.41	+8.313	167	11	26.5	8.63
998	5511	21	Ursæ Minoris,	η	5		21	57.21*	−1.839	13	54	4.8*	8.12
999	5512	14	Draconis,	η	3		21	58.36*	+0.820	28	8	42.4*	8.24
1000	5516	8	Ophiuchi,	φ	4½	16	22	33.61*	+3.430	106	16	50.9*	+ 8.30

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
951	−9.2468	−9.4406	−0.3701	−9.2376	−9.9605	+9.7224	+1.0338	−9.9255
952	8.5999	8.7948	+0.5578	+8.2367	−8.4133	−9.3676	1.0331	9.9258
953	8.5675	8.7625	0.5248	+7.9464	−9.3418	−9.1097	1.0330	9.9258
954	8.6568	8.8534	0.5967	+8.4458	+9.2480	−9.5186	1.0319	9.9263
955	8.6013	8.8029	0.3955	−8.2631	−9.8888	+9.3878	1.0283	9.9277
956	8.5831	8.7851	0.5481	+8.1603	−8.9138	−9.3029	1.0280	9.9278
957	8.5730	8.7935	0.4115	−8.1688	−9.8673	+9.3082	1.0146	9.9329
958	8.6848	8.9056	0.6240	+8.5324	+9.4703	−9.5598	1.0144	9.9330
959	8.5433	8.7651	0.5175	+7.8222	−9.4203	−8.9903	1.0137	9.9332
960	8.5588	8.7832	0.5409	+8.0800	−9.0941	−9.2307	1.0118	9.9339
961	8.6276	8.8520	0.5929	+8.4008	+9.2047	−9.4827	1.0117	9.9339
962	8.5570	8.7870	0.5436	+8.0964	−9.0362	−9.2447	1.0076	9.9354
963	8.6908	8.9212	0.2691	−8.5511	−9.9655	+9.5653	1.0073	9.9355
964	8.5557	8.7882	0.5442	+8.0992	−9.0212	−9.2470	1.0057	9.9360
965	8.5722	8.8064	0.5601	+8.2128	−8.1038	−9.3429	1.0045	9.9364
966	8.8136	9.0482	0.0609	−8.7465	−9.9841	+9.6349	1.0042	9.9365
967	8.8642	9.1108	0.7310	+8.8152	+9.7651	−9.6439	0.9951	9.9395
968	8.5648	8.8170	0.5657	+8.2297	+8.2695	−9.3536	0.9909	9.9408
969	8.5364	8.7895	0.5408	+8.0505	−9.0969	−9.2021	0.9902	9.9411
970	8.5164	8.7718	0.5145	+7.7417	−9.4486	−8.9115	0.9885	9.9416
971	8.6624	8.9188	0.2759	−8.5144	−9.9672	+9.5374	0.9877	9.9419
972	8.9331	9.1981	9.1222	−8.9009	−9.9899	+9.6467	0.9811	9.9438
973	8.5016	8.7690	0.4967	+7.2622	−9.5826	−8.4376	0.9792	9.9444
974	8.5016	8.7735	0.5100	+7.6434	−9.4880	−8.8153	0.9757	9.9454
975	8.6832	8.9605	0.6502	+8.5661	+9.5966	−9.5522	0.9715	9.9466
976	8.4881	8.7735	0.4997	+7.3653	−9.5632	−8.5402	0.9652	9.9483
977	9.1884	9.4747	0.9511	+9.1797	+9.8945	−9.6535	0.9644	9.9485
978	8.5242	8.8174	0.5602	+8.1538	−8.1072	−9.2864	0.9590	9.9500
979	8.4717	8.7763	0.4831	−6.8555	−9.6585	+8.0314	0.9499	9.9522
980	8.7734	9.0792	+9.9927	−8.7115	−9.9972	+9.5848	0.9489	9.9525
981	9.0932	9.4009	−0.2630	−9.0806	−9.9888	+9.6326	0.9474	9.9529
982	8.6323	8.9404	+0.2550	−8.4941	−9.9780	+9.5066	0.9470	9.9530
983	8.4941	8.8026	0.4225	−8.0178	−9.8505	+9.1682	0.9468	9.9530
984	8.4945	8.8031	0.5440	+8.0218	−9.0265	−9.1718	0.9467	9.9530
985	8.5328	8.8459	0.3694	−8.2477	−9.9229	+9.3558	0.9431	9.9539
986	8.4997	8.8144	0.5543	+8.0933	−8.6646	−9.2331	0.9417	9.9542
987	8.5452	8.8605	0.3530	−8.2945	−9.9368	+9.3884	0.9413	9.9543
988	8.5442	8.8601	0.3535	−8.2924	−9.9364	+9.3867	0.9408	9.9544
989	8.4786	8.8018	0.5398	+7.9712	−9.1196	−9.1252	0.9349	9.9558
990	8.4698	8.7937	0.4410	−7.8649	−9.8094	+9.0272	0.9343	9.9559
991	8.4548	8.7843	0.5085	+7.5557	−9.5000	−8.7284	0.9297	9.9570
992	8.4554	8.7852	0.5108	+7.6006	−9.4817	−8.7724	0.9294	9.9570
993	8.5515	8.8832	0.3289	−8.3383	−9.9533	+9.4125	0.9279	9.9573
994	8.4957	8.8281	0.5640	+8.1390	+7.9590	−9.2684	0.9273	9.9575
995	8.6926	9.0296	0.1140	−8.6088	−9.9976	+9.5375	0.9235	9.9583
996	8.5267	8.8659	0.5913	+8.2784	+9.1906	−9.3711	0.9216	9.9587
997	9.0966	9.4370	+0.9253	+9.0857	+9.8981	−9.6075	0.9207	9.9589
998	9.0611	9.4022	−0.2658	−9.0482	−9.9945	+9.6050	0.9201	9.9590
999	8.7681	9.1092	+9.9012	−8.7134	−0.0030	+9.5632	0.9201	9.9590
1000	−8.4571	−8.8012	+0.5348	+7.9048	−9.2074	−9.0631	+0.9176	−9.9595

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
1001	5519	9	Ophiuchi,	ω	5	16	23	15.10*	+3.546	111	8	25.8*	+ 8.14
1002	5520	10	Ophiuchi,	λ	4		23	21.16*	3.027	87	41	0.7*	8.27
1003	5523	30	Herculis,	g	5		23	43.05*	1.969	47	47	7.5*	8.09
1004	5525	27	Herculis,	β	2½		23	46.43	2.576	68	10	45.9	8.17
1005	5532	29	Herculis,	h	4½		25	35.41	2.806	78	11	6.6	8.08
1006	5536		Triang. Aust.,	η^λ	5		26	3.79	6.165	157	59	26.4	8.25
1007	5538		Scorpii,		5		26	30.81	3.932	124	56	29.5	8.09
1008	5539	23	Scorpii,	τ	3½		26	33.13*	+3.725	117	53	58.0*	7.94
1009	5545	15	Draconis,	A	4½		28	18.10*	−0.148	20	54	26.9*	7.79
1010	5547	12	Ophiuchi,		5		28	28.89	+3.144	91	59	59.1	8.12
1011	5548	13	Ophiuchi,	ζ	3½		28	54.21*	3.299	100	15	31.3*	7.73
1012	5552	35	Herculis,	σ	4		29	16.10*	1.932	47	15	3.1*	7.69
1013	5554		Aræ,		4		29	41.91	5.263	150	37	16.4	7.70
1014	5578		Triang. Aust.,	α	2		32	50.13	6.263	158	44	34.6	7.53
1015	5579		Ophiuchi,		5		32	54.17*	3.461	107	26	48.7*	7.41
1016	5596	42	Herculis,		5		34	40.64*	1.629	40	46	34.0*	7.31
1017	5604	40	Herculis,	ζ	3		35	37.99*	2.265	58	7	20.4*	6.79
1018	5609		Aræ,	η	4½		36	51.64	5.135	148	45	55.8	7.13
1019	5617	44	Herculis,	η	3		37	45.30*	2.054	50	47	22.3*	7.11
1020	5621	43	Herculis,	ι	5		38	38.05	2.876	81	8	22.9	6.93
1021	5628	18	Draconis,	g	5		39	53.42*	0.393	25	7	34.5*	6.92
1022	5632	26	Scorpii,	ε	3		40	27.43*	3.875	124	0	58.0*	7.12
1023	5637	20	Ophiuchi,		5		41	32.39*	3.312	100	30	46.6*	6.81
1024	5638		Scorpii,	μ^1	3		41	43.15	4.047	127	47	4.1	6.85
1025	5640		Scorpii,	μ^2	4		42	11.07	4.044	127	45	22.0	6.79
1026	5643		Draconis,		5		42	26.93	1.125	32	56	57.5	6.66
1027	5648	47	Herculis,	k	5		43	2.45	2.909	82	29	19.7	6.59
1028	5651		Scorpii,	ζ^1	4½		43	25.40	4.202	132	6	24.8	6.72
1029	5661		Scorpii,	ζ^2	3		44	2.33	4.197	132	5	56.3	6.90
1030	5666	50	Herculis,		5		44	47.81	2.338	59	56	4.4	6.46
1031	5667	52	Herculis,		5		44	50.79*	1.745	43	45	10.1*	6.53
1032	5683		Aræ,	ζ	3½		46	13.56	4.925	145	44	45.2	6.41
1033	5688	23	Ophiuchi,		5		46	34.93	3.203	95	54	14.5	6.41
1034	5692	25	Ophiuchi,	ι	4		46	54.81*	2.836	79	35	1.2*	6.33
1035	5693	53	Herculis,		5		47	16.85*	2.269	58	2	49.1*	6.26
1036	5697		Aræ,	ϵ^1	4		47	38.81	4.752	142	55	23.3	6.28
1037	5708	27	Ophiuchi,	κ	4		50	34.21*	2.838	80	23	15.8*	5.96
1038	5713		Aræ,	ϵ^2	5		51	10.97	4.765	143	0	14.9	5.99
1039	5731	58	Herculis,	ε	3		54	33.10*	2.294	58	50	58.2*	5.58
1040	5735		Scorpii,		5		54	57.65*	3.926	123	54	23.0*	5.67
1041	5740	19	Draconis,	h^1	5		55	12.74*	0.303	24	38	8.6*	5.58
1042	5747	59	Herculis,	d	5		56	4.07*	2.211	56	12	41.1*	5.49
1043	5765	60	Herculis,		5		58	25.43	2.781	77	2	55.6	5.30
1044	5778		Scorpii,	η	3½	17	1	24.62	+4.271	133	2	4.8	5.40
1045	5780	22	Ursæ Minoris,	ε	4		1	31.25*	−6.522	7	43	27.9*	5.06
1046	5781	35	Ophiuchi,	η	2½		1	46.73*	+3.435	105	32	2.3*	4.92
1047	5785	21	Draconis,	μ^1	4		2	13.66*	1.236	35	19	50.7*	4.97
1048	5788		Herculis,		5		2	42.98	2.123	53	52	0.0	4.98
1049	5802	37	Ophiuchi,		5		5	23.65	2.825	79	13	43.2	4.74
1050	5808	36	Ophiuchi,	A^1	4½	17	6	7.65*	+3.683	116	22	37.3*	+ 5.81

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
1001	−8.4667	−8.8143	+0.5493	+8.0237	−8.8797	−9.1696	+0.9147	−9.9601
1002	8.4363	8.7845	0.4802	−7.0429	−9.6726	+8.2187	0.9143	9.9602
1003	8.5648	8.9148	0.2929	−8.3921	−9.9708	+9.4378	0.9127	9.9605
1004	8.4665	8.8167	0.4119	−8.0367	−9.8706	+9.1805	0.9125	9.9605
1005	8.4357	8.7953	0.4494	−7.7469	−9.7870	+8.9137	0.9047	9.9621
1006	8.8506	9.2126	0.7856	+8.8177	+9.8374	−9.5676	0.9026	9.9625
1007	8.5087	8.8731	0.5942	+8.2667	+9.2307	−9.3564	0.9007	9.9628
1008	8.4759	8.8404	+0.5706	+8.1460	+8.6637	−9.2684	0.9005	9.9629
1009	8.8619	9.2357	−9.1841	−8.8324	−0.0066	+9.5610	0.8928	9.9643
1010	8.4139	8.7886	+0.4933	+6.9567	−9.6037	−8.1325	0.8919	9.9644
1011	8.4187	8.7957	0.5177	+7.6694	−9.4196	−8.8385	0.8900	9.9648
1012	8.5442	8.9231	0.2856	−8.3759	−9.9754	+9.4179	0.8884	9.9650
1013	8.7174	9.0986	0.7213	+8.6576	+9.7721	−9.5244	0.8864	9.9654
1014	8.8342	9.2324	0.7967	+8.8036	+9.8501	−9.5390	0.8718	9.9678
1015	8.4137	8.8122	0.5392	+7.8905	−9.1323	−9.0462	0.8715	9.9679
1016	8.5697	8.9781	0.2113	−8.4490	−9.9951	+9.4401	0.8630	9.9692
1017	8.4510	8.8648	0.3607	−8.1738	−9.9351	+9.2789	0.8584	9.9699
1018	8.6592	9.0799	0.7103	+8.5912	+9.7592	−9.4820	0.8523	9.9708
1019	8.4803	8.9061	0.3116	−8.2811	−9.9676	+9.3464	0.8478	9.9714
1020	8.3702	8.8012	0.4587	−7.5578	−9.7583	+8.7287	0.8433	9.9720
1021	8.7305	9.1688	9.5935	−8.6874	−0.0149	+9.4915	0.8368	9.9729
1022	8.4371	8.8787	0.5932	+8.1848	+9.2212	−9.2794	0.8339	9.9733
1023	8.3572	8.8053	0.5191	+7.6184	−9.4062	−8.7871	0.8282	9.9740
1024	8.4511	8.9003	0.6072	+8.2384	+9.3698	−9.3122	0.8272	9.9742
1025	8.4485	8.9004	0.6071	+8.2354	+9.3698	−9.3095	0.8247	9.9745
1026	8.6095	9.0631	0.0510	−8.5333	−0.0118	+9.4449	0.8233	9.9747
1027	8.3455	8.8027	0.4630	−7.4619	−9.7432	+8.6342	0.8201	9.9750
1028	8.4694	8.9289	0.6245	+8.2958	+9.4933	−9.3422	0.8180	9.9753
1029	8.4659	8.9292	0.6246	+8.2923	+9.4940	−9.3388	0.8147	9.9757
1030	8.3949	8.8629	0.3688	−8.0947	−9.9294	+9.2081	0.8105	9.9762
1031	8.4921	8.9603	0.2427	−8.3508	−9.9930	+9.3667	0.8102	9.9762
1032	8.5737	9.0506	0.6931	+8.4910	+9.7322	−9.4175	0.8024	9.9771
1033	8.3244	8.8036	0.5054	+7.3367	−9.5239	−8.5104	0.8004	9.9773
1034	8.3274	8.8087	0.4529	−7.5846	−9.7771	+8.7535	0.7985	9.9775
1035	8.3894	8.8730	0.3576	−8.1131	−9.9400	+9.2178	0.7964	9.9778
1036	8.5357	9.0217	0.6769	+8.4376	+9.6962	−9.3940	0.7943	9.9780
1037	8.3047	8.8098	0.4555	−7.5274	−9.7689	+8.6974	0.7769	9.9798
1038	8.5155	9.0246	0.6781	+8.4178	+9.7005	−9.3733	0.7732	9.9801
1039	8.3413	8.8736	0.3608	−8.0551	−9.9384	+9.1635	0.7520	9.9821
1040	8.3520	8.8871	0.5948	+8.0985	+9.2438	−9.1937	0.7494	9.9823
1041	8.6495	9.1863	9.4330	−8.6080	−0.0228	+9.4041	0.7478	9.9824
1042	8.3441	8.8871	0.3445	−8.0893	−9.9520	+9.1851	0.7421	9.9829
1043	8.2591	8.8192	0.4431	−7.6096	−9.8055	+8.7745	0.7262	9.9841
1044	8.3629	8.9457	+0.6313	+8.1970	+9.5354	−9.2370	0.7051	9.9857
1045	9.0976	9.6812	−0.8107	−9.0937	−0.0074	+9.3982	0.7044	9.9857
1046	8.2403	8.8259	+0.5353	+7.6681	−9.2011	−8.8280	0.7024	9.9858
1047	8.4587	9.0478	0.0949	−8.3702	−0.0175	+9.3085	0.6991	9.9861
1048	8.3100	8.9030	0.3273	−8.0806	−9.9647	+9.1639	0.6955	9.9863
1049	8.2046	8.8192	0.4508	−7.4762	−9.7838	+8.6445	0.6752	9.9876
1050	−8.2388	−8.8595	+0.5700	+7.8864	+8.6365	−9.0148	+0.6694	−9.9879

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
1051	5810		Apodis,	ζ	4	17	6	22.89	+6.258	157	36	19.4	+ 4.69
1052	5821	64	Herculis,	α	3½		7	48.53*	2.734	75	26	4.9*	4.48
1053	5823	22	Draconis,	ζ	3		8	21.92*	0.159	24	6	1.6*	4.47
1054	5828	65	Herculis,	δ	4		8	52.29	2.459	64	58	49.8	4.59
1055	5830	41	Ophiuchi,		4½		8	55.17	3.082	90	16	16.5	4.49
1056	5834	67	Herculis,	π	3½		9	49.51	2.088	53	1	5.5	4.32
1057	5842	68	Herculis,	u	4		11	47.24	2.212	56	44	5.1	4.17
1058	5844	40	Ophiuchi,	ξ	4½		12	1.04*	3.592	110	56	47.9*	4.36
1059	5845	53	Serpentis,	ν	4½		12	23.58*	3.371	102	41	22.4*	4.11
1060	5847	69	Herculis,	e	4½		12	30.03	2.067	52	32	52.1	4.02
1061	5850		Aræ,	γ	3		12	46.98	5.027	146	13	43.8	4.10
1062	5851	42	Ophiuchi,	θ	3½		12	48.07*	3.680	114	50	39.0*	4.15
1063	5852		Aræ,	β	3		12	50.58	4.966	145	22	50.8	4.19
1064	5859		Aræ,	κ¹	5		14	18.14	4.614	140	29	25.9	4.04
1065	5876	44	Ophiuchi,	b	5		17	12.69*	3.660	114	1	54.4*	3.80
1066	5877		Aræ,	δ	4		17	34.51	5.390	150	33	0.2	3.80
1067	5881	45	Ophiuchi,	d	4		17	46.77	3.819	119	43	32.9	3.87
1068	5886	75	Herculis,	ρ	4		18	30.68	2.073	52	42	45.7	3.59
1069	5893	49	Ophiuchi,	σ	4½		19	4.47*	2.977	85	43	30.2*	3.54
1070	5899		Aræ,	α	3		20	15.42	4.625	139	45	0.5	3.56
1071	5901	34	Scorpii,	υ	3½		20	34.31	4.071	127	10	5.6*	3.51
1072	5907	51	Ophiuchi,	c²	5		22	16.01*	3.660	113	50	28.2*	3.31
1073	5915	35	Scorpii,	λ	3		23	25.90	4.072	126	59	16.1*	3.14
1074	5922	76	Herculis,	λ	4½		24	40.61*	2.421	63	46	22.0*	3.01
1075	5935		Scorpii,	θ	3		26	32.80	4.301	132	53	45.6	3.06
1076	5937	23	Draconis,	β	2½		27	2.72*	1.353	37	35	8.2*	2.86
1077	5941	55	Ophiuchi,	α	2		27	58.35*	2.781	77	19	36.0*	2.98
1078	5949	55	Serpentis,	ξ	5		28	59.96*	3.433	105	17	56.1*	2.75
1079	5950	24	Draconis,	ν¹	5		29	13.33*	1.179	34	42	41.1*	2.66
1080	5951	25	Draconis,	ν²	5		29	18.79*	1.179	34	43	23.7*	2.67
1081	5953	57	Ophiuchi,	μ	5		29	41.67	3.259	98	1	18.9	2.63
1082	5959		Octantis,	σ	6		30	4.09	107.504	179	16	21.9	2.61
1083	5963		Pavonis,	η	4½		31	1.52	5.863	154	38	40.0	2.71
1084	5970		Scorpii,	κ	3		32	7.07	+4.146	128	56	47.4	2.51
1085	5972	27	Draconis,	f	5		32	34.21*	—0.256	21	46	11.5*	2.28
1086	5976	56	Serpentis,	ο	4½		32	59.17*	+3.369	102	47	24.5*	2.38
1087	5987	58	Ophiuchi,		5		34	26.58*	3.594	111	36	16.5*	2.16
1088	5990	85	Herculis,	ι	4		35	14.11*	1.713	43	54	41.2*	2.15
1089	5996	69	Ophiuchi,	β	3		36	3.68*	2.964	85	21	56.7*	1.92
1090	6004		Scorpii,	ι¹	3½		37	6.05	+4.200	130	3	47.3	2.14
1091	6006	28	Draconis,	ω	4		37	50.03*	—0.368	21	10	23.7*	1.66
1092	6008	3	Sagittarii,		5		38	7.11*	+3.767	117	46	4.8*	1.91
1093	6018		Scorpii,		4		39	39.09	4.083	126	59	29.2*	1.85
1094	6020	62	Ophiuchi,	γ	4		40	22.23*	3.004	87	13	55.2*	1.79
1095	6021	86	Herculis,	μ	4		40	35.46	+2.344	62	11	16.4	2.42
1096	6047	31	Draconis,	ψ¹	4½		44	36.63	—1.096	17	46	45.9*	1.60
1097	6074		Sagittarii,		5		49	27.22*	+3.852	120	13	55.4*	1.00
1098	6077	4	Sagittarii,		5		50	38.09*	3.661	113	47	48.3*	0.83
1999	6078	64	Ophiuchi,	ν	4		50	46.22*	3.304	99	45	1.7*	0.91
1100	6079	32	Draconis,	ξ	3½	17	50	56.33*	+1.036	33	6	8.0*	+ 0.73

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
1051	−8.6082	−9.2310	+0.7943	+8.5741	+9.8658	−9.3311	+0.6674	−9.9880
1052	8.1917	8.8267	0.4365	−7.5923	−9.8228	+8.7542	0.6559	9.9886
1053	8.5620	9.2018	9.1967	−8.5224	−0.0282	+9.3095	0.6513	9.9889
1054	8.2116	8.8558	0.3913	−7.8378	−9.9061	+8.9711	0.6471	9.9891
1055	8.1684	8.8130	0.4881	+5.8439	−9.6329	−7.0199	0.6467	9.9891
1056	8.2583	8.9110	0.3197	−8.0376	−9.9703	+9.1161	0.6391	9.9895
1057	8.2214	8.8920	0.3450	−7.9606	−9.9539	+9.0590	0.6220	9.9903
1058	8.1713	8.8440	0.5528	+7.7246	−8.7451	−8.8710	0.6199	9.9904
1059	8.1490	8.8252	0.5270	+7.4908	−9.3191	−8.6561	0.6166	9.9906
1060	8.2376	8.9148	0.3156	−8.0215	−9.9729	+9.0974	0.6156	9.9906
1061	8.3898	9.0697	0.7014	+8.3095	+9.7578	−9.2306	0.6131	9.9907
1062	8.1768	8.8568	0.5655	+7.8002	+8.2430	−8.9341	0.6129	9.9907
1063	8.3798	9.0602	0.6960	+8.2951	+9.7475	−9.2257	0.6125	9.9907
1064	8.3171	9.0116	0.6684	+8.2045	+9.6813	−9.1842	0.5991	9.9913
1065	8.1319	8.8557	0.5631	+7.7417	+7.5798	−8.8784	0.5708	9.9924
1066	8.3972	9.1248	0.7322	+8.3371	+9.8079	−9.2049	0.5672	9.9925
1067	8.1481	8.8778	0.5822	+7.8434	+9.0438	−8.9582	0.5651	9.9926
1068	8.1786	8.9161	0.3158	−7.9609	−9.9737	+9.0377	0.5576	9.9928
1069	8.0746	8.8182	0.4731	−6.9470	−9.7043	+8.1219	0.5517	9.9930
1070	8.2505	9.0070	0.6652	+8.1332	+9.6734	−9.1196	0.5391	9.9934
1071	8.1560	8.9161	0.6095	+7.9372	+9.3971	−9.0147	0.5357	9.9935
1072	8.0772	8.8567	0.5627	+7.6838	+7.2553	−8.8211	0.5168	9.9941
1073	8.1226	8.9159	0.6091	+7.9019	+9.3934	−8.9804	0.5033	9.9945
1074	8.0573	8.8659	0.3838	−7.7026	−9.9170	+8.8315	0.4884	9.9948
1075	8.1218	8.9544	0.6334	+7.9547	+9.5508	−8.9957	0.4649	9.9954
1076	8.1948	9.0341	0.1308	−8.0938	−0.0208	+9.0552	0.4584	9.9955
1077	7.9785	8.8304	0.4429	−7.3198	−9.8067	+8.4852	0.4461	9.9958
1078	7.9694	8.8356	0.5357	+7.3908	−9.1965	−8.5512	0.4321	9.9960
1079	8.1952	9.0645	0.0637	−8.1100	−0.0260	+9.0416	0.4289	9.9961
1080	8.1938	9.0644	0.0640	−8.1086	−0.0260	+9.0403	0.4277	9.9961
1081	7.9483	8.8244	0.5129	+7.0930	−9.4645	−8.2648	0.4223	9.9962
1082	9.8351	0.7167	2.0314	+9.8351	+9.9938	−9.1147	0.4169	9.9963
1083	8.2929	9.1888	0.7686	+8.2489	+9.8517	−9.0567	0.4029	9.9965
1084	8.0172	8.9299	+0.6173	+7.8155	+9.4571	−8.8824	0.3863	9.9968
1085	8.3316	9.2516	−9.4036	−8.2995	−0.0346	+9.0448	0.3792	9.9969
1086	7.9052	8.8318	+0.5279	+7.2503	−9.3084	−8.4155	0.3726	9.9970
1087	7.9019	8.8528	0.5559	+7.4680	−8.5752	−8.6125	0.3486	9.9973
1088	8.0156	8.9803	0.2279	−7.8732	−0.0074	+8.8903	0.3350	9.9975
1089	7.8434	8.8230	0.4717	−6.7508	−9.7100	+7.9254	0.3203	9.9976
1090	7.9390	8.9379	+0.6222	+7.7476	+9.4890	−8.8076	0.3011	9.9978
1091	8.2510	9.2641	−9.5623	−8.2206	−0.0354	+8.9545	0.2870	9.9980
1092	7.8562	8.8751	+0.5765	+7.5245	+8.9020	−8.6475	0.2814	9.9980
1093	7.8693	8.9198	0.6100	+7.6487	+9.4026	−8.7272	0.2500	9.9983
1094	7.7566	8.8228	0.4780	−6.4405	−9.6827	+7.6161	0.2344	9.9984
1095	7.8045	8.8757	+0.3743	−7.4734	−9.9288	+8.5962	0.2295	9.9984
1096	8.1659	9.3381	−0.0373	−8.1446	−0.0345	+8.8055	0.1290	9.9990
1097	7.5500	8.8869	+0.5853	+7.2520	+9.1076	−8.3646	9.9649	9.9995
1098	7.4737	8.8621	0.5634	+7.0795	+7.7782	−8.2170	9.9134	9.9996
1099	7.4351	8.8299	0.5185	+6.6639	−9.4125	−7.8337	9.9071	9.9997
1100	−7.6836	−9.0863	+0.0092	−7.6066	−0.0310	+8.5200	+9.8992	−9.9997

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.	Annual Variation.	North Polar Dist., Jan. 1, 1850.	Annual Variat'n
						h. m. s.	s.	° ′ ″	″
1101	6082	91	Herculis,	θ	4	17 51 6.67	+ 2.055	52 43 35.4*	+0.7
1102	6084	92	Herculis,	ξ	4	51 56.24	2.332	60 43 56.6	0.7
1103	6085	57	Serpentis,	ζ	5	52 33.77	3.169	93 40 31.0	0.6
1104	6087	94	Herculis,	ν	5	52 45.83*	2.296	59 47 42.3*	0.6
1105	6089	66	Ophiuchi,		5	52 50.09*	2.971	85 37 4.1	0.6
1106	6091	33	Draconis,	γ	2	53 7.45*	1.394	38 29 29.4*	0.6
1107	6092	67	Ophiuchi,		4	53 8.35	3.010	87 3 24.5	0.6
1108	6094	93	Herculis,		5	53 22.69*	2.667	73 14 12.0*	0.5
1109	6100		Pavonis,	π	5	54 7.20	5.715	153 39 52.2	0.5
1110	6104	69	Ophiuchi,	τ	5	54 55.01	3.269	98 10 27.9	0.4
1111	6105		Aræ,	θ	4	54 57.59	4.678	140 5 41.2	0.5
1112	6107		Sagittarii,	γ¹	4	55 26.54	3.840	119 34 53.0	0.4
1113	6110	96	Herculis,		5	55 58.44	+ 2.564	69 9 44.0	0.3
1114	6114	35	Draconis,		5	56 10.30*	− 2.702	13 1 16.2*	0.0
1115	6115	10	Sagittarii,	γ²	4	56 10.68*	+ 3.858	120 25 12.5*	0.5
1116	6123	70	Ophiuchi,		4½	57 52.35*	3.028	87 27 38.7*	1.2
1117	6126		Pavonis,	ι	4½	58 20.86	5.539	151 33 34.9	0.1
1118	6127		Sagittarii,		5	58 34.97	3.795	118 28 2.9	0.2
1119	6129		Herculis,		5	59 13.82*	1.562	41 32 25.7*	0.0
1120	6140		Telescopii,	ε	4½	18 0 5.90	4.456	135 58 24.7	+0.0
1121	6143	72	Ophiuchi,		4	0 14.41*	2.844	80 27 13.2*	−0.1
1122	6148		Pavonis,		5	1 23.31	5.639	153 5 6.9	0.1
1123	6150	103	Herculis,	ο	4	1 41.53*	2.341	61 15 16.8*	0.1
1124	6168	13	Sagittarii,	μ	3½	4 47.57*	3.587	111 5 33.8*	0.4
1125	6178	104	Herculis,	A	5	6 15.65	2.261	58 37 43.7*	0.6
1126	6186		Sagittarii,	η	4	7 28.60	+ 4.056	126 48 8.2*	0.4
1127	6206	40	Draconis,		5	11 15.40	− 4.459	10 1 33.4	1.0
1128	6208	41	Draconis,		5	11 21.57	− 4.466	10 1 21.7	1.0
1129	6209	19	Sagittarii,	δ	3½	11 23.43*	+ 3.842	119 53 10.6*	0.9
1130	6218		Lyræ,		5	12 21.41*	1.915	49 7 11.7*	1.0
1131	6223	105	Herculis,		5	13 0.36*	2.471	65 36 45.7*	1.1
1132	6224	36	Draconis,		5	13 1.90*	0.345	25 39 11.1*	1.1
1133	6229	58	Serpentis,	η	4	13 33.14*	3.102	92 56 0.3*	0.5
1134	6233	20	Sagittarii,	ε	3	14 12.94*	3.987	124 27 1.0*	1.1
1135	6235	1	Lyræ,	κ	4½	14 36.38*	2.103	54 0 0.2*	1.3
1136	6240		Telescopii,	α	4	15 50.92	4.451	136 2 42.3	1.1
1137	6250		Telescopii,	ζ	4½	17 15.01	4.597	139 8 42.5	1.0
1138	6253		Pavonis,	ν	5	17 21.98	5.644	152 21 55.6	1.3
1139	6255		Draconis,		5	17 42.50*	1.535	40 57 7.5*	1.5
1140	6263	22	Sagittarii,	λ	4	18 42.75*	3.707	115 29 56.7*	1.4
1141	6278		Telescopii,	δ¹	5	20 38.47	4.445	136 0 34.3	1.6
1142	6279		Sagittarii,		5	20 38 96*	+ 3.419	104 39 23.9*	1.7
1143	6281	23	Ursæ Minoris,	δ	3	20 43.54*	−19.293	3 24 10.0*	1.8
1144	6282		Telescopii,	δ²	5	20 56.19	+ 4.438	135 51 13.5	1.7
1145	6289	39	Draconis,	b	5	21 42.93*	+ 0.876	31 17 5.6*	1.9
1146	6296		Coronæ Aust.,	θ	5	22 47.97	+ 4.301	132 24 52.2	1.8
1147	6297	43	Draconis,	φ	5	22 54.25	− 0.848	18 44 36.9	2.0
1148	6302	44	Draconis,	χ	4½	23 45.53	− 1.073	17 19 59.1	1.7
1149	6315		Pavonis,	ζ	4	25 29.29	+ 7.045	161 32 44.6	2.0
1150	6350		Draconis,		5	18 30 32.05	+ 1.360	37 45 51.0	−2.6

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
1101	−7.5117	−8.9228	+0.3126	−7.2939	−9.9780	+8.3708	+9.8908	−9.9997
1102	7.4294	8.8830	0.3658	−7.1186	−9.9381	+8.2354	9.8484	9.9997
1103	7.3359	8.8246	0.4992	+6.1428	−9.5670	−7.3180	9.8133	9.9998
1104	7.3864	8.8871	0.3603	−7.0881	−9.9434	+8.2008	9.8014	9.9998
1105	7.3199	8.8250	0.4725	−6.2031	−9.7068	+7.3779	9.7970	9.9998
1106	7.5068	9.0296	0.1430	−7.4004	−0.0222	+8.3706	9.7792	9.9998
1107	7.3006	8.8243	0.4774	−6.0112	−9.6855	+7.1867	9.7784	9.9998
1108	7.3035	8.8426	0.4262	−6.7636	−9.8475	+7.9208	9.7630	9.9998
1109	7.5860	9.1768	0.7612	+7.5385	+9.8472	−8.3616	9.7114	9.9999
1110	7.1743	8.8282	0.5136	+6.3271	−9.4585	−7.4988	9.6482	9.9999
1111	7.3589	9.0166	0.6692	+7.2438	+9.6889	−8.2271	9.6444	9.9999
1112	7.1831	8.8845	0.5831	+6.8765	+9.0652	−7.9920	9.6008	9.9999
1113	7.0979	8.8532	+0.4086	−6.6490	−9.8816	+7.7957	9.5468	9.9999
1114	7.6946	9.4711	−0.4330	−7.6833	−0.0302	+8.2122	9.5257	9.9999
1115	7.1105	8.8882	+0.5861	+6.8149	+9.1209	−7.9267	9.5245	9.9999
1116	6.7914	8.8243	0.4788	−5.4379	−9.6793	+6.6136	9.2693	0.0000
1117	7.0040	9.1461	0.7434	+6.9481	+9.8278	−7.8020	9.1601	0.0000
1118	6.6710	8.8799	0.5793	+6.3492	+8.9786	−7.4693	9.0933	0.0000
1119	−6.5311	9.0023	0.1936	−6.4053	−0.0155	+7.4030	+8.8310	0.0000
1120	+5.6169	8.9819	0.6487	−5.4736	+9.6206	+6.4917	−7.9372	0.0000
1121	5.8480	8.8300	0.4542	+5.0677	−9.7737	−6.2377	8.3202	0.0000
1122	6.9506	9.1681	0.7562	−6.9008	+9.8422	+7.7327	9.0847	0.0000
1123	6.7499	8.8810	0.3687	+6.4319	−9.9352	−7.5509	9.1711	0.0000
1124	7.1746	8.8539	0.5546	−6.7307	−8.6542	+7.8767	9.6227	9.9999
1125	7.3290	8.8924	0.3533	+7.0455	−9.9498	−8.1529	9.7386	9.9998
1126	7.4339	8.9202	+0.6096	−7.2113	+9.4000	+8.2909	9.8157	9.9998
1127	8.2742	9.5826	−0.6515	+8.2675	−0.0250	−8.6844	9.9933	9.9995
1128	8.2783	9.5827	−0.6518	+8.2716	−0.0249	−8.6883	9.9972	9.9995
1129	7.5821	8.8854	+0.5841	−7.2796	+9.0846	+8.3937	9.9984	9.9995
1130	7.6772	8.9447	0.2822	+7.4931	−9.9919	−8.5477	0.0340	9.9994
1131	7.6183	8.8638	0.3919	+7.2341	−9.9071	−8.3696	0.0560	9.9993
1132	7.9421	9.1868	9.4642	+7.8970	−0.0366	−8.7095	0.0568	9.9993
1133	7.5960	8.8237	0.4968	−6.3050	−9.5824	+7.4805	0.0737	9.9992
1134	7.7000	8.9068	0.6005	−7.4526	+9.3153	+8.5449	0.0946	9.9992
1135	7.7201	8.9151	0.3225	+7.4893	−9.9721	−8.5733	0.1063	9.9991
1136	7.8220	8.9815	0.6488	−7.6792	+9.6201	+8.6967	0.1417	9.9990
1137	7.8844	9.0070	0.6639	−7.7632	+9.6724	+8.7549	0.1784	9.9988
1138	8.0367	9.1563	0.7495	−7.9841	+9.8339	+8.8265	0.1813	9.9988
1139	7.8949	9.0061	0.1860	+7.7730	−0.0160	−8.7656	0.1897	9.9987
1140	7.7799	8.8670	0.5689	−7.4139	+8.5729	+8.5455	0.2137	9.9986
1141	7.9362	8.9804	0.6484	−7.7932	+9.6182	+8.8110	0.2562	9.9982
1142	7.7924	8.8365	+0.5339	−7.1955	−9.2248	+8.3573	0.2563	9.9982
1143	9.0062	0.0487	−1.2861	+9.0054	−0.0086	−8.9550	0.2580	9.9982
1144	7.9411	8.9792	+0.6476	−7.7970	+9.6152	+8.8160	0.2623	9.9982
1145	8.0845	9.1065	+9.9446	+8.0163	−0.0319	−8.9078	0.2783	9.9981
1146	7.9527	8.9535	+0.6321	−7.7817	+9.5451	+8.8260	0.2993	9.9979
1147	8.3160	9.3148	−9.9293	+8.2923	−0.0341	−8.9753	0.3012	9.9978
1148	8.3647	9.3475	−0.0757	+8.3445	−0.0329	−8.9947	0.3171	9.9977
1149	8.3687	9.3208	+0.8483	−8.3458	+9.9060	+9.0223	0.3475	9.9973
1150	+8.1602	−9.0330	+0.1335	+8.0581	−0.0209	−9.0212	−0.4255	−9.9961

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.	Annual Variation.	North Polar Dist., Jan. 1, 1850.	Annual Variation.
						h. m. s.	s.	° ′ ″	″
1151	6352		Pavonis,		5	18 30 43.39	+5.937	155 0 9.9	− 2.70
1152	6355	3	Lyræ,	α	1	31 51.56*	2.032	51 21 10.9*	3.06
1153	6360		Pavonis,	θ	5	33 51.52	5.888	155 13 31.3	2.80
1154	6361	2	Aquilæ,		5	34 3.69	3.289	99 11 27.5	2.97
1155	6371	27	Sagittarii,	φ	4½	36 16.97*	3.758	117 8 22.3*	3.13
1156	6383		Pavonis,	λ	5	38 19.00	5.620	152 21 1.9	3.33
1157	6387	110	Herculis,		5	39 12.47	2.585	69 35 36.3	3.06
1158	6390	4	Lyræ,	ε^1	5	39 22.12*	1.986	50 29 2.1*	3.50
1159	6391	5	Lyræ,	ε^2	5	39 24.46*	1.988	50 32 28.1*	3.51
1160	6392	6	Lyræ,	ζ^1	5	39 36.35	2.064	52 32 52.4	3.52
1161	6395	46	Draconis,	c	5	39 43.41*	1.160	34 36 39.0*	3.46
1162	6405		Pavonis,	κ	5	41 28.09	6.259	157 24 48.9	3.53
1163	6419		Draconis,		5	43 21.55	1.339	37 10 29.9	3.77
1164	6429	10	Lyræ,	β	3	44 32.50*	2.215	56 48 30.4*	3.85
1165	6434	32	Sagittarii,	ν^1	5	45 6.65*	3.627	112 55 25.9*	3.93
1166	6440	34	Sagittarii,	σ	3	45 57.78*	3.729	116 28 38.8*	3.92
1167	6441	35	Sagittarii,	ν^2	5	46 2.95*	3.632	112 51 9.8*	4.02
1168	6452		Draconis,		5	48 13.02	1.349	37 13 11.1	4.19
1169	6453	113	Herculis,		5	48 25.24	2.535	67 32 29.0*	4.27
1170	6460	63	Serpentis,	θ^1	4½	48 45.58*	2.982	85 59 14.1*	4.34
1171	6461	37	Sagittarii,	ξ^2	4	48 46.64*	3.583	111 17 55.0*	4.24
1172	6462		Serpentis,		5	48 47.04*	2.982	85 59 19.7*	4.36
1173	6463	47	Draconis,	o	5	48 58.86*	0.886	30 47 37.8*	4.25
1174	6466	12	Lyræ,	δ^2	5	49 15.63	+2.099	53 17 18.3	4.32
1175	6469		Draconis,		5	49 28.82	−1.449	16 5 22.1	4.43
1176	6475	13	Lyræ,		5	50 46.16*	+1.825	46 14 56.7*	4.41
1177	6478	50	Draconis,		5	51 10.70	−1.901	14 44 49.7	4.46
1178	6487	13	Aquilæ,	ε	3½	52 48.91*	+2.723	75 7 53.9*	4.48
1179	6489	38	Sagittarii,	ζ	3½	53 3.77*	3.828	120 5 20.3*	4.57
1180	6491	14	Lyræ,	γ	3	53 20.01	2.244	57 30 45.3	4.65
1181	6496	48	Draconis,		5	54 12.66*	1.018	32 22 57.5*	4.63
1182	6507	39	Sagittarii,	o	4½	55 41.43*	+3.600	111 57 21.0*	4.80
1183	6510	52	Draconis,	υ	5	56 13.01*	−0.708	18 54 15.0*	4.92
1184	6511		Coronæ Aust.,	γ	5	56 16.72	+4.072	127 16 22.1	4.50
1185	6521	40	Sagittarii,	τ	4	57 34.27*	3.755	117 53 2.7*	4.76
1186	6523		Coronæ Aust.,	δ	5	57 53.97	4.184	130 43 23.8	4.87
1187	6526	16	Aquilæ,	λ	3	58 17.35	3.187	95 6 10.8	4.98
1188	6528	17	Aquilæ,	ζ	3	58 30.90*	2.755	76 21 19.9*	5.01
1189	6535		Coronæ Aust.,	α	4½	59 15.91	4.098	128 7 54.4	4.97
1190	6541		Coronæ Aust.,	β	5	59 42.46	4.143	129 34 23.6	5.02
1191	6548	41	Sagittarii,	π	4½	19 0 50.33*	3.575	111 15 24.4*	5.27
1192	6564	20	Aquilæ,		5	4 32.40*	3.260	98 11 7.6*	5.59
1193	6575	42	Sagittarii,	ψ	5	6 20.44	3.687	115 30 33.8	5.74
1194	6581	20	Lyræ,	η	5	8 39.21	2.042	51 6 32.7	5.97
1195	6583	53	Draconis,		5	8 50.09*	1.136	33 23 41.3*	5.98
1196	6584	43	Sagittarii,	d	5	8 51.29*	3.519	109 12 53.2*	5.98
1197	6589	1	Vulpeculæ,		5	9 46.18	2.581	68 52 11.9	6.07
1198	6595	25	Aquilæ,	ω	5	10 46.70	2.818	78 40 13.1	6.16
1199	6599	21	Lyræ,	θ	5	11 9.66*	2.080	52 7 50.0*	6.13
1200	6601	54	Draconis,		5	19 11 14.32*	+1.079	32 33 8.7*	− 6.06

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
1151	+8.3240	−9.1941	+0.7718	−8.2813	+9.8542	+9.0833	−0.4282	−9.9961
1152	8.0729	8.9270	0.3036	+7.8685	−9.9811	−8.9372	0.4439	9.9958
1153	8.3696	9.1969	0.7735	−8.3276	+9.8550	+9.1260	0.4701	9.9952
1154	8.0000	8.8247	0.5165	−7.2034	−9.4320	+8.3739	0.4727	9.9952
1155	8.0723	8.8691	0.5737	−7.7314	+8.8082	+8.8569	0.5000	9.9945
1156	8.3785	9.1512	0.7472	−8.3259	+9.8274	+9.1685	0.5234	9.9939
1157	8.0831	8.8457	0.4117	+7.6256	−9.8752	−8.7735	0.5333	9.9936
1158	8.1695	8.9302	0.2976	+7.9731	−9.9830	−9.0365	0.5351	9.9936
1159	8.1695	8.9298	0.2981	+7.9727	−9.9829	−9.0364	0.5355	9.9936
1160	8.1596	8.9177	0.3142	+7.9436	−9.9747	−9.0194	0.5376	9.9935
1161	8.3063	9.0630	0.0654	+8.2217	−0.0242	−9.1521	0.5389	9.9934
1162	8.4947	9.2324	0.7947	−8.4600	+9.8701	+9.2205	0.5574	9.9929
1163	8.3170	9.0349	0.1268	+8.2183	−0.0190	−9.1757	0.5765	9.9922
1164	8.1871	8.8930	0.3449	+7.9255	−9.9544	−9.0242	0.5881	9.9918
1165	8.1510	8.8512	0.5593	−7.7415	−8.2480	+8.8818	0.5935	9.9915
1166	8.1714	8.8632	0.5709	−7.8205	+8.6857	+8.9485	0.6015	9.9912
1167	8.1595	8.8506	0.5590	−7.7488	−8.2878	+8.8893	0.6023	9.9912
1168	8.3621	9.0326	0.1301	+8.2632	−0.0176	−9.2209	0.6220	9.9903
1169	8.1798	8.8484	0.4032	+7.7619	−9.8890	−8.9037	0.6238	9.9902
1170	8.1496	8.8151	0.4741	+6.9946	−9.7001	−8.1696	0.6269	9.9901
1171	8.1794	8.8447	0.5539	−7.7396	−8.6929	+8.8849	0.6270	9.9901
1172	8.1498	8.8151	0.4741	+6.9947	−9.7001	−8.1697	0.6270	9.9901
1173	8.4412	9.1047	9.9436	+8.3752	−0.0260	−9.2606	0.6288	9.9900
1174	8.2489	8.9098	+0.3215	+8.0254	−9.9694	−9.1055	0.6312	9.9899
1175	8.7121	9.3710	−9.1635	+8.6947	−0.0248	−9.3135	0.6331	9.9898
1176	8.3070	8.9544	+0.2605	+8.1468	−9.9948	−9.1817	0.6441	9.9893
1177	8.7634	9.4072	−0.2747	+8.7489	−0.0226	−9.3307	0.6475	9.9891
1178	8.1974	8.8271	+0.4354	+7.6067	−9.8256	−8.7680	0.6610	9.9884
1179	8.2475	8.8750	0.5826	−7.9477	+9.0512	+9.0609	0.6630	9.988
1180	8.2607	8.8860	0.3507	+7.9908	−9.9488	−9.0930	0.6651	9.988
1181	8.4649	9.0828	0.0091	+8.3915	−0.0226	−9.2965	0.6721	9.9877
1182	8.2379	8.8437	+0.5555	−7.8107	−8.5977	+8.9541	0.6835	9.9871
1183	8.6987	9.3002	−9.8553	+8.6746	−0.0246	−9.3613	0.6876	9.9868
1184	8.3089	8.9099	+0.6083	−8.0911	+9.3840	+9.1680	0.6880	9.9868
1185	8.2730	8.8637	0.5747	−7.9429	+8.8414	+9.0654	0.6977	9.9862
1186	8.3422	8.9303	0.6217	−8.1567	+9.4812	+9.2124	0.7001	9.9860
1187	8.2263	8.8114	0.5033	−7.1755	−9.5392	+8.3498	0.7029	9.9858
1188	8.2387	8.8220	0.4404	+7.6114	−9.8129	−8.7751	0.7046	9.9857
1189	8.3359	8.9135	0.6112	−8.1266	+9.4074	+9.1984	0.7100	9.9853
1190	8.3479	8.9221	0.6168	−8.1521	+9.4489	+9.2151	0.7131	9.9851
1191	8.2734	8.8390	0.5530	−7.8328	−8.7372	+8.9783	0.7211	9.9845
1192	8.2723	8.8109	0.5126	−7.4257	−9.4664	+8.5973	0.7461	9.9825
1193	8.3240	8.8500	0.5661	−7.9581	+8.3222	+9.0897	0.7577	9.9816
1194	8.4027	8.9130	0.3097	+8.2006	−9.9721	−9.2678	0.7722	9.9802
1195	8.5543	9.0633	0.0543	+8.4759	−0.0155	−9.3927	0.7733	9.9801
1196	8.3200	8.8289	0.5460	−7.8374	−8.9796	+8.9886	0.7734	9.9801
1197	8.3309	8.8337	0.4112	+7.8878	−9.8743	−9.0337	0.7790	9.9796
1198	8.3153	8.8114	0.4495	+7.6085	−9.7873	−8.7761	0.7850	9.9790
1199	8.4117	8.9053	0.3182	+8.1998	−9.9671	−9.2731	0.7873	9.9787
1200	+8.5786	−9.0718	+0.0321	+8.5044	−0.0154	−9.4113	−0.7878	−9.9787

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
1201	6608		Sagittarii,	β^1	3½	19	11	50.53	+4.328	134	44	8.0	— 5.99
1202	6610		Sagittarii,	β^2	4		12	21.92	4.331	135	4	36.4	6.03
1203	6612	57	Draconis,	δ	3		12	30.45*	0.041	22	36	8.3*	6.31
1204	6619	44	Sagittarii,	ρ^1	5		12	58.18*	3.489	108	7	28.7*	6.34
1205	6622		Sagittarii,	α	4		13	29.33	4.179	130	53	34.4	6.06
1206	6623	1	Cygni,	κ	4		13	37.94*	1.389	36	54	23.9*	6.42
1207	6644	31	Aquilæ,	b	5		17	49.06*	2.865	78	22	18.3	7.37
1208	6646	30	Aquilæ,	δ	3½		17	56.05*	3.027	87	10	48.6*	6.80
1209	6649		Telescopii,	μ	4		18	23.67	+4.901	145	24	42.2	6.53
1210	6650	60	Draconis,	τ	4½		18	24.87	—1.096	16	55	30.9	6.81
1211	6662	58	Draconis,	π	4		19	53.25*	+0.332	24	34	26.3*	6.87
1212	6674	6	Vulpeculæ,	α	4		22	27.88	2.495	65	38	6.3*	6.97
1213	6690	6	Cygni,	β	3		24	40.37*	2.420	62	21	7.2*	7.26
1214	6697	10	Cygni,	ι^2	5		25	55.37*	1.516	38	35	17.5*	7.47
1215	6701	38	Aquilæ,	μ	4½		26	45.67*	2.934	82	56	8.4*	7.32
1216	6703	37	Aquilæ,	k	5		26	51.35	3.313	100	52	59.9	7.41
1217	6706	52	Sagittarii,	h^2	4½		27	34.39*	3.663	115	12	33.7*	7.48
1218	6713	39	Aquilæ,	κ	4		28	49.24	3.233	97	21	23.0	7.60
1219	6715	41	Aquilæ,	ι	5		28	57.70	3.111	91	36	52.7	7.60
1220	6729	44	Aquilæ,	σ	5		31	47.36*	2.965	84	56	24.7*	7.83
1221	6734	13	Cygni,	θ	4		32	25.18*	+1.611	40	7	27.6*	8.08
1222	6735	61	Draconis,	σ	5		32	38.53*	—0.107	20	35	38.1*	6.06
1223	6739	5	Sagittæ,	α	4		33	23.61	+2.685	72	19	36.6	7.98
1224	6740	12	Cygni,	ϕ	4		33	27.16	2.368	60	11	22.3	8.01
1225	6742	55	Sagittarii,	e^2	5		33	56.10*	3.440	106	28	13.8*	8.03
1226	6744	6	Sagittæ,	β	5		34	18.82	2.697	72	52	1.2	8.04
1227	6748		Cygni,		5		35	18.15	1.348	35	22	34.5	8.10
1228	6771	15	Cygni,		5		38	52.09	2.166	53	0	16.7	8.47
1229	6772	50	Aquilæ,	γ	3		39	7.67*	2.857	79	44	54.7*	8.42
1230	6779	18	Cygni,	δ	3½		40	17.23*	1.876	45	13	58.7*	8.50
1231	6780		Cygni,		5		40	17.99	1.158	32	20	22.1	8.50
1232	6783	7	Sagittæ,	δ	4		40	42.16	2.684	71	49	54.7	8.58
1233	6784	17	Cygni,	χ	5		40	44.09	2.275	56	37	4.1	8.02
1234	6794	8	Sagittæ,	ζ	5		42	19.25	2.668	71	13	48.0	8.73
1235	6801		Pavonis,	ϵ	4		43	8.80	7.094	163	17	48.4	8.64
1236	6802	53	Aquilæ,	α	1½		43	27.78*	2.925	81	31	26.3*	9.13
1237	6811	55	Aquilæ,	η	4		44	49.86	3.060	89	22	31.6	8.82
1238	6812		Sagittarii,	ι	4½		44	54.98	4.179	132	15	26.6	8.82
1239	6817		Cygni,		5		45	28.47*	2.058	49	46	45.0*	8.91
1240	6825	59	Aquilæ,	ξ	5		46	58.67*	2.912	81	55	23.2*	8.97
1241	6827	13	Vulpeculæ,		5		47	5.23	2.552	66	18	28.5	9.11
1242	6832	59	Sagittarii,	b	5		47	44.09*	3.696	117	33	44.7*	9.09
1243	6833	60	Aquilæ,	β	3½		47	56.66*	2.952	83	57	51.5*	8.56
1244	6849	22	Cygni,		5		50	30.19	2.144	51	54	32.9	9.30
1245	6851	21	Cygni,	η	5		50	41.10	2.253	55	18	47.0	9.27
1246	6857		Cygni,		5		52	1.19	2.081	50	1	58.3	9.42
1247	6858	12	Sagittæ,	γ	4½		52	5.22	2.669	70	54	41.0	9.50
1248	6866	14	Vulpeculæ,		5		52	44.48	2.576	67	18	16.4	9.45
1249	6870	62	Sagittarii,	c	4½		53	25.57*	3.705	118	7	19.1*	9.58
1250	6873		Pavonis,	δ	4	19	53	56.99	+5.969	156	33	18.1	— 8.50

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
1201	+8.4615	−8.9507	+0.6366	−8.3090	+9.5578	+9.3365	−0.7913	−9.9783
1202	8.4671	8.9530	0.6380	−8.3172	+9.5643	+9.3422	0.7943	9.9780
1203	8.7321	9.2171	8.2672	+8.6974	−0.0188	−9.4582	0.7952	9.9779
1204	8.3416	8.8236	0.5424	−7.8345	−9.0682	+8.9885	0.7978	9.9776
1205	8.4440	8.9227	0.6201	−8.2600	+9.4672	+9.3146	0.8008	9.9773
1206	8.5448	9.0226	0.1404	+8.4477	−0.0083	−9.4023	0.8016	9.9772
1207	8.3555	8.8074	0.4489	+7.6599	−9.7890	−8.8269	0.8248	9.9745
1208	8.3476	8.7988	0.4784	+7.0395	−9.6811	−8.2151	0.8254	9.9744
1209	8.5954	9.0439	+0.6900	−8.5110	+9.7236	+9.4411	0.8278	9.9741
1210	8.8855	9.3339	−0.0287	+8.8663	−0.0118	−9.5065	0.8279	9.9741
1211	8.7384	9.1780	+9.5088	+8.6972	−0.0150	−9.4922	0.8357	9.9731
1212	8.4111	8.8357	0.3986	+8.0266	−9.8929	−9.1621	0.8489	9.9713
1213	8.4342	8.8462	0.3834	+8.1008	−9.9123	−9.2242	0.8599	9.9697
1214	8.5926	8.9977	0.1794	+8.4856	−9.9992	−9.4567	0.8659	9.9687
1215	8.3949	8.7953	0.4649	+7.4849	−9.7364	−8.6576	0.8699	9.9681
1216	8.4000	8.7998	0.5198	−7.6760	−9.3993	+8.8442	0.8704	9.9680
1217	8.4390	8.8349	0.5628	−8.0683	+7.4150	+9.2009	0.8738	9.9675
1218	8.4049	8.7940	0.5093	−7.5123	−9.4940	+8.6848	0.8796	9.9665
1219	8.4021	8.7905	0.4921	−6.8521	−9.6103	+8.0280	0.8803	9.9664
1220	8.4166	8.7898	0.4715	+7.3620	−9.7105	−8.5364	0.8932	9.9642
1221	8.6085	8.9784	+0.2072	+8.4919	−9.9926	−9.4772	0.8960	9.9637
1222	8.8724	9.2412	−9.3023	+8.8437	−0.0058	−9.5660	0.8969	9.9635
1223	8.4430	8.8078	+0.4281	+7.9252	−9.8402	−9.0803	0.9003	9.9629
1224	8.4839	8.8484	0.3743	+8.1803	−9.9206	−9.2948	0.9005	9.9629
1225	8.4425	8.8046	0.5357	−7.8951	−9.1937	+9.0530	0.9027	9.9625
1226	8.4457	8.8058	0.4302	+7.9149	−9.8355	−9.0713	0.9043	9.9621
1227	8.6676	9.0226	0.1297	+8.5790	−9.9987	−9.5177	0.9086	9.9613
1228	8.5429	8.8798	0.3336	+8.3223	−9.9510	−9.4007	0.9235	9.9583
1229	8.4533	8.7890	0.4550	+7.7036	−9.7698	−8.8727	0.9246	9.9580
1230	8.5998	8.9297	0.2717	+8.4475	−9.9759	−9.4748	0.9293	9.9570
1231	8.7228	9.0526	0.0637	+8.6496	−9.9989	−9.5540	0.9294	9.9570
1232	8.4749	8.8028	0.4271	+7.9688	−9.8417	−9.1227	0.9310	9.9567
1233	8.5311	8.8588	0.3567	+8.2717	−9.9347	−9.3695	0.9311	9.9566
1234	8.4829	8.8028	0.4250	+7.9905	−9.8458	−9.1428	0.9375	9.9552
1235	9.0039	9.3198	0.8502	−8.9852	+9.8679	+9.6198	0.9407	9.9545
1236	8.4684	8.7828	0.4611	+7.6369	−9.7496	−8.8082	0.9420	9.9542
1237	8.4690	8.7768	0.4854	+6.5064	−9.6471	−7.6824	0.9473	9.9529
1238	8.6000	8.9074	0.6191	−8.4277	+9.4503	+9.4731	0.9476	9.9528
1239	8.5886	8.8934	0.3134	+8.3986	−9.9590	−9.4576	0.9497	9.9523
1240	8.4815	8.7791	0.4626	+7.6292	−9.7445	−8.8009	0.9555	9.9509
1241	8.5158	8.8129	0.4060	+8.1198	−9.8786	−9.2577	0.9559	9.9508
1242	8.5323	8.8264	0.5673	−8.1976	+8.4440	+9.3214	0.9583	9.9501
1243	8.4832	8.7763	0.4690	+7.5050	−9.7204	−8.6787	0.9591	9.9499
1244	8.5942	8.8754	0.3308	+8.3844	−9.9485	−9.4565	0.9685	9.9474
1245	8.5758	8.8561	0.3524	+8.3310	−9.9349	−9.4221	0.9691	9.9473
1246	8.6111	8.8854	0.3183	+8.4189	−9.9541	−9.4795	0.9739	9.9459
1247	8.5204	8.7943	0.4252	+8.0350	−9.8444	−9.1865	0.9741	9.9459
1248	8.5332	8.8041	0.4112	+8.1196	−9.8695	−9.2606	0.9765	9.9452
1249	8.5551	8.8230	0.5681	−8.2285	+8.5065	+9.3500	0.9789	9.9445
1250	+8.9027	−9.1681	+0.7620	−8.8652	+9.8019	+9.6411	−0.9807	−9.9440

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation
						h.	m.	s.	s.	°	′	″	″
1251	6877		Sagittarii,		5	19	54	48.47*	+ 3.840	122	28	22.1*	− 9.5
1252	6879	15	Vulpeculæ,		5		54	55.40*	2.469	62	39	28.0*	9.7
1253	6882		Vulpeculæ,		5		55	23.47	2.543	65	36	44.2	9.7
1254	6905	64	Draconis,	e	5		59	52.41	0.651	25	35	54.4	10.0
1255	6926	67	Draconis,	ρ	4	20	2	7.69*	0.300	22	33	14.1*	10.2
1256	6932	66	Draconis,		5		3	9.17*	0.964	28	26	19.7*	10.3
1257	6934	65	Aquilæ,	θ	3½		3	33.76*	3.103	91	15	44.8*	10.3
1258	6937	28	Cygni,	b^2	5		3	51.43*	2.228	53	35	56.7*	10.4
1259	6952	67	Aquilæ,	ρ	5		7	20.25	2.779	75	15	21.9	10.6
1260	6959		Cygni,		5		8	21.19	1.671	38	59	10.0	10.6
1261	6965	31	Cygni,	o^2	4		8	54.58*	1.890	43	42	40.6*	10.7
1262	6966		Vulpeculæ,		5		8	54.68	2.540	64	51	43.2	10.7
1263	6972	5	Capricorni,	a^1	4		9	19.76*	3.334	102	58	4.0*	10.7
1264	6973	23	Vulpeculæ,		4½		9	33.36	2.485	62	38	32.4	10.8
1265	6974	6	Capricorni,	a^2	3		9	43.67*	3.339	103	0	20.7*	10.7
1266	6976	33	Cygni,		4½		9	54.48*	1.401	33	53	23.0*	10.8
1267	6979	24	Vulpeculæ,		5		10	22.11	2.569	65	47	14.0	10.8
1268	6983	32	Cygni,		4½		10	49.97*	1.854	42	44	39.2*	10.8
1269	6991	8	Capricorni,	ν	5		12	20.30*	3.337	103	13	36.7*	10.9
1270	6995	9	Capricorni,	β	3½		12	34.69*	+ 3.380	105	15	3.4*	11.0
1271	6999		Ursæ Minoris,	λ	5		13	1.76*	−53.184	1	8	22.0*	11.0
1272	7004		Pavonis,	α	2		13	45.03	+ 4.807	147	12	35.3	11.0
1273	7005	1	Cephei,	κ	4½		13	50.39*	− 1.862	12	44	34.7*	11.0
1274	7022	37	Cygni,	γ	3		16	50.81*	+ 2.153	50	13	15.4*	11.3
1275	7027		Cygni,		5		17	25.78	2.126	49	27	4.6	11.3
1276	7029	39	Cygni,		5		17	52.25	2.395	58	17	26.5	11.4
1277	7031	10	Capricorni,	π	5		18	43.79*	3.446	108	41	57.1*	11.4
1278	7042	11	Capricorni,	ρ	5		20	17.88*	3.433	108	18	19.4*	11.5
1279	7058	69	Aquilæ,		5		21	48.62	3.145	93	22	49.1	11.6
1280	7067	41	Cygni,		4½		23	15.99	2.450	60	7	46.2	11.7
1281	7085	45	Cygni,	ω^2	5		25	24.79	1.858	41	33	2.5	11.9
1282	7088	2	Delphini,	ε	4		26	2.66*	2.868	79	12	11.2*	11.9
1283	7091	46	Cygni,	ω^2	5		26	40.95	1.851	41	17	1.5	11.9
1284	7096		Indi,	α	3		26	59.94	4.264	137	48	34.0	12.0
1285	7098	2	Cephei,	θ	5		27	3.34*	1.020	27	30	32.1*	11.9
1286	7106		Pavonis,	υ	5		28	7.79	5.642	157	17	0.3	11.9
1287	7107	4	Delphini,	ζ	5		28	17.73*	2.807	75	50	23.9*	12.1
1288	7121	6	Delphini,	β	4		30	30.86*	2.813	75	55	25.0*	12.2
1289	7122	71	Aquilæ,		5		30	35.24*	3.104	91	37	31.1	12.2
1290	7129		Pavonis,	β	3		31	22.68	5.524	156	44	9.6	12.2
1291	7134	15	Capricorni,	υ	5		31	30.26*	3.426	108	39	45.9*	12.3
1292	7137	8	Delphini,	θ	4½		31	39.29	2.833	77	12	26.7	12.3
1293	7149	9	Delphini,	α	3½		32	40.26*	2.791	74	36	49.4*	12.4
1294	7165		Pavonis,	σ	4½		35	0.51	5.823	159	19	1.5	12.6
1295	7171	50	Cygni,	α	1		36	19.12*	2.044	45	15	12.4*	12.6
1296	7173	11	Delphini,	δ	4		36	27.50*	2.804	75	27	36.8	12.6
1297	7177	16	Capricorni,	ψ	4½		37	12.31*	+ 3.570	115	48	21.7*	12.5
1298	7178	75	Draconis,		5½		37	25.20*	− 3.382	9	5	41.9*	12.7
1299	7184		Ursæ Minoris,		5		37	47.97	−41.226	1	20	4.6	12.7
1300	7196	2	Aquarii,	ε	4½	20	39	33.13*	+ 3.259	100	2	28.6*	−12.8

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a'	b'	c'	d'
1251	+8.5792	−8.8408	+0.5817	−8.3091	+9.0204	+9.4114	−0.9837	−9.9431
1252	8.5572	8.8183	0.3917	+8.2193	−9.8967	−9.3440	0.9841	9.9429
1253	8.5480	8.8070	0.4048	+8.1638	−9.8790	−9.2993	0.9857	9.9425
1254	8.8869	9.1260	9.8149	+8.8421	−9.9862	−9.6537	1.0008	9.9377
1255	8.9460	9.1753	9.4738	+8.9114	−9.9831	−9.6713	1.0081	9.9352
1256	8.8553	9.0801	9.9776	+8.7994	−9.9832	−9.6533	1.0114	9.9340
1257	8.5345	8.7576	0.4908	−6.8775	−9.6181	+8.0535	1.0127	9.9336
1258	8.6296	8.8514	0.3474	+8.4030	−9.9336	−9.4848	1.0136	9.9332
1259	8.5607	8.7677	0.4428	+7.9663	−9.8034	−9.1279	1.0244	9.9292
1260	8.7505	8.9532	0.2230	+8.6410	−9.9692	−9.6159	1.0275	9.9280
1261	8.7114	8.9118	0.2759	+8.5704	−9.9597	−9.5860	1.0292	9.9273
1262	8.5941	8.7945	0.4048	+8.2223	−9.8764	−9.3552	1.0292	9.9273
1263	8.5634	8.7620	0.5225	−7.9144	−9.3679	+9.0792	1.0304	9.9268
1264	8.6043	8.8020	0.3956	+8.2666	−9.8885	−9.3912	1.0311	9.9266
1265	8.5646	8.7616	0.5226	−7.9169	−9.3672	+9.0817	1.0316	9.9264
1266	8.8075	9.0037	0.1435	+8.7267	−9.9742	−9.6491	1.0322	9.9261
1267	8.5952	8.7895	0.4089	+8.2081	−9.8700	−9.3442	1.0335	9.9256
1268	8.7249	8.9172	0.2680	+8.5908	−9.9602	−9.5986	1.0349	9.9250
1269	8.5727	8.7588	0.5230	−7.9322	−9.3631	+9.0966	1.0393	9.9232
1270	8.5773	8.7624	+0.5284	−7.9973	−9.2978	+9.1578	1.0400	9.9229
1271	0.2644	0.4477	−1.7254	+0.2643	−9.9267	−9.7390	1.0413	9.9223
1272	8.8314	9.0117	+0.6815	−8.7561	+9.6689	+9.6658	1.0434	9.9214
1273	9.2218	9.4017	−0.2699	+9.2110	−9.9588	−9.7306	1.0437	9.9213
1274	8.6882	8.8558	+0.3325	+8.4943	−9.9358	−9.5560	1.0522	9.9175
1275	8.6947	8.8599	0.3276	+8.5077	−9.9377	−9.5645	1.0538	9.9167
1276	8.6469	8.8103	0.3783	+8.3676	−9.9045	−9.4734	1.0550	9.9162
1277	8.6026	8.7625	0.5369	−8.1086	−9.1688	+9.2611	1.0574	9.9150
1278	8.6059	8.7594	0.5356	−8.1029	−9.1903	+9.2564	1.0616	9.9130
1279	8.5881	8.7356	0.4962	−7.3587	−9.5857	+8.5340	1.0657	9.9109
1280	8.6531	8.7948	0.3888	+8.3504	−9.8922	−9.4646	1.0695	9.9090
1281	8.7751	8.9082	0.2685	+8.6492	−9.9496	−9.6470	1.0751	9.9060
1282	8.6062	8.7368	0.4572	+7.8788	−9.7611	−9.0471	1.0767	9.9051
1283	8.7806	8.9087	0.2670	+8.6565	−9.9489	−9.6520	1.0784	9.9042
1284	8.7737	8.9006	0.6286	−8.6435	+9.4804	+9.6467	1.0792	9.9038
1285	8.9365	9.0631	0.0060	+8.8843	−9.9595	−9.7250	1.0793	9.9037
1286	9.0169	9.1393	0.7497	−8.9819	+9.7550	+9.7447	1.0820	9.9022
1287	8.6175	8.7392	0.4474	+8.0060	−9.7896	−9.1687	1.0824	9.9019
1288	8.6229	8.7359	0.4480	+8.0089	−9.7878	−9.1717	1.0879	9.8987
1289	8.6100	8.7227	0.4914	−7.0627	−9.6142	+8.2387	1.0881	9.8986
1290	9.0152	9.1248	0.7424	−8.9784	+9.7436	+9.7510	1.0901	9.8975
1291	8.6355	8.7446	0.5349	−8.1406	−9.2000	+9.2933	1.0904	9.8973
1292	8.6233	8.7319	0.4520	+7.9686	−9.7766	−9.1337	1.0907	9.8970
1293	8.6307	8.7353	0.4443	+8.0545	−9.7973	−9.2147	1.0932	9.8955
1294	9.0724	9.1679	0.7659	−9.0435	+9.7607	+9.7676	1.0988	9.8920
1295	8.7721	8.8625	0.3100	+8.6197	−9.9318	−9.6472	1.1018	9.8900
1296	8.6380	8.7278	0.4474	+8.0378	−9.7888	−9.1997	1.1022	9.8898
1297	8.6712	8.7582	+0.5528	−8.3100	−8.7308	+9.4405	1.1039	9.8886
1298	9.4272	9.5134	−0.5299	+9.4218	−9.9202	−9.7967	1.1044	9.8883
1299	0.2599	0.3443	−1.6152	+0.2598	−9.8931	−9.8031	1.1055	9.8876
1300	+8.6377	−8.7156	+0.5122	−7.8791	−9.4676	+9.0485	−1.1093	−9.8850

CATALOGUE OF 1500 STARS.

No.	B.A.C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist.. Jan. 1, 1850			Annual Variation
						h.	m.	s.	s.	°	′	″	″
1301	7200	12	Delphini,	γ	4	20	39	42.05	+2.785	74	24	44.9	−12.7
1302	7201	3	Aquarii,		4		39	49.25*	3.173	95	34	24.3*	12.8
1303	7204	53	Cygni,	ε	3		40	8.55	2.426	56	35	20.4	13.2
1304	7207		Microscopii,	α	4½		40	35.59	3.790	124	19	51.1*	12.7
1305	7213	54	Cygni,	λ	5		41	34.06	2.333	54	3	27.9	13.0
1306	7215		Cephei,		5		41	37.72*	1.495	32	57	24.9*	12.8
1307	7220	3	Cephei,	η	3½		42	13.97*	1.232	28	44	33.6*	13.8
1308	7228		Indi,	β	4		43	2.81	4.757	149	0	54.8	13.0
1309	7239	6	Aquarii,	μ	4½		44	33.56*	3.246	99	32	33.3*	13.1
1310	7250		Octantis,	α	4½		46	23.07	7.614	167	35	17.5	12.7
1311	7253	57	Cygni,		5		47	56.44*	2.120	46	10	43.1*	13.4
1312	7256	32	Vulpeculæ,		4½		48	10.05*	2.557	62	30	36.5*	13.4
1313	7277	58	Cygni,	ν	4		51	35.04*	2.234	49	24	28.8*	13.6
1314	7281		Cephei,		5		52	16.14*	+1.605	33	41	18.5*	13.6
1315	7291	76	Draconis,		5		53	7.61*	−3.832	8	1	43.9*	13.7
1316	7299		Cephei,		5		54	13.17*	−2.422	10	0	47.2*	13.8
1317	7305	22	Capricorni,	η	5		55	51.59*	+3.430	110	26	40.9*	13.9
1318	7333	62	Cygni,	ξ	4		59	28.68*	2.180	46	40	5.7*	14.1
1319	7336	61	Cygni,		5½	21	0	10.71	2.691	51	59	8.0	17.4
1320	7344	13	Aquarii,	ν	5		1	25.03*	3.276	101	58	32.7*	14.2
1321	7345	63	Cygni,	f^2	5		1	26.08*	2.063	42	57	8.7*	14.2
1322	7350	5	Equulei,	γ	5		3	2.89	2.923	80	28	11.1	14.2
1323	7368	64	Cygni,	ζ	3		6	33.22*	2.550	60	23	9.4*	14.5
1324	7372	7	Equulei,	δ	4½		7	10.57	2.927	80	35	52.5	14.3
1325	7374	29	Capricorni,		5		7	26.34*	3.334	105	47	29.4*	14.6
1326	7377		Cephei,		5		7	59.05*	1.542	30	37	44.3*	14.6
1327	7380	8	Equulei,	α	4½		8	19.45*	+3.005	85	22	10.1*	14.6
1328	7381	77	Draconis,		5½		8	22.65	−1.046	12	28	59.6	14.7
1329	7385	65	Cygni,	τ	5		8	48.33*	+2.391	52	35	33.0*	15.2
1330	7386	4	Piscis Aust.,		5		8	49.71*	3.665	122	47	45.8*	14.6
1331	7398	67	Cygni,	σ	4½		11	31.69	2.353	51	13	54.3	14.8
1332	7399	66	Cygni,	υ	4½		11	45.12	2.462	55	43	49.3	14.9
1333	7407	32	Capricorni,	ι	5		13	53.20*	3.357	107	28	12.1*	15.0
1334	7409		Pavonis,	γ	3		13	58.74	5.081	156	2	27.5	15.7
1335	7416	5	Cephei,	α	3		14	59.74*	1.439	28	2	55.5*	15.0
1336	7418	1	Pegasi,		4		15	9.08	2.776	70	50	4.3	15.1
1337	7423		Indi,	γ	5		15	31.42	4.337	145	18	18.9	14.9
1338	7428	6	Cephei,		5		16	15.07	1.259	25	45	47.3	15.1
1339	7445	34	Capricorni,	ζ	4		18	5.65*	3.443	113	3	27.5*	15.3
1340	7478	22	Aquarii,	β	3		23	39.51*	3.168	96	13	41.7*	15.6
1341	7480	71	Cygni,	g	5		23	54.95*	2.206	44	7	8.0*	15.6
1342	7493	8	Cephei,	β	3		26	42.32*	0.807	20	5	49.7*	15.6
1343	7495		Cephei,		5		26	52.03	1.647	30	12	3.9	15.7
1344	7503	73	Cygni,	ρ	4½		28	20.60*	2.250	45	4	10.0*	15.7
1345	7506	39	Capricorni,	ε	5		28	40.46*	+3.375	110	8	6.5*	15.8
1346	7510		Cephei,		5½		29	6.18	−1.389	10	7	48.8	15.9
1347	7514	23	Aquarii,	ξ	5		29	45.77*	+3.202	98	31	26.1*	15.8
1348	7522	4	Pegasi,		5		31	1.09	3.004	84	54	10.9	15.9
1349	7525	40	Capricorni,	γ	4		31	46.40*	3.341	107	20	13.2*	16.0
1350	7539	41	Capricorni,		5	21	33	27.80	+3.435	113	56	16.3	−16.0

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	*b*	*c*	*d*	*a′*	*b′*	*c′*	*d′*
1301	+8.6476	−8.7249	+0.4448	+8.0769	−9.7953	−9.2367	−1.1096	−9.8847
1302	8.6336	8.7105	0.5011	−7.6210	−9.5532	+8.7950	1.1099	9.8845
1303	8.7108	8.7864	0.3794	+8.4516	−9.8942	−9.5493	1.1106	9.8840
1304	8.7164	8.7903	0.5762	−8.4677	+8.8615	+9.5607	1.1116	9.8833
1305	8.7272	8.7974	0.3678	+8.4958	−9.9022	−9.5802	1.1138	9.8817
1306	8.9000	8.9700	0.1762	+8.8238	−9.9420	−9.7355	1.1139	9.8817
1307	8.9549	9.0226	0.0858	+8.8978	−9.9420	−9.7560	1.1153	9.8807
1308	8.9271	8.9916	0.6773	−8.8603	+9.6294	+9.7480	1.1171	9.8793
1309	8.6481	8.7068	0.5105	−7.8677	−9.4816	+9.0377	1.1204	9.8769
1310	9.3137	9.3654	0.8816	−9.3034	+9.8042	+9.8118	1.1243	9.8738
1311	8.7911	8.8368	0.3257	+8.6314	−9.9179	−9.6658	1.1276	9.8712
1312	8.7018	8.7467	0.4072	+8.3661	−9.8620	−9.4901	1.1281	9.8708
1313	8.7764	8.8083	0.3486	+8.5898	−9.9064	−9.6463	1.1352	9.8649
1314	8.9142	8.9435	+0.2056	+8.8344	−9.9287	−9.7545	1.1366	9.8637
1315	9.5149	9.5410	−0.5832	+9.5106	−9.8929	−9.8318	1.1383	9.8621
1316	9.4219	9.4438	−0.3833	+9.4153	−9.8972	−9.8316	1.1405	9.8602
1317	8.6937	8.7094	+0.5352	−8.2369	−9.1909	+9.3847	1.1437	9.8572
1318	8.8106	8.8126	0.3378	+8.6471	−9.9042	−9.8650	1.1507	9.8505
1319	8.7773	8.7766	0.3678	+8.5668	−9.8904	−9.6393	1.1520	9.8492
1320	8.6856	8.6803	0.5145	−8.0026	−9.4451	+9.1692	1.1544	9.8468
1321	8.8427	8.8373	0.3142	+8.7072	−9.9085	−9.7166	1.1544	9.8468
1322	8.6851	8.6736	0.4645	+7.9041	−9.7352	−9.0742	1.1574	9.8436
1323	8.7463	8.7214	0.4063	+8.4401	−9.8553	−9.5554	1.1638	9.8367
1324	8.6925	8.6652	0.4652	+7.9056	−9.7323	−9.0758	1.1649	9.8355
1325	8.7038	8.6755	0.5223	−8.1386	−9.3646	+9.2979	1.1654	9.8349
1326	8.9809	8.9506	0.1848	+8.9156	−9.9074	−9.7988	1.1662	9.8338
1327	8.6900	8.6584	+0.4767	+7.5970	−9.6880	−8.7717	1.1669	9.8331
1328	9.3539	9.3222	−0.0173	+9.3435	−9.8800	−9.8544	1.1670	9.8330
1329	8.7895	8.7561	+0.3758	+8.5730	−9.8793	−9.6491	1.1678	9.8321
1330	8.7649	8.7314	0.5630	−8.4986	+7.5052	+9.5993	1.1678	9.8321
1331	8.8023	8.7585	0.3711	+8.5990	−9.8798	−9.6670	1.1725	9.8265
1332	8.7774	8.7328	0.3910	+8.5280	−9.8668	−9.6212	1.1729	9.8260
1333	8.7187	8.6659	0.5250	−8.1961	−9.3322	+9.3517	1.1765	9.8215
1334	9.0897	9.0366	0.7045	−9.0506	+9.6329	+9.8353	1.1766	9.8213
1335	9.0277	8.9707	0.1511	+8.9735	−9.8954	−9.8219	1.1783	9.8191
1336	8.7251	8.6675	0.4416	+8.2413	−9.7967	−9.3927	1.1786	9.8188
1337	8.9456	8.8866	0.6376	−8.8606	+9.4679	+9.7920	1.1792	9.8180
1338	9.0640	9.0022	0.0988	+9.0185	−9.8914	−9.8327	1.1804	9.8164
1339	8.7413	8.6724	0.5366	−8.3342	−9.1617	+9.4741	1.1834	9.8123
1340	8.7165	8.6262	0.5000	−7.7518	−9.5599	+8.9253	1.1922	9.7997
1341	8.8716	8.7803	0.3429	+8.7276	−9.8774	−9.7464	1.1926	9.7991
1342	9.1824	9.0803	9.9057	+9.1552	−9.8642	−9.8673	1.1968	9.7925
1343	9.0172	8.9144	0.2168	+8.9538	−9.8773	−9.8315	1.1971	9.7921
1344	8.8709	8.7624	0.3524	+8.7199	−9.8699	−9.7460	1.1993	9.7885
1345	8.7488	8.6390	+0.5278	−8.2857	−9.2929	+9.4344	1.1998	9.7877
1346	9.4768	9.3653	−0.1784	+9.4700	−9.8316	−9.8913	1.2004	9.7867
1347	8.7279	8.6138	+0.5042	−7.8988	−9.5299	+9.0701	1.2013	9.7850
1348	8.7266	8.6076	0.4769	+7.6752	−9.6866	−8.8496	1.2032	9.7819
1349	8.7461	8.6241	0.5214	−8.2203	−9.3705	+9.3762	1.2042	9.7800
1350	+8.7674	−8.6387	+0.5346	−8.3756	−9.1889	+9.5127	−1.2066	−9.7758

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annu Variati
						h.	m.	s.	s.	°	′	″	″
1351	7542	9	Cephei,		5	21	33	53.84*	+1.609	28	35	36.1*	—16.
1352	7543	43	Capricorni,	κ	5		34	16.47*	3.364	109	32	49.9*	16.
1353	7557	9	Piscis Aust.,	ι	4½		36	0.02*	3.600	123	42	25.2*	16.
1354	7560	80	Cygni,	π^1	4½		36	46.26*	2.122	39	29	36.7*	16.
1355	7561	8	Pegasi,	ε	2½		36	49.11*	2.951	80	48	37.0*	16.
1356	7567	9	Pegasi,		4½		37	24.73	2.844	73	20	4.7	16.
1357	7568	78	Cygni,	μ^1	5		37	26.08*	2.673	61	55	58.3*	16.
1358	7571	10	Pegasi,	κ	4		37	51.26	2.712	65	2	32.6	16.
1359	7580	49	Capricorni,	δ	3½		38	45.30*	3.323	106	48	18.5*	16.
1360	7583	10	Piscis Aust.,	θ	5		38	55.41	3.547	121	35	24.2*	16.
1361	7588	11	Cephei,		4½		39	42.86*	0.908	19	22	43.0*	16.
1362	7595	10	Cephei,	ν	4½		41	7.53*	1.731	29	34	12.4*	16.
1363	7597	78	Draconis,		5		41	13.03*	0.762	18	22	1.6*	16.
1364	7598	81	Cygni,	π^2	5		41	15.38*	2.210	41	22	58.4*	16.
1365	7607	14	Pegasi,		5		43	12.63	2.648	60	31	19.9	16.
1366	7610		Cephei,		5		44	21.68*	1.096	20	32	37.1*	16.
1367	7613		Gruis,	γ	3		44	49.99	3.672	128	4	3.9	16.
1368	7618	51	Capricorni,	μ	5		45	6.72*	3.285	104	15	18.1*	16.
1369	7633		Indi,	δ	5		47	40.33	4.129	145	42	8.6	16.
1370	7634		Indi,	κ^1	5		47	51.53	4.289	149	43	26.4	16.
1371	7657	12	Piscis Aust.,	η	5		52	12.57	3.469	119	10	15.0	17.
1372	7672	31	Aquarii,	o	5		55	33.31	3.110	92	52	37.2	17.
1373	7684		Gruis,	λ	5		57	3.47	3.650	130	15	55.1	17.
1374	7686	16	Cephei,		5		57	5.37*	0.885	17	32	0.0*	17.
1375	7688	34	Aquarii,	α	3		58	4.64*	3.085	91	2	47.4*	17.
1376	7689	22	Pegasi,	ν	5		58	6.89	3.031	85	40	22.3	17.
1377	7691	33	Aquarii,	ι	4½		58	19.77*	3.252	104	35	42.3*	17.
1378	7692		Gruis,	α	2		58	45.09	3.824	137	41	3.2	17.
1379	7699	18	Cephei,		5		59	22.29	1.770	27	36	34.2	17.
1380	7700	17	Cephei,	ξ	5		59	26.04*	1.738	26	6	6.3*	17.
1381	7706	24	Pegasi,	ι	4	22	0	1.84	2.788	65	23	7.4	17.
1382	7721	27	Pegasi,	π^1	5		2	35.11*	2.655	57	33	31.0*	17.
1383	7723	26	Pegasi,	θ	4		2	37.97*	3.033	84	32	17.3*	17.
1384	7731	29	Pegasi,	π^2	4		3	19.76*	2.658	57	33	21.8*	17.
1385	7746		Lacertæ,		5		5	19.65	2.304	39	55	0.6	17.
1386	7749	21	Cephei,	ζ	4		5	39.53*	2.071	32	32	12.9*	17.
1387	7756		Gruis,	μ^1	5		6	33.93	3.667	132	5	29.4	17.
1388	7758	24	Cephei,		5		6	54.77	1.170	18	23	50.5	17.
1388	7765		Lacertæ,		5		7	26.87	2.574	51	1	43.4	17.
1390	7767		Tucanæ,	α	3		8	10.83	4.194	151	0	14.5	17.
1391	7773	43	Aquarii,	θ	4½		8	54.85*	3.175	98	31	41.6*	17.
1392	7777	1	Lacertæ,		5		9	26.24	2.605	52	59	47.4	17.
1393	7778	23	Cephei,	ε	4½		9	31.13*	2.197	33	42	11.1*	17.
1394	7788	30	Pegasi,		5		12	54.79	3.025	84	57	46.5	17.
1395	7790	47	Aquarii,		5		13	19.93	3.319	112	20	49.7	17.
1396	7795	48	Aquarii,	γ	3		13	54.42*	3.106	92	8	28.5*	17.
1397	7796	31	Pegasi,		4½		14	8.29	2.955	78	32	52.7.	17.
1398	7800	2	Lacertæ,		5		14	50.22*	2.465	44	13	1.9*	18.
1399	7808		Tucanæ,	δ	5		16	35.73	4.348	155	43	41.9	17.
1400	7814	52	Aquarii,	π	5	22	17	36.99*	+3.068	89	22	55.4*	—18.

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
1351	+9.0490	−8.9186	+0.2070	+8.9925	−9.8635	−9.8485	−1.2073	−9.7747
1352	8.7553	8.6234	0.5254	−8.2798	−9.3228	+9.4301	1.2078	9.7737
1353	8.8118	8.6731	0.5557	−8.5561	−8.5416	+9.6522	1.2102	9.7692
1354	8.9295	8.7877	0.3267	+8.8169	−9.8618	−9.7965	1.2112	9.7672
1355	8.7386	8.5966	0.4690	+7.9419	−9.7175	−9.1124	1.2113	9.7671
1356	8.7524	8.6081	0.4529	+8.2100	−9.7655	−9.3674	1.2121	9.7655
1357	8.7882	8.6437	0.4241	+8.4607	−9.8176	−9.5825	1.2121	9.7655
1358	8.7770	8.6308	0.4328	+8.4022	−9.8052	−9.5358	1.2127	9.7643
1359	8.7546	8.6048	0.5190	−8.2156	−9.3950	+9.3728	1.2139	9.7619
1360	8.8055	8.6551	0.5497	−8.5247	−8.8274	+9.6311	1.2142	9.7615
1361	9.2160	9.0624	9.9476	+9.1907	−9.8364	−9.8876	1.2152	9.7594
1362	9.0455	8.8862	0.2377	+8.9849	−9.8510	−9.8542	1.2171	9.7555
1363	9.2404	9.0808	9.8912	+9.2177	−9.8305	−9.8923	1.2172	9.7553
1364	8.9187	8.7588	0.3438	+8.7939	−9.8536	−9.7903	1.2172	9.7552
1365	8.8017	8.6339	0.4226	+8.4937	−9.8158	−9.6096	1.2198	9.7498
1366	9.1977	9.0252	0.0335	+9.1692	−9.8291	−9.8905	1.2213	9.7465
1367	8.8474	8.6730	0.5627	−8.6374	+7.0792	+9.7096	1.2219	9.7452
1368	8.7575	8.5819	0.5131	−8.1488	−9.4532	+9.3113	1.2222	9.7444
1369	8.9962	8.8101	0.6171	−8.9133	+9.3147	+9.8402	1.2254	9.7371
1370	9.0448	8.8579	0.6354	−8.9811	+9.4014	+9.8597	1.2256	9.7365
1371	8.8115	8.6064	0.5398	−8.4994	−9.0828	+9.6166	1.2309	9.7236
1372	8.7570	8.5377	0.4921	−7.4577	−9.6101	+8.6332	1.2348	9.7132
1373	8.8756	8.6498	0.5622	−8.6861	−6.9031	+9.7447	1.2365	9.7085
1374	9.2793	9.0534	9.9582	+9.2586	−9.7909	−9.9136	1.2365	9.7084
1375	8.7594	8.5292	0.4890	−7.0209	−9.6280	+8.1970	1.2376	9.7052
1376	8.7606	8.5303	0.4799	+7.6383	−9.6730	−8.8131	1.2377	9.7051
1377	8.7739	8.5426	0.5115	−8.1752	−9.4660	+9.3371	1.2379	9.7044
1378	8.9319	8.6988	0.5811	−8.8008	+8.9085	+9.8051	1.2384	9.7030
1379	9.0948	8.8589	0.2520	+9.0423	−9.8104	−9.8843	1.2391	9.7010
1380	9.1174	8.8813	0.2306	+9.0707	−9.8074	−9.8902	1.2392	9.7008
1381	8.8028	8.5641	0.4416	+8.4225	−9.7801	−9.5572	1.2398	9.6989
1382	8.8379	8.5880	0.4239	+8.5674	−9.7988	−9.6698	1.2425	9.6904
1383	8.7663	8.5161	0.4783	+7.7448	−9.6800	−8.9189	1.2426	9.6902
1384	8.8387	8.5855	0.4243	+8.5683	−9.7978	−9.6707	1.2433	9.6879
1385	8.9598	8.6976	0.3625	+8.8446	−9.8105	−9.8280	1.2454	9.6810
1386	9.0368	8.7731	0.3155	+8.9626	−9.8040	−9.8694	1.2458	9.6799
1387	8.8979	8.6301	0.5616	−8.7242	−7.5563	+9.7708	1.2467	9.6767
1388	9.2696	9.0002	0.0669	+9.2468	−9.7683	−9.9221	1.2471	9.6755
1389	8.8786	8.6068	0.4083	+9.6772	−9.8021	−9.7440	1.2476	9.6736
1390	9.0845	8.8094	0.6234	−9.0264	+9.3002	+9.8880	1.2483	9.6710
1391	8.7756	8.4971	0.5002	−7.9467	−9.5564	+9.1180	1.2491	9.6683
1392	8.8690	8.5880	0.4155	+8.6485	−9.7964	−9.7269	1.2496	9.6665
1393	9.0272	8.7459	0.3307	+8.9472	−9.7969	−9.8675	1.2497	9.6662
1394	8.7764	8.4792	0.4797	+7.7199	−9.6739	−8.8943	1.2530	9.6536
1395	8.8090	8.5099	0.5208	−8.3890	−9.3632	+9.5312	1.2534	9.6521
1396	8.7759	8.4741	0.4904	−7.3484	−9.6201	+8.5242	1.2539	9.6499
1397	8.7846	8.4816	0.4698	+8.0824	−9.7108	−9.2498	1.2541	9.6490
1398	8.9330	8.6268	0.3913	+8.7884	−9.7924	−9.8079	1.2548	9.6463
1399	9.1642	8.8495	0.6398	−9.1240	+9.3406	+9.9140	1.2564	9.6395
1400	+8.7791	−8.4594	+0.4864	+6.8120	−9.6421	−7.9880	−1.2574	−9.6355

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
1401	7815	3	Lacertæ,	β	4½	22	17	40.35*	+2.343	38	31	16.7*	—17.9
1402	7820	4	Lacertæ,		5		18	26.49	2.416	41	16	57.9	18.1
1403	7828		Gruis,	δ^1	4		20	17.06	3.624	134	15	35.5	18.1
1404	7830		Gruis,	δ^2	5		20	46.75	3.632	134	30	56.6	18.0
1405	7832	55	Aquarii,	ζ	4		21	6.37*	3.092	90	47	8.1*	18.2
1406	7840	57	Aquarii,	σ	5		22	42.20*	3.184	101	26	37.3*	18.3
1407	7841		Tucanæ,	ν	5		22	47.98	4.118	152	45	4.1	18.1
1408	7842	17	Piscis Aust.,	β	4		22	57.92*	3.437	123	6	50.6*	18.2
1409	7845	5	Lacertæ,		5		23	18.85	2.503	43	3	33.9	18.3
1410	7848	27	Cephei,	δ	4½		23	36.60*	+2.213	32	21	5.3*	18.3
1411	7851		Ursæ Minoris,		5½		24	30.09*	—3.501	4	38	59.3*	18.3
1412	7855	7	Lacertæ,	α	4		25	7.35*	+2.457	40	29	14.4*	18.3
1413	7857	28	Cephei,	.	5½		25	31.26	0.537	11	58	41.8	18.3
1414	7864	59	Aquarii,	υ	5		26	28.91	3.296	111	28	26.5	18.3
1415	7868	62	Aquarii,	η	4		27	38.77*	3.087	90	53	20.3*	18.4
1416	7886		Octantis,	β	5		30	21.23	6.688	172	9	53.6	18.5
1417	7896	31	Cephei,		5		32	3.90*	1.486	17	8	4.8*	18.6
1418	7898	18	Piscis Aust.,	ε	4		32	21.01*	3.338	117	49	27.6*	18.5
1419	7902	30	Cephei,		5		33	20.56*	2.114	27	11	39.3*	18.6
1420	7904		Gruis,	β	3		33	41.22	3.626	137	40	1.8	18.6
1421	7908	42	Pegasi,	ζ	3		33	58.89*	2.990	79	57	0.3*	18.6
1422	7914	43	Pegasi,	ο	5		34	43.24*	2.809	61	28	25.8*	18.6
1423	7923	44	Pegasi,	η	3		35	58.57*	2.805	60	33	43.0*	18.7
1424	7925		Gruis,	η	5		36	23.50	3.731	144	17	18.3	18.4
1425	7943	46	Pegasi,	ξ	5		39	12.02	2.993	78	35	38.4	18.3
1426	7945	47	Pegasi,	λ	4½		39	18.66	2.883	67	13	19.4	18.8
1427	7946		Gruis,	ε	4		39	28.04	3.666	142	6	14.5	18.8
1428	7958	48	Pegasi,	μ	4		42	46.10*	2.888	66	11	20.8*	18.9
1429	7961		Cephei,		5		43	34.66	2.443	34	53	31.3	18.9
1430	7966	22	Piscis Aust.,	γ	5		44	10.71	3.361	123	40	14.8*	19.0
1431	7967	32	Cephei,	ι	4		44	21.28*	2.114	24	35	15.6*	18.8
1432	7970	73	Aquarii,	λ	4		44	47.10*	3.133	98	22	34.8*	19.0
1433	7973		Cephei,		5		45	31.53	2.304	29	5	59.3	19.0
1434	7980	76	Aquarii,	δ	3		46	40.95*	+3.195	106	37	1.4*	19.0
1435	7990		Cephei,		5½		47	55.54*	—0.006	7	38	32.0*	19.1
1436	7992	24	Piscis Aust.,	α	1		49	21.03*	+3.335	120	24	57.1*	18.9
1437	8008		Gruis,	ζ	5		52	0.19	+3.610	143	33	22.8	19.2
1438	8023	1	Andromedæ,	ο	4		55	1.79*	+2.745	48	28	44.3*	19.2
1439	8026		Cephei,		5½		55	25.07	—0.146	6	27	23.7	19.3
1440	8031	4	Piscium,	β	5		56	14.67*	+3.057	86	59	11.3*	19.2
1441	8032	53	Pegasi,	β	2		56	30.53*	2.898	62	43	46.8*	19.4
1442	8034	54	Pegasi,	α	2		57	17.51*	2.985	75	36	2.8*	19.3
1443	8039		Cephei,		5		57	50.89	2.251	23	35	55.2	19.3
1444	8043		Gruis,	θ	5		58	24.79	3.414	134	19	43.8	19.2
1445	8051	55	Pegasi,		5		59	27.03	3.021	81	23	58.4	19.3
1446	8052	56	Pegasi,		4½		59	48.85	2.916	65	20	25.0	19.3
1447	8062	88	Aquarii,	c^2	4½	23	1	26.57	3.212	111	59	6.2	19.4
1448	8067		Gruis,	ι	5		1	50.78	3.419	136	3	23.8	19.7
1449	8069	89	Aquarii,	c^3	5		1	53.85	3.219	113	16	7.2	19.4
1450	8074	33	Cephei,	π	5	23	3	8.53*	+1.882	15	25	22.1*	—19.3

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
1401	+8.9847	—8.6649	+0.3702	+8.8782	—9.7841	—9.8486	—1.2574	—9.6353
1402	8.9604	8.6367	0.3834	+8.8363	—9.7843	—9.8318	1.2581	9.6322
1403	8.9264	8.5937	0.5587	—8.7702	—8.2201	+9.8013	1.2598	9.6248
1404	8.9288	8.5936	0.5590	—8.7745	—8.1847	+9.8037	1.2602	9.6228
1405	8.7822	8.4454	0.4883	—6.9192	—9.6317	+8.0953	1.2605	9.6214
1406	8.7923	8.4474	0.5027	—8.0898	—9.5361	+9.2572	1.2619	9.6148
1407	9.1229	8.7776	0.6169	—9.0718	+9.2232	+9.9086	1.2619	9.6144
1408	8.8607	8.5146	0.5352	—8.5982	—9.1433	+9.6973	1.2621	9.6137
1409	8.9498	8.6019	0.3953	+8.8135	—9.7749	—9.8239	1.2624	9.6122
1410	9.0559	8.7064	+0.3442	+8.9826	—9.7613	—9.8871	1.2626	9.6110
1411	9.8762	9.5219	—0.5535	+9.8747	—9.6419	—9.9598	1.2634	9.6069
1412	8.9731	8.6160	+0.3875	+8.8543	—9.7692	—9.8428	1.2639	9.6045
1413	9.4688	9.1096	9.7370	+9.4592	—9.6831	—9.9524	1.2642	9.6028
1414	8.8179	8.4537	0.5158	—8.3815	—9.4125	+9.5264	1.2650	9.5986
1415	8.7877	8.4174	0.4884	—6.9785	—9.6312	+8.1546	1.2660	9.5934
1416	9.6552	9.2706	0.8309	—9.6512	+9.5031	+9.9618	1.2681	9.5812
1417	9.3219	8.9280	0.1605	+9.3022	—9.6863	—9.9475	1.2694	9.5733
1418	8.8447	8.4492	0.5230	—8.5138	—9.3214	+9.6365	1.2697	9.5719
1419	9.1322	8.7312	0.3243	+9.0813	—9.7216	—9.9173	1.2704	9.5672
1420	8.9641	8.5612	0.5576	—8.8328	—8.3075	+9.8372	1.2707	9.5656
1421	8.7993	8.3948	0.4748	+8.0411	—9.6914	—9.2105	1.2709	9.5642
1422	8.8493	8.4407	0.4482	+8.5284	—9.7442	—9.6482	1.2714	9.5606
1423	8.8541	8.4384	0.4472	+8.5456	—9.7436	—9.6617	1.2724	9.5544
1424	9.0282	8.6101	0.5722	—8.9377	+8.5611	+9.8800	1.2727	9.5524
1425	8.8050	8.3708	0.4739	+8.1011	—9.6935	—9.2686	1.2747	9.5382
1426	8.8317	8.3968	0.4589	+8.4196	—9.7272	—9.5604	1.2747	9.5376
1427	9.0082	8.5724	0.5637	—8.9054	+7.6990	+9.8698	1.2748	9.5368
1428	8.8374	8.3819	0.4587	+8.4435	—9.7247	—9.5809	1.2771	9.5194
1429	9.0419	8.5814	0.3879	+8.9558	—9.7132	—9.8893	1.2776	9.5150
1430	8.8794	8.4153	0.5263	—8.6233	—9.2579	+9.7196	1.2780	9.5117
1431	9.1806	8.7154	0.3271	+9.1394	—9.6770	—9.9346	1.2781	9.5107
1432	8.8048	8.3369	0.4961	—7.9681	—9.5824	+9.1396	1.2784	9.5083
1433	9.1136	8.6411	0.3624	+9.0550	—9.6902	—9.9181	1.2789	9.5042
1434	8.8198	8.3400	+0.5046	—8.2762	—9.5131	+9.4337	1.2796	9.4976
1435	9.6783	9.1905	—8.0682	+9.6744	—9.5615	—9.9743	1.2804	9.4904
1436	8.8672	8.3702	+0.5197	—8.5716	—9.3436	+9.6834	1.2813	9.4820
1437	9.0307	8.5160	+0.5563	—8.9362	—8.3541	+9.8861	1.2828	9.4659
1438	8.9319	8.3963	+0.4377	+8.7534	—9.7078	—9.8038	1.2845	9.4467
1439	9.7555	9.2171	—9.3316	+9.7527	—9.5120	—9.9798	1.2847	9.4442
1440	8.8075	8.2632	+0.4845	+7.5283	—9.6511	—8.7037	1.2852	9.4387
1441	8.8582	8.3121	0.4597	+8.5193	—9.7084	—9.6442	1.2853	9.4370
1442	8.8213	8.2695	0.4739	+8.2170	—9.6880	—9.3792	1.2858	9.4317
1443	9.2053	8.6494	0.3523	+9.1674	—9.6224	—9.9459	1.2861	9.4279
1444	8.9535	8.3935	0.5337	—8.7979	—9.0966	+9.8285	1.2864	9.4241
1445	8.8135	8.2457	0.4797	+7.9883	—9.6704	—9.1594	1.2869	9.4169
1446	8.8503	8.2798	0.4641	+8.4707	—9.7013	—9.6052	1.2871	9.4143
1447	8.8424	8.2594	0.5061	—8.4157	—9.4883	+9.5590	1.2879	9.4027
1448	8.9685	8.3823	0.5339	—8.8258	—9.0810	+9.8432	1.2881	9.3997
1449	8.8466	8.2601	0.5072	—8.4433	—9.4765	+9.5825	1.2881	9.3994
1450	+9.3856	—8.7893	+0.2743	+9.3697	—9.5467	—9.9706	—1.2887	—9.3902

CATALOGUE OF 1500 STARS.

No.	B. A. C.	Constellation.			Mag.	Right Ascension, Jan. 1, 1850.			Annual Variation.	North Polar Dist., Jan. 1, 1850.			Annual Variation.
						h.	m.	s.	s.	°	′	″	″
1451	8082	7	Andromedæ,		5	23	5	41.49*	+2.724	41	24	44.8*	−19.58
1452	8085	90	Aquarii,	φ	5		6	33.14*	3.114	96	51	24.1*	19.35
1453	8093		Tucanæ,		5		7	47.32	3.423	152	47	51.9	22.39
1454	8098		Tucanæ,	γ	4		8	38.54	3.554	149	3	28.2	19.55
1455	8105	6	Piscium,	γ	4½		9	23.36*	3.110	87	32	11.1*	19.61
1456	8109	93	Aquarii,	ψ^2	5		10	6.54	3.128	100	0	2.8*	19.56
1457	8113		Sculptoris,	γ	5		10	42.66*	3.260	123	20	54.9*	19.52
1458	8114	8	Andromedæ,		5		10	48.22*	2.756	41	48	13.4*	19.61
1459	8116	95	Aquarii,	ψ^3	5		11	9.27*	3.128	100	25	47.9*	19.62
1460	8131	62	Pegasi,	τ	5		13	13.18*	2.961	67	4	47.7*	19.65
1461	8144	98	Aquarii,	b^1	5		15	5.29	3.163	110	55	5.3	19.60
1462	8160	68	Pegasi,	υ	5		17	53.91*	2.986	67	25	15.4*	19.79
1463	8161	99	Aquarii,	b^2	5		18	9.77	3.163	111	27	46.2	19.70
1464	8162	4	Cassiopeæ,		5		18	11.95*	2.627	28	32	24.4*	19.74
1465	8177	10	Piscium,	θ	5		20	21.61*	3.042	84	26	40.0*	19.73
1466	8180		Cephei,		5		20	57.42*	2.463	20	27	54.1*	19.77
1467	8182	70	Pegasi,	q	5		21	34.24	3.028	78	3	57.3	19.83
1468	8188		Cassiopeæ,		5		23	7.67*	2.747	32	16	40.8*	19.82
1469	8201		Sculptoris,	β	5		24	54.82	3.238	128	38	50.6	19.72
1470	8202	101	Aquarii,	b^4	5		25	25.38	3.152	111	44	33.5	19.87
1471	8210		Phœnicis,	ι	5		26	59.21	3.247	133	26	38.6	19.70
1472	8213		Ursæ Minoris,		5½		27	48.99*	0.073	3	31	11.8*	19.88
1473	8224	16	Andromedæ,	λ	4½		30	14.43*	2.912	44	21	15.3*	19.50
1474	8229	17	Andromedæ,	ι	4		30	47.57*	2.918	47	33	42.7*	19.94
1475	8232	102	Aquarii,	ω^1	5		32	0.13	3.119	105	3	4.1	19.89
1476	8233	17	Piscium,	ι	4½		32	14.34*	3.112	85	11	10.7*	19.47
1477	8237	19	Andromedæ,	κ	4½		33	1.99*	2.927	46	29	44.8*	19.94
1478	8238	35	Cephei,	γ	3		33	14.02*	2.391	13	12	16.6*	20.09
1479	8240	103	Aquarii,	A^1	5		33	47.62	3.123	108	51	18.8	19.87
1480	8242	104	Aquarii,	A^2	5		33	58.44	3.126	108	38	50.5	20.00
1481	8243	18	Piscium,	λ	5		34	23.55*	3.064	89	2	42.4*	19.80
1482	8255	106	Aquarii,	ι^1	5		36	25.17	3.123	109	6	31.3	19.98
1483	8256	78	Pegasi,		5		36	27.33	3.106	61	28	5.6	19.96
1484	8261	20	Andromedæ,	ψ	5		38	36.99*	2.948	44	24	44.2*	19.96
1485	8268	5	Cassiopeæ,	τ	5		39	44.77*	2.891	32	11	0.2*	20.04
1486	8273		Cephei,		5		40	46.33	2.813	23	1	34.8	19.99
1487	8275		Sculptoris,	δ	5		41	6.39*	3.141	118	57	33.4*	19.89
1488	8290		Octantis,	γ^1	5		43	5.70	3.715	175	51	9.9	19.98
1489	8314		Cephei,		5		47	35.90	2.852	16	25	24.9	20.14
1490	8319		Octantis,	γ^2	5		49	8.23	3.474	173	0	18.4	19.79
1491	8323		Tucanæ,	η	5		49	39.75	3.170	155	7	50.5	20.11
1492	8328	27	Piscium,		5		50	59.58*	3.072	94	23	18.0*	19.92
1493	8331	28	Piscium,	ω	4½		51	36.63*	3.080	83	58	1.8*	19.96
1494	8334		Tucanæ,	ε	5		52	4.39	3.179	156	24	43.1	19.94
1495	8344		Cassiopeæ,		5		53	59.60*	3.009	29	36	44.6*	20.04
1496	8346	29	Piscium,		5		54	8.17*	3.075	93	51	44.5*	20.07
1497	8349	30	Piscium,		4½		54	15.96*	3.082	96	50	52.0*	20.03
1498	8358	2	Ceti,		4		56	3.12*	3.082	108	10	14.9*	20.07
1499	8366		Cassiopeæ,		5		57	23.00*	3.044	29	31	17.2*	20.05
1500	8368	33	Piscium,		5	23	57	39.39*	+3.076	96	32	48.7*	−20.10

CATALOGUE OF 1500 STARS.

No.	Logarithms of				Logarithms of			
	a	b	c	d	a′	b′	c′	d′
1451	+8.9911	−8.3740	+0.4337	+8.8661	−9.6658	−9.8627	−1.2899	−9.3706
1452	8.8151	8.1908	0.4925	−7.8921	−9.6056	+9.0650	1.2903	9.3638
1453	9.1525	8.5177	0.5633	−9.1016	+7.3979	+9.9377	1.2909	9.3538
1454	9.1018	8.4596	0.5522	−9.0351	−8.5119	+9.9223	1.2912	9.3468
1455	8.8136	8.1648	0.4854	+7.4471	9.6464	−8.6228	1.2915	9.3405
1456	8.8202	8.1650	0.4944	−8.0599	9.5910	+9.2294	1.2919	9.3344
1457	8.8919	8.2312	0.5130	−8.6321	9.3895	+9.7300	1.2921	9.3292
1458	8.9900	8.3285	0.4397	+8.8624	9.6514	−9.8623	1.2921	9.3284
1459	8.8212	8.1565	0.4945	−8.0790	9.5894	+9.2478	1.2923	9.3253
1460	8.8505	8.1665	0.4707	+8.4410	9.6800	−9.5813	1.2931	9.3069
1461	8.8451	8.1429	0.5011	−8.3978	9.5262	+9.5443	1.2938	9.2894
1462	8.8512	8.1202	0.4726	+8.4355	9.6728	−9.5769	1.2949	9.2617
1463	8.8478	8.1141	0.5005	−8.4112	9.5283	+9.5561	1.2949	9.2590
1464	9.1374	8.4033	0.4191	+9.0812	9.5643	−9.9365	1.2950	9.2587
1465	8.8194	8.0617	0.4840	+7.8054	9.6519	−8.9794	1.2957	9.2358
1466	9.2740	8.5096	0.3915	+9.2457	9.4921	−9.9654	1.2959	9.2293
1467	8.8273	8.0558	0.4805	+8.1428	9.6619	−9.3094	1.2961	9.2224
1468	9.0907	8.3010	0.4364	+9.0178	9.5648	−9.9215	1.2966	9.2047
1469	8.9262	8.1145	0.5097	−8.7217	9.3869	+9.7904	1.2971	9.1833
1470	8.8510	8.0329	0.4984	−8.4197	9.5408	+9.5638	1.2973	9.1770
1471	8.9584	8.1200	0.5122	−8.7958	9.3353	+9.8329	1.2977	9.1570
1472	0.0315	9.1823	8.3892	+0.0306	9.2214	−9.9949	1.2979	9.1466
1473	8.9757	8.0917	0.4614	+8.8300	9.5977	−9.8507	1.2986	9.1123
1474	8.9523	8.0600	0.4647	+8.7815	9.6086	−9.8256	1.2987	9.1041
1475	8.8358	7.9249	0.4934	−8.2503	9.5883	+9.4112	1.2990	9.0859
1476	8.8223	7.9077	0.4853	+7.7461	9.6459	−8.9207	1.2990	9.0822
1477	8.9604	8.0331	0.4656	+8.7982	9.5973	−9.8348	1.2992	9.0697
1478	9.4622	8.5316	0.3814	+9.4505	9.3274	−9.9854	1.2993	9.0664
1479	8.8450	7.9051	0.4945	−8.3545	9.5726	+9.5066	1.2994	9.0573
1480	8.8445	7.9016	0.4944	−8.3493	9.5740	+9.5020	1.2994	9.0543
1481	8.8213	7.8712	0.4869	+7.0434	9.6393	−8.2194	1.2995	9.0472
1482	8.8462	7.8601	0.4939	−8.3613	9.5758	+9.5127	1.2999	9.0116
1483	8.8778	7.8911	0.4766	+8.5569	9.6338	−9.6768	1.2999	9.0109
1484	8.9770	7.9482	0.4689	+8.8309	9.5686	−9.8520	1.3003	8.9692
1485	9.0958	8.0433	0.4599	+9.0233	9.4854	−9.9259	1.3005	8.9458
1486	9.2300	8.1548	0.4482	+9.1940	9.3924	−9.9624	1.3007	8.9232
1487	8.8804	7.7975	0.4958	−8.5655	−9.5312	+9.6835	1.3007	8.9156
1488	9.7278	8.5965	0.5862	−9.7245	+8.2810	+9.9954	1.3010	8.8674
1488	9.3719	8.1056	0.4511	+9.3538	−9.2420	−9.9813	1.3016	8.7331
1490	9.7378	8.4139	0.5547	−9.7346	7.7709	+9.9963	1.3017	8.6756
1491	9.1997	7.8542	0.5053	−9.1574	9.1511	+9.9573	1.3018	8.6541
1492	8.8249	7.4194	0.4878	−7.7086	9.6332	+8.8834	1.3019	8.5942
1493	8.8260	7.3897	0.4865	+7.8477	9.6389	−9.0213	1.3019	8.5634
1494	9.2214	7.7605	0.5020	−9.1835	9.1523	+9.9619	1.3020	8.5388
1495	9.1299	7.5487	0.4784	+9.0691	9.3751	−9.9391	1.3021	8.4186
1496	8.8248	7.2327	0.4876	−7.6532	9.6348	+8.8283	1.3021	8.4078
1497	8.8269	7.2250	0.4878	−7.9033	9.6313	+9.0763	1.3021	8.3980
1498	8.8461	7.0822	0.4883	−8.3400	9.6095	+9.4939	1.3022	8.2360
1499	9.1313	7.1898	0.4834	+9.0709	9.3499	−9.9396	1.3022	8.0585
1500	+8.8267	−6.8360	+0.4875	−7.8837	−9.6334	+9.0570	−1.3022	−8.0092

Secular Variation of the Annual Precession in Right Ascension.

Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.
	s.		s		s.		s.		s.
2	+ 0.0488	213	+0.0729	631	—0.1203	827	+ 0.5413	1235	— 0.1614
5	— 0.2212	219	+0.1047	632	—0.0380	829	+ 0.9674	1250	— 0.0962
9	— 0.0587	225	+0.0404	640	—0.0693	831	+ 0.4714	1255	— 0.0352
10	— 0.0709	228	+0.0355	645	—0.0361	838	+ 0.0435	1271	—29.3200
12	— 0.0951	231	+0.0352	648	—0.0725	844	+ 0.0421	1272	— 0.0603
16	+ 0.0467	235	+0.0376	653	—0.1161	845	+ 0.0424	1273	— 0.1670
18	+ 0.0673	272	+0.0405	654	—0.1470	848	+ 0.0521	1284	— 0.0411
19	— 0.0475	277	+0.0703	656	—0.0670	850	+ 0.0372	1286	— 0.1205
20	— 0.0475	288	+0.0977	662	—0.0917	852	+ 0.0498	1290	— 0.1171
21	— 0.0469	289	+0.0421	664	—0.0386	853	+ 0.1202	1294	— 0.1449
23	+ 0.0468	293	+0.0372	668	—0.0653	855	+ 0.0860	1298	— 0.3856
27	+ 0.0529	300	+0.2235	671	—0.0848	856	+ 0.0860	1299	—22.6920
28	— 0.0384	303	+0.0668	675	—0.0390	857	+ 0.4195	1308	— 0.0746
31	+ 0.0370	331	+0.0833	692	—0.0579	858	+ 0.1099	1310	— 0.3630
38	+ 0.0581	405	—0.0864	694	—0.1159	859	+ 0.0455	1315	— 0.5136
39	+ 0.0439	411	—1.4765	698	+0.0418	874	+ 0.0387	1316	— 0.3064
43	+ 0.0412	422	—0.0366	703	—0.0381	878	+ 0.0376	1328	— 0.1722
44	+ 0.0686	424	—0.0434	705	+0.0532	879	+ 0.0361	1334	— 0.1221
46	+ 1.2222	425	—0.1054	710	—0.0458	880	+ 0.1009	1337	— 0.0658
51	+ 0.0360	447	—0.4520	711	+0.1154	882	+ 0.0436	1342	— 0.0368
53	+ 0.0408	452	—0.0375	714	+0.0601	889	+ 0.0403	1346	— 0.2683
57	+ 0.0565	463	—0.0366	715	+0.0356	892	+ 0.7265	1361	— 0.0354
60	+11.4276	471	—0.0812	718	+0.0385	893	+ 0.0461	1363	— 0.0426
64	+ 0.0396	495	—0.3909	719	—0.1319	894	+ 0.0535	1369	— 0.0682
65	+ 0.1173	498	—0.0641	720	+0.0500	895	+ 0.1385	1370	— 0.0837
66	+ 0.0748	511	—0.0423	721	—0.0448	896	+ 0.0735	1373	— 0.0356
73	+ 0.0399	512	—0.1185	727	+0.1732	897	+ 0.0438	1374	— 0.0389
74	+ 0.1390	521	—0.0392	728	+0.0642	903	+ 0.0435	1378	— 0.0476
75	+ 0.0426	523	—0.0619	732	+0.0558	905	+ 0.0379	1387	— 0.0381
81	+ 0.0462	525	—0.0502	735	+0.0652	907	+ 0.0724	1390	— 0.0880
86	+ 0.0508	529	—0.0770	736	+0.0651	910	+ 0.0729	1399	— 0.1149
92	+ 0.0968	532	—0.1041	740	+0.0389	913	+ 0.1115	1403	— 0.0408
101	+ 0.1606	533	—0.1458	743	+0.0516	924	+ 0.0357	1404	— 0.0411
103	+ 0.1830	534	—0.0799	744	+0.1123	929	+ 0.1913	1407	— 0.0945
108	+ 0.0374	536	—0.1623	748	—0.0582	934	+ 0.1244	1411	— 1.1224
117	+ 0.0664	537	—0.1008	750	+0.0961	938	+ 0.0858	1413	— 0.0879
118	+ 0.1288	555	—0.2169	751	+0.0383	951	+ 0.2015	1416	— 0.6890
127	+ 0.0491	557	—0.0459	754	+0.0395	967	+ 0.0789	1420	— 0.0456
137	+ 0.0660	558	—0.1376	757	+0.0961	972	+ 0.0381	1424	— 0.0599
143	+ 0.0568	560	—0.0358	758	+0.0628	975	+ 0.0369	1427	— 0.0540
150	+ 0.0579	562	—0.0448	760	+0.7414	977	+ 0.3318	1435	— 0.2170
151	+ 0.0675	563	—0.1317	762	+0.0579	981	+ 0.1234	1437	— 0.0556
158	+ 0.0482	566	—0.1350	766	+0.2314	997	+ 0.2495	1439	— 0.2923
160	+ 0.0395	567	—0.0551	768	+0.2321	998	+ 0.1151	1444	— 0.0378
162	+ 0.0708	568	—0.1049	771	+0.1328	1006	+ 0.0923	1448	— 0.0400
166	+ 0.1108	572	—0.0928	775	+0.0450	1009	+ 0.0381	1453	— 0.0811
171	+ 0.2783	574	—0.0632	783	+0.0355	1013	+ 0.0541	1454	— 0.0671
172	+ 0.0470	575	—0.0627	786	+0.1105	1014	+ 0.0922	1464	+ 0.0359
176	+ 0.0762	581	—0.0372	794	+1.3966	1018	+ 0.0452	1466	+ 0.0455
177	+ 0.0717	586	—0.8153	801	+0.0567	1045	+ 0.2928	1472	— 0.4747
178	+ 0.0599	589	—0.1052	805	+0.0526	1051	+ 0.0543	1478	+ 0.0703
180	+ 0.1967	592	—0.1730	807	+0.0360	1082	+21.1441	1485	+ 0.0404
181	+ 0.0425	593	—0.0578	808	+0.0371	1143	— 0.6157	1486	+ 0.0569
183	+ 0.0421	605	—0.1100	814	+0.0450	1149	— 0.0371	1488	— 0.3599
188	+ 0.0404	608	—0.0839	817	+0.0369	1162	— 0.0418	1489	+ 0.0843
189	+ 0.0886	610	—0.0652	818	+0.0409	1175	— 0.0459	1490	— 0.3254
190	+ 0.1602	622	—1.7282	820	+0.2860	1177	— 0.0587	1491	— 0.0704
193	+ 0.1011	623	—0.1180	821	+0.0424	1209	— 0.0359	1494	— 0.0735
210	+ 0.0832	624	—0.0404	822	+0.0820	1210	— 0.0600	1495	+ 0.0501
212	+ 0.5112	630	—0.9968	824	+0.0358	1222	— 0.0400	1499	+ 0.0517

Secular Variation of the Annual Precession in North Polar Distance.

Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.	Star.	Sec. Var.
	″		″		″		″		″		″
60	+0.713	297	+0.479	413	+0.621	592	+0.511	1028	— 0.580	1133	—0.457
150	0.430	298	0.587	415	0.501	622	+0.713	1029	0.581	1134	0.580
151	0.447	300	1.367	418	0.632	630	+0.544	1032	0.683	1136	0.648
166	0.549	305	0.483	421	0.533	827	—0.508	1033	0.443	1137	0.671
172	0.462	308	0.579	422	0.907	829	—0.646	1036	0.659	1138	0.817
176	0.529	310	0.444	423	0.486	831	—0.511	1038	0.664	1140	0.539
177	0.523	312	0.625	424	0.938	857	—0.622	1040	0.551	1141	0.646
178	0.503	313	0.553	425	1.273	892	+0.496	1044	— 0.604	1142	—0.497
181	0.468	317	0.592	427	0.469	894	—0.443	1045	+ 0.913	1143	+2.807
183	0.481	322	0.450	428	0.450	895	0.577	1046	— 0.484	1144	—0.645
188	0.493	324	0.542	429	0.611	896	0.490	1050	0.527	1146	0.622
189	0.603	325	0.432	431	0.568	897	0.441	1051	0.883	1149	1.022
190	0.725	326	0.461	433	0.747	903	0.449	1055	0.438	1151	0.855
191	0.440	327	0.569	443	0.505	905	0.438	1058	0.510	1153	0.856
192	0.476	328	0.516	447	1.852	913	0.607	1059	0.480	1154	0.474
193	0.636	329	0.450	449	0.582	914	0.451	1061	0.718	1155	0.540
210	0.631	331	1.143	450	0.538	920	0.462	1062	0.525	1156	0.804
211	0.454	333	0.438	452	0.743	924	0.476	1063	0.709	1162	0.894
212	1.160	334	0.461	454	0.430	934	0.696	1064	0.666	1165	0.519
216	0.489	335	0.561	455	0.483	935	0.455	1065	0.524	1166	0.532
218	0.475	336	0.441	456	0.640	937	0.432	1066	0.774	1167	0.518
225	0.553	340	0.475	458	0.585	938	0.629	1067	0.548	1171	0.510
226	0.441	341	0.476	462	0.481	942	0.434	1070	0.665	1179	0.543
228	0.542	345	0.439	464	0.685	947	0.465	1071	0.585	1182	0.509
231	0.557	346	0.475	465	0.499	948	0.450	1072	0.526	1184	0.574
233	0.518	347	0.517	467	0.559	952	0.444	1073	0.585	1185	0.531
235	0.574	348	0.556	470	0.577	954	0.486	1075	0.620	1186	0.591
240	0.477	349	0.435	471	0.874	956	0.436	1078	0.496	1187	0.450
241	0.441	352	0.438	472	0.516	958	0.526	1081	0.471	1189	0.576
246	0.449	355	0.603	475	0.448	960	0.435	1082	15.545	1190	0.583
248	0.450	356	0.534	476	0.529	961	0.491	1083	0.849	1191	0.503
250	0.453	359	0.604	480	0.524	962	0.439	1084	0.600	1192	0.456
251	0.469	360	0.780	481	0.466	964	0.441	1086	0.488	1193	0.515
252	0.445	361	0.730	482	0.503	965	0.458	1087	0.521	1196	0.490
254	0.460	364	0.548	488	0.503	967	0.683	1089	0.430	1201	0.601
255	0.451	367	0.519	490	0.485	968	0.468	1090	0.608	1202	0.602
256	0.451	369	0.472	491	0.497	969	0.442	1092	0.548	1204	0.483
258	0.453	371	0.717	495	1.299	975	0.576	1093	0.592	1205	0.577
261	0.460	374	0.641	499	0.483	977	1.157	1094	0.437	1209	0.673
262	0.440	375	0.648	511	0.580	978	0.472	1097	0.561	1216	0.448
266	0.450	376	0.595	512	0.773	984	0.458	1098	0.533	1217	0.495
269	0.450	380	0.481	514	0.448	986	0.470	1099	0.481	1218	0.436
270	0.486	382	0.517	516	0.457	989	0.457	1103	0.460	1225	0.459
272	0.674	383	0.532	523	0.605	994	0.485	1105	0.433	1235	0.930
277	0.807	384	0.519	525	0.561	996	0.517	1107	0.438	1238	0.544
278	0.442	385	0.772	526	0.506	997	1.117	1109	0.841	1242	0.480
279	0.449	386	0.499	529	0.611	1000	0.455	1110	0.476	1249	0.475
280	0.439	388	0.786	532	0.653	1001	0.471	1111	0.681	1250	0.741
281	0.467	389	0.966	537	0.625	1006	0.817	1112	0.558	1251	—0.488
282	0.661	391	0.497	557	0.453	1007	0.526	1115	0.562	1271	+6.481
284	0.432	392	0.808	558	0.599	1008	0.498	1116	0.439	1272	—0.585
285	0.539	394	0.558	562	0.439	1011	0.443	1117	0.808	1284	—0.497
286	0.468	395	0.529	563	0.568	1013	0.709	1118	0.554	1286	—0.654
287	0.563	396	0.773	566	0.565	1014	0.848	1120	0.650	1290	—0.637
289	0.737	399	0.673	567	0.447	1015	0.469	1122	0.832	1294	—0.663
290	0.597	400	0.528	568	0.522	1018	0.700	1124	0.523	1299	+4.641
291	0.582	404	0.518	572	0.493	1022	0.538	1126	— 0.593	1308	—0.526
293	0.724	405	1.512	575	0.440	1023	0.454	1127	+ 0.653	1310	—0.830
294	0.500	408	0.470	586	0.895	1024	0.556	1128	+ 0.653	1334	—0.490
296	+0.655	411	+4.450	589	+0.452	1025	—0.557	1129	— 0.559		

Name.	Symbol.	Distance from the Sun.			Eccentricity.	Sidereal Revolution.	Syn Re ti
		Mean.	Greatest.	Least.			
						Days.	D
Mercury	☿	0.3870984	0.4666927	0.3075041	0.2056178	87.9692824	115
Venus	♀	0.7233317	0.7282636	0.7183998	0.0068183	224.7007754	583
Earth	♁	1.0000000	1.0167751	0.9832249	0.0167751	365.2563744	
Mars	♂	1.523691	1.6657795	1.3816025	0.0932528	686.9794561	779
Asteroids.							
Jupiter	♃	5.202767	5.453663	4.951871	0.0482235	4332.5848032	398
Saturn	♄	9.538850	10.073278	9.004422	0.0560265	10759.2197106	378
Uranus	♅	19.18239	20.07630	18.28848	0.0466006	30686.8205556	369
Neptune	♆	30.03627	30.29816	29.77438	0.0087193	60126.722	367

Name.	Longitude of the Perihelion.	Annual Variation.	Longitude of Ascending Node.	Annual Variation.	Inclination of Orbit.	Annual Variation.	Mean daily Motion.
	° ′ ″	″	° ′ ″	″	° ′ ″	″	′ ″
Mercury	74 57 27.0	+ 5.81	46 23 55.0	—10.07	7 0 13.3	+0.18	245 32.6
Venus	124 14 25.6	— 3.24	75 11 29.8	—20.50	3 23 31.4	+0.07	96 7.8
Earth	100 11 27.0	+11.24					59 8.3
Mars	333 6 38.4	+15.46	48 16 18.0	—25.22	1 51 5.7	—0.01	31 26.7
Asteroids.							
Jupiter	11 45 32.8	+ 6.65	98 48 37.8	—15.90	1 18 42.4	—0.23	4 59.3
Saturn	89 54 41.2	+19.31	112 16 34.2	—19.54	2 29 29.9	—0.15	2 0.6
Uranus	168 5 24	+ 2.28	73 8 47.8	—36.05	0 46 29.2	+0.03	42.4
Neptune	47 17 58.0		130 10 12.3		1 46 59.0		21.6

Name.	Time of Rotation.	Diameter.		Volume.	Mass.	Density.	Light at		Gravity.	o
		Apparent.	In Miles.				Perihelion.	Aphelion.		
	h. m. s.	″								
Sun	607 48	1923.64	889,614	1415225	354936	0.250			28.36	4
Mercury	24 5 28	6.69	3,093	0.0595	0.0729	1.225	10.58	4.59	0.48	
Venus	23 21 21	17.10	7,917	0.9960	0.9101	0.908	1.94	1.91	0.90	
Earth	23 56 4		7,926	1.0000	1.0000	1.000	1.034	0.967	1.00	
Mars	24 37 22	5.8	4,251	0.1364	0.1324	0.972	0.524	0.360	0.49	
Asteroids.										
Jupiter	9 55 26	38.4	92,301	1491.0	338.718	0.227	0.0408	0.0336	2.45	
Saturn	10 14	17.1	75,181	772.0	101.364	0.131	0.0123	0.0099	1.09	
Uranus		4.1	36,270	86.5	14.251	0.167	0.0027	0.0025	0.76	
Neptune		2.4	33,660	76.6	18.900	0.321	0.0011	0.0011	1.36	

The preceding elements of Neptune are for the beginning of 1854; the others are for t ginning of 1840.

Elements of the Moon.

Mean distance from the earth	59.96435 terrestrial radii.
Mean sidereal revolution	27.321661418 days.
Mean synodical revolution	29.530588715 days.
Mean longitude January 1, 1801	118° 17′ 8″.3.
Mean longitude of perigee at do.	266° 10′ 7″.5.
Mean longitude of ascending node at do.	13° 53′ 17″.7.
Mean inclination of orbit	5° 8′ 47″.9.
Mean revolution of nodes	6798.279 days.
Mean revolution of perigee	3232.575343 days.
Eccentricity of orbit	0.0548442.
Diameter of the moon	2153 miles.
Density, that of the earth being 1	0.5657.
Mass, that of the earth being 1	0.011399.

Elements of the Satellites of Mars.

Satellite.	Sidereal Revolution.	Daily Motion.	Distance from Mars.		Inclination to Ecliptic.
			Apparent.	In Miles.	
	h. m.	°	″		° ′
1. Phobos	7 39.3	1128.794	12.95	5820	26 6
2. Deimos	30 17.9	285.165	32.35	14600	26 6

Elements of the Satellites of Jupiter.

Sat.	Sidereal Revolution.	Distance in Radii of Jupiter.	Orbit inclined to Jupiter's Equator.	Diameter.		Mass, that of Jupiter being 1.
				Apparent.	In Miles.	
	d. h. m. s.		° ′ ″	″		
1	1 18 27 33.505	6.04853	0 0 7	1.015	2436	.000017328
2	3 13 13 42.040	9.62347	0 1 6	0.911	2187	.000023235
3	7 3 42 33.360	15.35024	0 5 3	1.488	3573	.000088497
4	16 16 32 11.271	26.99835	0 0 24	1.273	3057	.000042659

Elements of the Satellites of Saturn.

Sat.	Sidereal Revolution.	Distance in Radii of Saturn.	Eccentricity.	Longitude of Peri-Saturnium.	Mean Longitude.	Epoch.
	d. h. m. s.			° ′ ″	° ′ ″	
1	0 22 36 17.7	3.1408	0.06889	104 42	264 16 36	1789.705
2	1 8 53 2.7	4.0319	Uncertain.		67 56 25	1789.705
3	1 21 18 33.0	4.9926	0.0051	184 36	158 31 0	1836.308
4	2 17 44 51.2	6.399	0.02	42 30	327 40 48	1836.0
5	4 12 25 11.1	8.932	0.02269	95	353 44 0	1836.0
6	15 22 41 24.9	20.706	0.029223	244 35 50	137 21 24	1830.0
7	21 4 20	25.029	0.115	295	32	1849.0
8	79 7 54 40.8	64.359	0.0282	351 25	269 37 48	1790.0

Elements of the Satellites of Uranus.

Sat.	Sidereal Revolution.	Daily Motion.	Mean apparent Distance.	Mean Distance in Miles.
	Days.	°	″	
1	2.52035	142.8373	13.54	119994
2	4.14397	86.8732	19.28	170863
3	8.705886	41.35133	33.88	288600
4	13.463263	26.73943	45.20	385000

Elements of the Satellite of Neptune.

Sidereal revolution	5*d.* 21*h.* 2*m.* 44*s.*
Apparent mean distance	16″.98.
True mean distance	235,800 miles.
Orbit inclined to the plane of ecliptic	145°

TABLE XXXV.—ELEMENTS OF THE ASTEROIDS.

No.	Name.	Discovered			App't Magn.	Mean Distance.	Eccentricity.	Period.
		When.	By whom.	Where.				Days.
1	Ceres	1801, Jan. 1	Piazzi	Palermo	7.7	2.765938	0.080257	1680.2
2	Pallas	1802, March 28	Olbers	Bremen	7.9	2.770386	0.239367	1684.2
3	Juno	1804, Sept. 1	Harding	Lilienthal	8.7	2.668678	0.256176	1592.3
4	Vesta	1807, March 29	Olbers	Bremen	6.6	2.361339	0.090204	1325.3
5	Astræa	1845, Dec. 8	Hencke	Driessen	10.0	2.576500	0.189992	1510.5
6	Hebe	1847, July 1	Hencke	Driessen	8.6	2.425418	0.201657	1379.6
7	Iris	1847, Aug. 13	Hind	London	8.6	2.386147	0.230832	1346.3
8	Flora	1847, Oct. 18	Hind	London	8.9	2.201386	0.156704	1193.0
9	Metis	1848, April 25	Graham	Markree	8.9	2.385730	0.123321	1345.9
10	Hygeia	1849, April 12	Gasparis	Naples	9.8	3.149373	0.100557	2041.4
11	Parthenope	1850, May 11	Gasparis	Naples	9.5	2.452588	0.098887	1402.9
12	Victoria	1850, Sept. 13	Hind	London	9.6	2.332811	0.218920	1301.4
13	Egeria	1850, Nov. 2	Gasparis	Naples	9.9	2.575625	0.084873	1509.8
14	Irene	1851, May 20	Hind	London	9.7	2.589368	0.165230	1521.9
15	Eunomia	1851, July 29	Gasparis	Naples	9.1	2.644180	0.187357	1570.4
16	Psyche	1852, March 17	Gasparis	Naples	10.1	2.922752	0.134225	1825.0
17	Thetis	1852, April 17	Luther	Bilk	9.9	2.473710	0.126865	1421.0
18	Melpomene	1852, June 24	Hind	London	9.5	2.296060	0.217078	1270.7
19	Fortuna	1852, Aug. 22	Hind	London	9.7	2.441368	0.157922	1393.3
20	Massilia	1852, Sept. 19	Gasparis	Naples	9.3	2.409386	0.143696	1366.0
21	Lutetia	1852, Nov. 15	Goldschmidt	Paris	10.3	2.435431	0.162045	1388.2
22	Calliope	1852, Nov. 16	Hind	London	10.6	2.495579	0.103630	1439.9
23	Thalia	1852, Dec. 15	Hind	London	11.0	2.628824	0.231732	1556.8
24	Themis	1853, April 5	Gasparis	Naples	11.6	3.149947	0.117504	2041.9
25	Phocæa	1853, April 6	Chacornac	Marseilles	10.5	2.401060	0.252533	1358.9
26	Proserpina	1853, May 5	Luther	Bilk	10.7	2.656079	0.087521	1581.1
27	Euterpe	1853, Nov. 8	Hind	London	9.9	2.347305	0.172896	1313.5
28	Bellona	1854, March 1	Luther	Bilk	9.8	2.775177	0.154507	1688.6
29	Amphitrite	1854, March 1	Marth	London	9.1	2.554866	0.072383	1491.5
30	Urania	1854, July 22	Hind	London	10.1	2.364199	0.127174	1327.8
31	Euphrosyne	1854, Sept. 1	Ferguson	W'sh'gton	11.6	3.156158	0.216013	2048.0
32	Pomona	1854, Oct. 26	Goldschmidt	Paris	11.0	2.589039	0.080617	1521.6
33	Polyhymnia	1854, Oct. 28	Chacornac	Paris	11.4	2.867075	0.336987	1773.1
34	Circe	1855, April 6	Chacornac	Paris	11.5	2.688302	0.108253	1609.9
35	Leucothea	1855, April 19	Luther	Bilk	12.1	2.981229	0.221025	1880.1
36	Atalanta	1855, Oct. 5	Goldschmidt	Paris	12.9	2.748705	0.297900	1664.5
37	Fides	1855, Oct. 5	Luther	Bilk	10.5	2.641907	0.174798	1568.4
38	Leda	1856, Jan. 12	Chacornac	Paris	10.9	2.739685	0.155576	1656.3
39	Lætitia	1856, Feb. 8	Chacornac	Paris	9.3	2.769387	0.111075	1683.3
40	Harmonia	1856, March 31	Goldschmidt	Paris	9.1	2.267148	0.046085	1246.8
41	Daphne	1856, May 23	Goldschmidt	Paris	10.2	2.400337	0.202488	1358.3
42	Isis	1856, May 23	Pogson	Oxford	9.2	2.439997	0.225627	1392.1
43	Ariadne	1857, April 15	Pogson	Oxford	10.3	2.203838	0.167565	1194.9
44	Nysa	1857, May 27	Goldschmidt	Paris	9.5	2.424163	0.149334	1378.6
45	Eugenia	1857, June 27	Goldschmidt	Paris	11.0	2.721430	0.082427	1639.8
46	Hestia	1857, Aug. 16	Pogson	Oxford	10.8	2.530335	0.166149	1470.1
47	Aglaia	1857, Sept. 15	Luther	Bilk	12.1	2.881472	0.129497	1786.5
48	Doris	1857, Sept. 19	Goldschmidt	Paris	11.3	3.104474	0.075801	1997.9
49	Pales	1857, Sept. 19	Goldschmidt	Paris	12.2	3.085916	0 237804	1980.0
50	Virginia	1857, Oct. 4	Ferguson	W'sh'gton	12.6	2.648628	0.286957	1574.4
51	Nemausa	1858, Jan. 22	Laurent	Nismes	10.0	2.367790	0.066998	1330.8
52	Europa	1858, Feb. 4	Goldschmidt	Paris	10.5	3.099883	0.100938	1993.4
53	Calypso	1858, April 4	Luther	Bilk	11.0	2.618545	0.206715	1547.7
54	Alexandra	1858, Sept. 10	Goldschmidt	Paris	10.5	2.707653	0.199006	1627.3
55	Pandora	1858, Sept. 10	Searle	Albany	11.1	2.759831	0.142086	1674.6
56	Melete	1857, Sept. 9	Goldschmidt	Paris	12.1	2.583455	0.227018	1516.6

No.	Longitude of Perihelion.			Longitude of ascending Node.			Inclination of Orbit.			Mean daily Motion.	Mean Longitude at Epoch.			Epoch, Washington mean Time.
	°	′	″	°	′	″	°	′	″	″	°	′	″	
1	149	26	13.1	80	49	54.7	10	36	27.8	771.3333	346	48	15.4	1859, Sept. 7.0000
2	122	7	38.4	172	38	32.7	34	42	29.8	769.4780	224	28	25.5	1858, May 28.7860
3	54	0	55.8	170	58	22.0	13	3	9.8	813.8848	104	2	31.1	1858, Jan. 28.7860
4	250	35	29.4	103	21	10.3	7	8	9.1	977.8432	218	26	1.1	1858, April 22.7860
5	134	35	35.7	141	24	48.5	5	19	35.2	857.9486	80	56	2.7	1849, Dec. 30.7488
6	15	2	23.4	138	35	19.5	14	46	35.4	939.3481	124	54	18.6	1857, Feb. 12.7488
7	41	29	15.3	259	46	16.1	5	28	1.4	962.6335	322	34	38.8	1858, July 18.7488
8	32	54	28.3	110	17	48.6	5	53	8.0	1086.3310	68	48	32.0	1848, Jan. 0.7488
9	71	3	55.6	68	31	31.6	5	36	0.6	962.8856	128	8	12.7	1858, June 29.7488
10	227	47	58.8	287	38	34.2	3	47	9.3	634.8491	354	47	47.6	1851, Sept. 16.7488
11	316	10	7.1	125	3	41.1	4	36	57.9	923.7824	283	56	41.9	1858, June 26.7488
12	301	39	24.7	235	34	41.7	8	23	19.4	995.8341	7	42	5.0	1850, Dec. 30.7488
13	119	12	59.0	43	17	55.7	16	33	6.7	858.3861	138	44	42.6	1851, Dec. 5.0000
14	179	28	21.9	86	40	4.5	9	7	7.4	851.5608	67	12	20.6	1857, Nov. 19.7488
15	27	31	8.1	293	56	15.8	11	43	39.0	825.2220	238	54	5.1	1859, May 11.0000
16	13	16	14.8	150	35	34.0	3	4	6.5	710.0987	50	51	42.0	1860, Nov. 20.0000
17	259	22	51.2	125	27	13.3	5	35	40.7	911.9760	210	1	24.3	1856, April 3.7488
18	15	11	48.0	150	4	33.3	10	8	58.3	1019.8395	304	33	25.3	1859, July 2.0000
19	30	22	50.2	211	29	28.7	1	32	28.8	930.1578	148	28	55.8	1858, March 2.7488
20	98	28	37.6	206	41	27.6	0	41	7.3	948.7396	195	16	53.9	1858, April 20.7488
21	327	2	45.2	80	27	23.3	3	5	11.1	933.5610	41	24	9.0	1853, Jan. 1.7488
22	58	16	41.1	66	36	54.7	13	44	51.9	714.9070	76	59	2.0	1852, Dec. 30.7488
23	123	58	40.6	67	38	34.4	10	13	13.6	832.4617	280	7	33.7	1859, July 10.0000
24	137	54	9.7	36	10	30.3	0	49	1.8	634.6753	30	2	41.5	1856, Sept. 24.7488
25	302	46	9.0	214	4	54.6	21	35	53.6	953.6780	294	46	13.5	1857, July 9.7488
26	235	17	26.8	45	53	14.6	3	35	40.3	819.6815	181	21	20.9	1857, March 19.7488
27	87	39	0.0	93	44	45.0	1	35	31.1	986.6260	260	43	32.7	1859, June 13.7488
28	122	22	48.3	144	43	5.4	9	22	30.8	767.4862	159	3	36.8	1854, Feb. 27.7488
29	56	39	6.6	356	26	51.8	6	7	49.6	868.8694	293	11	23.8	1859, July 8.7488
30	31	23	24.7	308	13	46.3	2	5	56.9	976.0689	19	30	24.4	1858, Oct. 8.7488
31	93	51	6.6	31	25	23.0	26	25	12.4	632.8031	53	49	50.3	1854, Dec. 30.7488
32	193	33	42.5	220	48	1.4	5	28	49.1	851.7238	134	30	20.0	1860, Jan. 24.7488
33	340	51	46.1	9	16	9.2	1	56	41.5	730.8833	266	47	55.8	1858, April 13.7488
34	149	58	35.1	184	47	10.8	5	26	33.2	904.9883	193	36	37.2	1855, April 9.4488
35	198	51	53.9	355	57	26.3	8	12	10.7	689.3084	173	36	11.3	1860, Feb. 14.0000
36	42	22	25.0	359	8	48.4	18	42	9.5	778.6000	36	19	53.2	1855, Dec. 30.7488
37	66	5	35.8	8	10	23.4	3	7	19.3	826.2860	42	34	30.3	1855, Dec. 30.7488
38	100	40	28.4	296	27	47.3	6	58	31.9	782.4484	112	55	7.2	1855, Dec. 30.7488
39	1	58	57.7	157	19	31.0	10	20	50.7	769.8940	146	44	19.7	1855, Dec. 31.7488
40	2	1	50.9	93	32	2.9	4	15	48.4	1039.4100	222	12	9.1	1856, June 30.7488
41	230	21	29.8	180	5	50.8	15	48	23.0	954.1100	202	28	48.5	1856, May 31.2488
42	317	59	49.2	84	30	43.8	8	34	31.8	930.9425	247	46	26.0	1860, Jan. 0.7488
43	277	13	54.8	264	29	12.7	3	27	47.6	1084.5177	224	4	55.7	1857, April 16.7488
44	111	37	52.5	131	1	16.6	3	41	40.8	940.0780	278	9	28.1	1857, Dec. 31.7488
45	230	1	10.2	148	5	2.5	6	34	57.9	790.3345	294	33	47.6	1857, Dec. 31.7488
46	354	31	9.9	181	26	35.3	2	17	48.6	881.5347	197	15	32.5	1860, March 25.2488
47	313	43	9.0	4	30	39.9	5	0	14.4	725.4135	2	48	17.2	1858, Feb. 6.7488
48	77	37	40.9	185	14	1.8	6	29	42.8	648.6710	9	54	48.6	1857, Dec. 31.7488
49	32	49	57.9	290	29	59.9	3	8	30.5	654.5316	10	30	20.3	1858, Oct. 31.7488
50	10	0	12.4	173	32	18.7	2	47	53.6	823.1440	31	41	25.6	1857, Dec. 30.7488
51	175	41	27.7	175	39	8.2	9	36	37.9	973.8489	154	23	36.1	1857, Dec. 30.7488
52	102	14	26.0	129	56	57.2	7	24	34.9	650.1127	136	26	0.9	1857, Dec. 31.7488
53	92	28	10.7	144	4	18.7	5	6	59.0	837.3700	162	27	23.0	1858, April 8.2488
54	293	56	0.4	313	50	17.5	11	47	9.0	796.3741	346	21	55.2	1858, Dec. 29.7488
55	11	26	7.4	10	57	29.3	7	13	30.2	773.8975	28	26	11.4	1858, Dec. 29.7488
56	294	58	25.7	194	52	31.4	7	56	2.3	854.4862	330	53	38.0	1857, Sept. 12.7488

TABLE XXXV.—ELEMENTS OF THE ASTEROIDS.

No.	Name.	Discovered.			App't Magn.	Mean Distance.	Eccentricity.	Period.
		When.	By whom.	Where.				Days.
57	Mnemosyne	1859, Sept. 22	Luther	Bilk	10.9	3.157288	0.104116	2049.1
58	Concordia	1860, March 24	Luther	Bilk	11.6	2.699318	0.041619	1619.8
59	Elpis	1860, Sept. 12	Chacornac	Paris	11.3	2.713215	0.117133	1632.3
60	Echo	1860, Sept. 15	Ferguson	Washington	12.2	2.392879	0.185428	1352.0
61	Danaë	1860, Sept. 19	Goldschmidt	Paris	11.7	3.003977	0.168435	1901.7
62	Erato	1860, Oct. 10	Förster	Berlin	11.8	3.128731	0.170159	2021.3
63	Ausonia	1861, Feb. 10	Gasparis	Naples	9.9	2.395709	0.125375	1354.4
64	Angelina	1861, March 4	Tempel	Marseilles	10.3	2.680963	0.128206	1603.3
65	Cybele	1861, March 8	Tempel	Marseilles	11.3	3.420520	0.120312	2310.6
66	Maia	1861, April 9	Tuttle	Camb'ge, U.S.	12.7	2.663544	0.133916	1587.7
67	Asia	1861, April 17	Pogson	Madras	11.6	2.420899	0.184439	1375.8
68	Leto	1861, April 29	Luther	Bilk	10.3	2.782166	0.187489	1695.0
69	Hesperia	1861, April 29	Schiaparelli	Milan	12.0	2.971693	0.173831	1871.1
70	Panopæa	1861, May 5	Goldschmidt	Paris	11.1	2.613270	0.183129	1543.0
71	Niobe	1861, Aug. 13	Luther	Bilk	10.8	2.756160	0.173729	1671.3
72	Feronia	1861, May 29	Peters	Clinton, U. S.	13	2.266077	0.119784	1245.9
73	Clytie	1862, April 7	Tuttle	Camb'ge, U. S.	13	2.666582	0.042750	1590.4
74	Galatea	1862, Aug. 29	Tempel	Marseilles	11	2.778516	0.238405	1691.6
75	Eurydice	1862, Sept. 22	Peters	Clinton, U. S.	11	2.670834	0.306746	1594.2
76	Freia	1862, Oct. 21	D'Arrest	Copenhagen	12	3.386430	0.187177	2276.2
77	Frigga	1862, Nov. 12	Peters	Clinton, U. S.	11	2.671924	0.135814	1595.2
78	Diana	1863, March 15	Luther	Bilk	10	2.623642	0.203982	1552.2
79	Eurynome	1863, Sept. 14	Watson	Ann Arbor	10	2.444173	0.195333	1395.7
80	Sappho	1864, May 2	Pogson	Madras	10	2.297090	0.200478	1271.6
81	Terpsichore	1864, Sept. 30	Tempel	Marseilles	12.1	2.854377	0.211215	1761.4
82	Alcmene	1864, Nov. 27	Luther	Bilk	12.4	2.759818	0.225938	1674.6
83	Beatrix	1865, April 26	Gasparis	Naples	10.0	2.431644	0.085150	1384.9
84	Clio	1865, Aug. 25	Luther	Bilk	11.8	2.361823	0.235611	1325.7
85	Io	1865, Sept. 19	Peters	Clinton, U. S.	11.3	2.653895	0.191154	1579.5
86	Semele	1866, Jan. 4	Tietjen	Berlin	14.0	3.103545	0.207730	1997.0
87	Sylvia	1866, May 16	Pogson	Madras	11.5	3.492706	0.081178	2384.1
88	Thisbe	1866, June 15	Peters	Clinton, U. S.	10.0	2.768707	0.165061	1682.7
89	Julia	1866, Aug. 6	Stephan	Marseilles	10.2	2.549470	0.180031	1486.8
90	Antiope	1866, Oct. 1	Luther	Bilk	11.0	3.157634	0.201945	2049.4
91	Ægina	1866, Nov. 4	Stephan	Marseilles	11.8	2.491248	0.065948	1436.2
92	Undina	1867, July 7	Peters	Clinton, U. S.	10.8	3.191261	0.104082	2082.2
93	Minerva	1867, Aug. 24	Watson	Ann Arbor	11.6	2.753810	0.133263	1669.1
94	Aurora	1867, Sept. 6	Watson	Ann Arbor	11	3.163792	0.090143	2055.4
95	Arethusa	1867, Nov. 23	Luther	Bilk	10	3.069284	0.148424	1964.0
96	Ægle	1868, Feb. 17	Coggia	Marseilles	11	3.055836	0.141558	1951.1
97	Clotho	1868, Feb. 17	Tempel	Marseilles	11	2.668621	0.256895	1592.3
98	Ianthe	1868, April 18	Peters	Clinton, U. S.	12	2.684535	0.189190	1606.5
99	Atrophos	1868, May 28	Borrelly	Marseilles	13	2.833978	0.242667	1742.5
100	Hecate	1868, July 11	Watson	Ann Arbor	12	3.098111	0.156622	1991.7
101	Helena	1868, Aug. 15	Watson	Ann Arbor	10	2.573119	0.139404	1507.6
102	Miriam	1868, Aug. 22	Peters	Clinton, U. S.	11	2.662367	0.254181	1586.7
103	Hera	1868, Sept. 8	Watson	Ann Arbor	10	2.706424	0.081475	1626.2
104	Clymene	1868, Sept. 13	Watson	Ann Arbor	11	3.179809	0.197340	2071.0
105	Artemis	1868, Sept. 16	Watson	Ann Arbor	12.0	2.379945	0.176197	1341.0
106	Dione	1868, Oct. 10	Watson	Ann Arbor	12.3	3.162304	0.183184	2054.0
107	Camilla	1868, Nov. 17	Pogson	Madras	12.0	3.380094	0.048269	2269.7
108	Hecuba	1869, April 2	Luther	Bilk	10.6	3.213411	0.100390	2104.0
109	Felicitas	1869, Oct. 9	Peters	Clinton, U. S.	10.0	2.694057	0.299886	1615.1
110	Lydia	1870, April 22	Borrelly	Marseilles	11.0	2.693090	0.080626	1614.2
111	Ate	1870, Aug. 14	Peters	Clinton, U. S.	11.0	2.575614	0.101397	1509.8
112	Iphigenia	1870, Sept. 20	Peters	Clinton, U. S.	11.0	2.435656	0.126398	1388.4

No.	Longitude of Perihelion.	Longitude of Ascending Node.	Inclination of Orbit.	Mean daily Motion.	Mean Longitude at Epoch.	Epoch, Berlin mean Time.
	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	
57	52 53 13.0	200 5 25.1	15 8 1.6	632.4633	28 35 25.6	1860, Jan. 1.0
58	184 36 39.2	161 15 47.3	5 1 50.1	800.0652	186 5 46.9	1860, April 13.5
59	18 19 21.7	170 20 0.9	8 37 12.5	793.9265	317 18 5.3	1864, July 31.0
60	98 19 58.8	192 0 49.1	3 34 27.0	958.5739	47 55 13.7	1864, Oct. 19.7512
61	342 44 12.7	334 16 57.9	18 16 32.9	681.4933	73 6 25.4	1862, Jan. 0.0
62	34 2 2.8	126 11 32.8	2 12 18.4	641.1423	160 46 29.3	1862, Dec. 29.0
63	269 27 50.5	338 6 6.7	5 47 17.1	956.8777	342 57 46.4	1862, Nov. 19.0
64	123 38 6.1	311 9 14.1	1 19 52.2	808.2954	119 24 20.2	1865, Jan. 7.0
65	258 20 36.9	158 53 34.8	3 28 9.8	560.8775	180 18 11.6	1861, Jan. 0.0
66	38 13 4.7	8 13 12.5	3 2 24.6	816.2381	188 42 32.5	1861, May 27.2512
67	306 19 46.8	202 42 22.5	5 59 33.0	941.9796	144 53 4.9	1864, Jan. 1.0
68	345 10 5.0	44 53 14.3	7 57 9.2	764.5970	71 51 56.6	1863, Nov. 16.0
69	109 6 25.4	187 1 7.2	8 28 19.4	692.6300	163 53 30.4	1861, June 3.0117
70	300 3 30.3	48 14 42.6	11 38 30.2	839.9060	248 44 41.8	1861, May 28.0
71	221 58 46.8	316 18 48.4	23 18 29.5	775.4436	343 0 14.1	1862, Jan. 0.0
72	307 53 59.3	207 44 6.6	5 23 54.9	1040.1468	233 42 39.2	1865, Jan. 0.0
73	59 58 32.8	7 33 43.0	2 24 39.1	814.8434	15 17 28.6	1864, Oct. 4
74	7 19 35.1	197 59 43.6	3 58 18.9	766.1030	33 9 32.9	1863, Feb. 15.0
75	334 25 8.1	359 55 42.9	5 0 0.8	812.8981	18 12 19.5	1863, Jan. 0.0
76	92 52 4.3	213 3 21.2	2 1 51.5	569.3699	45 8 24.6	1862, Oct. 24.5
77	58 10 41.8	2 8 39.0	2 27 56.2	812.4010	204 24 52.0	1865, Jan. 0.0
78	121 54 8.2	338 58 57.0	8 38 29.4	834.9295	160 3 35.8	1863, March 9.0
79	44 20 33.1	206 42 40.0	4 36 50.5	928.5574	45 50 13.3	1864, Jan. 1.0372
80	354 55 25.6	218 29 12.6	8 36 22.1	1019.1535	258 0 27.4	1864, May 5.0
81	48 29 28.8	2 32 6.0	7 55 36.9	735.7660	21 56 27.2	1864, Oct. 6.0
82	131 19 40.8	26 56 50.7	2 51 15.1	773.9020	103 51 54.6	1865, Feb. 16.0
83	192 43 16.4	27 33 42.8	5 0 15.8	935.7507	310 53 27.7	1866, June 11.0
84	339 11 58.1	327 22 1.1	9 22 25.5	977.5422	353 48 43.6	1865, Nov. 13.0
85	322 34 31.5	203 55 58.8	11 53 15.8	820.6933	352 28 17.6	1870, Jan. 0.0
86	28 39 4.3	87 57 14.8	4 47 31.3	648.9624	39 8 17.6	1866, Jan. 20.0
87	337 21 30.0	76 23 41.3	10 51 22.0	543.5800	230 48 53.6	1866, Jan. 0.0
88	308 40 46.8	277 42 51.5	5 14 34.8	770.1788	304 56 2.3	1866, Aug. 4.5
89	353 15 59.1	311 33 31.4	16 12 1.8	871.6300	345 2 53.9	1866, Oct. 29.0
90	294 2 27.3	71 0 23.4	2 17 25.1	632.3591	346 8 36.5	1866, Oct. 18.0
91	68 9 8.1	11 36 7.0	2 9 12.1	902.3620	61 6 19.5	1867, Feb. 2.0
92	334 29 39.8	102 50 56.1	9 56 22.0	622.3906	278 39 56.0	1866, Jan. 0.0
93	276 39 54.8	5 2 28.0	8 35 34.9	776.4367	343 27 53.6	1867, Oct. 2.0
94	45 22 32.5	4 32 9.3	8 5 27.0	630.5129	25 53 12.0	1867, Nov. 28.0
95	28 54 36.4	244 20 50.3	12 52 6.5	659.8598	68 18 15.7	1868, Feb. 12.0
96	165 14 32.7	322 49 25.9	16 5 41.7	664.2200	151 49 14.7	1868, March 1.5
97	65 33 36.0	160 36 34.8	11 44 58.4	813.9103	126 8 50.0	1868, Jan. 0.0
98	147 43 7.5	354 16 43.2	15 32 35.1	806.6830	150 25 16.9	1868, Jan. 0.0
99	239 9 29.9	41 35 0.7	14 0 31.1	743.7370	229 2 12.9	1868, May 28
100	307 29 47.3	128 25 11.0	6 22 33.3	650.6701	304 19 58.0	1868, July 1.0
101	328 40 51.0	343 35 0.1	10 4 19.5	859.6400	346 33 18.1	1868, Sept. 14.0
102	355 9 10.2	211 32 43.2	5 6 3.3	816.7800	303 28 54.0	1868, Jan. 0.0
103	321 16 47.8	136 15 45.9	5 24 22.3	796.9178	350 3 29.0	1868, Sept. 1.0372
104	62 11 55.4	43 46 42.1	2 53 26.7	625.7560	18 24 14.2	1868, Sept. 14.0
105	242 36 17.8	187 54 1.8	21 38 59.0	966.3980	348 54 8.0	1868, Oct. 13.0
106	27 12 33.8	63 14 47.9	4 38 30.5	630.9588	19 26 18.6	1868, Oct. 11.0
107	273 46 0.0	178 17 0.0	10 46 0.0	570.9800	40 11 0.0	1869, Jan. 1.0
108	175 4 29.6	352 19 55.9	4 24 16.3	615.9664	191 25 52.3	1869, May 15.5
109	55 56 3.2	4 56 4.4	8 2 56.1	802.4102	34 58 26.8	1869, Oct. 9.0
110	357 27 40.1	57 4 24.3	5 59 49.9	802.8426	190 45 1.3	1870, April 22.5
111	122 53 7.3	306 26 28.4	5 1 21.4	858.3920	328 10 28.3	1870, Sept. 0.0
112	337 43 25.8	324 4 37.2	2 36 52.0	933.4300	10 32 51.7	1870, Oct. 23.5

No.	Name.	Discovered.			App't Magn.	Mean Distance.	Eccen- tricity.	Period
		When.	By whom.	Where.				Days.
113	Amalthea	1871, March 12	Luther	Bilk	11.7	2.376719	0.085967	1338.3
114	Cassandra	1871, July 23	Peters	Clinton, N. Y.	10.5	2.675815	0.140112	1598.7
115	Thyra	1871, Aug. 6	Watson	Ann Arbor	10.6	2.379158	0.194072	1340.3
116	Sirona	1871, Sept. 8	Peters	Clinton, N. Y.	11.3	2.766876	0.143284	1681.0
117	Lomia	1871, Sept. 12	Borrelly	Marseilles	11.0	2.990711	0.022884	1889.1
118	Peitho	1872, March 15	Luther	Bilk	11.7	2.438687	0.160669	1391.0
119	Althea	1872, April 3	Watson	Ann Arbor	10.2	2.582372	0.081481	1515.7
120	Lachesis	1872, April 10	Borrelly	Marseilles	11.6	3.120324	0.047600	2013.2
121	Hermione	1872, May 12	Watson	Ann Arbor	11.1	3.456775	0.122787	2347.4
122	Gerda	1872, July 31	Peters	Clinton, N. Y.	11.4	3.219864	0.037504	2110.3
123	Brunhilda	1872, July 31	Peters	Clinton, N. Y.	11.2	2.691851	0.114994	1613.1
124	Alcestis	1872, Aug. 23	Peters	Clinton, N. Y.	10.0	2.628587	0.078360	1553.0
125	Liberatrix	1872, Sept. 11	Pr. Henry	Paris	10.6	3.035245	0.346754	1931.4
126	Velleda	1872, Nov. 5	Pa. Henry	Paris	11.2	2.439932	0.106124	1392.0
127	Johanna	1872, Nov. 5	Pr. Henry	Paris	11.2	2.755037	0.065939	1670.2
128	Nemesis	1872, Nov. 25	Watson	Ann Arbor	11.0	2.751358	0.125720	1666.9
129	Antigone	1873, Feb. 6	Peters	Clinton, N. Y.	11.5	2.875560	0.206979	1781.0
130	Electra	1873, Feb. 17	Peters	Clinton, N. Y.	10.7	3.122899	0.207748	2015.7
131	Vala	1873, May 25	Peters	Clinton, N. Y.	9.5	2.419506	0.081767	1374.6
132	Æthra	1873, June 13	Watson	Ann Arbor	11.6	2.602541	0.379926	1533.5
133	Cyrene	1873, Aug. 16	Watson	Ann Arbor	10.7	3.060745	0.137442	1955.8
134	Sophrosyne	1873, Sept. 27	Luther	Bilk	10.6	2.562797	0.115801	1498.5
135	Hertha	1874, Feb. 18	Peters	Clinton, N. Y.	12.5	2.429271	0.204639	1382.9
136	Austria	1874, March 18	Palisa	Pola	11.0	2.286897	0.084841	1263.1
137	Melibœa	1874, April 21	Palisa	Pola	11.6	3.133423	0.208583	2025.9
138	Tolosa	1874, May 19	Perrotin	Toulouse	12.2	2.449147	0.162283	1399.9
139	Juewa	1874, Oct. 10	Watson	Peking	11.0	2.873869	0.202313	1779.5
140	Siwa	1874, Oct. 13	Palisa	Pola	12.0	2.731631	0.216039	1649.0
141	Lumen	1875, Jan. 13	Pa. Henry	Paris	11.4	2.709467	0.223314	1629.0
142	Polana	1875, Jan. 28	Palisa	Pola	13.0	2.381590	0.099778	1342.4
143	Adria	1875, Feb. 23	Palisa	Pola	12.3	2.756779	0.070931	1671.8
144	Vibilia	1875, June 3	Peters	Clinton, N. Y.	10.1	2.642348	0.228620	1568.8
145	Adeona	1875, June 3	Peters	Clinton, N. Y.	10.2	2.693881	0.212663	1614.9
146	Lucina	1875, June 8	Borrelly	Marseilles	11.5	2.722463	0.069509	1640.7
147	Protogeneia	1875, July 11	Schulhof	Vienna	12.0	3.134216	0.019315	2026.7
148	Gallia	1875, Aug. 7	Pr. Henry	Paris	10.8	2.769810	0.184674	1683.7
149	Medusa	1875, Sept. 21	Perrotin	Toulouse	13.0	2.132744	0.119369	1137.6
150	Nuwa	1875, Oct. 18	Watson	Ann Arbor	12.0	2.980671	0.130572	1879.6
151	Abundantia	1875, Nov. 1	Palisa	Pola	11.3	2.599166	0.053642	1530.5
152	Atala	1875, Nov. 2	Pa. Henry	Paris	11.5	3.136173	0.086253	2028.6
153	Hilda	1875, Nov. 2	Palisa	Pola	12	3.950390	0.163113	2867.8
154	Bertha	1875, Nov. 4	Pr. Henry	Paris	12	3.220994	0.100081	2111.4
155	Scylla	1875, Nov. 8	Palisa	Pola	12	2.912675	0.244593	1815.6
156	Xantippe	1875, Nov. 22	Palisa	Pola	12	3.037540	0.263701	1933.6
157	Dejanira	1875, Dec. 1	Borrelly	Marseilles	12	2.585652	0.219895	1518.6
158	Coronis	1876, Jan. 4	Knorre	Berlin	11	2.990143	0.292005	1888.5
159	Æmilia	1876, Jan. 26	Pa. Henry	Paris	12	3.125244	0.115535	2018.0
160	Una	1876, Feb. 20	Peters	Clinton, N. Y.	11	2.754181	0.063683	1669.5
161	Athor	1876, April 18	Watson	Ann Arbor	11	2.375976	0.132903	1337.7
162	Laurentia	1876, April 21	Pr. Henry	Paris	12	3.021100	0.165346	1917.9
163	Erigone	1876, April 26	Perrotin	Toulouse	12	2.354513	0.149061	1319.6
164	Eva	1876, July 12	Pa. Henry	Paris	12	2.552525	0.320751	1489.5
165	Loreley	1876, Aug. 9	Peters	Clinton, N. Y.	11	3.128643	0.072570	2021.3
166	Rhodope	1876, Aug. 15	Peters	Clinton, N. Y.	11	2.719938	0.238576	1638.4
167	Urda	1876, Aug. 28	Peters	Clinton, N. Y.	12	3.218607	0.311923	2109.1
168	Sibylla	1876, Sept. 28	Watson	Ann Arbor	11	3.378104	0.067252	2267.8

No.	Longitude of Perihelion.			Longitude of Ascending Node.			Inclination of Orbit.			Mean daily Motion.	Mean Longitude at Epoch.			Epoch, Berlin mean Time.
	°	′	″	°	′	″	°	′	″	″	°	′	″	
113	199	1	51.4	123	9	43.4	5	2	19.8	968.3664	132	37	36.7	1876, Jan. 19.0
114	153	5	51.1	164	24	12.1	4	54	31.2	810.6292	152	42	57.7	1874, Jan. 0.0
115	42	59	52.7	309	5	3.8	11	34	38.7	966.8779	170	59	25.9	1877, March 25.0
116	152	46	53.2	64	25	41.6	3	35	12.6	770.9425	44	58	58.6	1876, Oct. 23.0
117	48	45	40.4	349	38	42.5	14	57	33.2	686.0326	358	9	24.4	1871, Sept. 15.5
118	77	30	56.6	47	29	35.2	7	48	8.1	931.6920	160	32	49.4	1872, March 24.5
119	11	29	27.6	203	56	41.2	5	45	5.4	855.0239	296	51	0.8	1877, July 3.0
120	214	34	43.1	342	50	20.4	7	1	12.4	643.7347	146	27	38.6	1877, Feb. 8.5
121	0	55	19.5	77	0	5.2	7	35	2.9	552.0773	164	45	36.1	1877, March 5.0
122	208	52	46.8	179	0	31.3	1	36	17.7	614.1156	185	41	38.6	1876, April 5.0
123	72	56	37.2	308	27	55.9	6	27	25.4	803.3968	294	42	10.4	1876, July 8.5
124	244	39	2.3	188	25	11.5	2	55	51.8	832.5744	300	33	29.6	1876, Aug. 17.0
125	251	17	6.9	171	16	22.7	6	4	44.2	670.9900	316	2	35.9	1872, Sept. 12.0
126	347	45	50.3	23	7	10.0	2	56	9.0	930.9792	137	41	6.8	1874, Jan. 0.0
127	122	37	15.0	31	46	37.6	8	16	40.3	775.9173	346	25	1.5	1876, Sept. 5.5
128	16	34	12.1	76	30	40.0	6	15	31.2	777.4729	245	18	6.4	1875, April 25.0
129	241	9	50.7	138	1	59.7	12	10	52.2	727.6507	170	19	52.7	1878, Jan. 0.0
130	20	32	35.8	146	1	4.8	22	54	49.5	642.9388	312	46	19.1	1875, Dec. 21.0
131	258	34	55.3	65	15	27.0	4	38	37.0	942.7941	150	34	9.7	1876, Feb. 25.0
132	152	24	7.7	260	2	20.6	24	59	59.3	845.1041	171	35	10.3	1877, Feb. 13.0
133	248	23	46.3	321	11	55.9	7	13	33.5	662.6219	207	16	4.8	1877, April 4.0
134	67	34	19.7	346	22	19.0	11	35	48.4	864.8392	3	51	30.5	1877, Sept. 13.5
135	319	49	52.3	343	58	52.7	2	18	38.4	937.1120	175	3	23.5	1874, Feb. 25.0
136	316	35	33.0	186	9	10.0	9	33	14.4	1025.9742	129	37	55.3	1877, Jan. 31.5
137	310	19	34.8	204	18	4.7	13	46	27.4	639.7040	228	40	25.3	1874, April 21.5
138	311	39	7.9	54	52	14.8	3	13	53.7	925.7298	160	7	30.4	1877, Feb. 20.5
139	140	59	41.8	1	39	45.2	10	15	1.3	728.2932	43	33	18.4	1874, Nov. 3.0
140	300	33	21.6	107	2	20.5	3	11	37.7	785.9111	101	23	26.5	1876, Jan. 5.5
141	22	33	40.7	318	58	44.8	11	32	44.5	795.5750	120	15	44.9	1875, Feb. 25.0
142	229	41	38.8	292	38	50.3	2	19	0.5	965.3960	146	49	22.7	1875, March 4.5
143	222	38	1.2	333	41	57.6	11	29	25.4	775.1830	163	49	6.2	1875, March 25.5
144	8	30	57.3	76	44	51.6	4	50	47.7	826.0790	289	57	37.0	1875, June 29.5
145	118	7	47.0	77	43	29.8	14	23	56.3	802.4890	205	18	3.7	1875, Jan. 0.0
146	216	3	1.3	84	14	17.2	13	14	46.0	789.8850	257	3	17.7	1875, June 21.5
147	32	39	41.4	251	20	36.8	1	54	17.4	639.4596	310	23	58.4	1875, Sept. 3.5
148	35	57	17.6	145	8	51.0	25	21	24.3	769.7300	0	14	31.2	1875, Oct. 8.0
149	246	41	14.6	160	7	46.4	1	5	54.1	1139.1950	342	16	21.0	1875, Sept. 30.5
150	352	45	17.0	207	32	33.5	2	9	1.4	689.5030	16	32	24.6	1875, Nov. 2.5
151	142	11	33.0	38	46	26.7	6	16	18.6	846.7335	151	14	57.9	1877, Jan. 0.5
152	84	22	35.0	41	29	6.0	12	12	30.0	638.8540	55	41	54.0	1875, Dec. 17.0
153	285	1	35.3	228	20	28.0	7	50	53.4	451.9050	34	58	42.3	1875, Dec. 19.0
154	168	41	18.5	37	35	43.1	20	48	54.9	613.7940	53	31	35.4	1875, Nov. 22.5
155	82	1	8.0	42	52	3.0	14	4	20.0	713.7875	61	5	55.0	1875, Nov. 8.5
156	155	57	38.5	246	10	51.2	7	28	38.1	670.2300	82	29	12.1	1875, Nov. 27.5
157	109	12	26.3	62	24	52.7	11	49	47.2	853.3920	89	2	57.2	1875, Dec. 26.5
158	355	10	25.2	282	48	56.1	1	23	22.8	686.2270	73	43	27.9	1876, Jan. 4.5
159	100	40	13.2	135	5	17.7	6	5	23.3	642.2152	137	24	17.6	1877, Jan. 4.5
160	191	16	8.8	12	2	50.4	3	32	0.4	776.2790	145	11	8.6	1876, Jan. 0.0
161	312	55	58.7	18	32	47.5	9	9	50.7	968.8220	230	44	58.0	1876, May 21.5
162	147	44	25.3	38	15	5.0	6	2	53.0	675.7100	202	23	28.0	1876, May 19.0
163	93	17	4.2	158	49	33.9	4	40	34.9	982.1000	207	40	54.6	1876, May 27.5
164	2	45	34.0	77	27	10.0	24	48	4.0	870.0700	298	50	48.0	1876, July 19.5
165	338	23	22.2	304	0	30.9	11	10	12.5	641.1684	19	18	52.2	1876, Sept. 23.5
166	261	37	45.4	129	14	43.5	11	40	35.8	790.9842	222	5	25.6	1876, Sept. 12.5
167	32	39	22.2	170	7	25.4	1	42	14.5	614.4750	317	43	27.4	1876, Jan. 0.0
168	156	7	0.6	209	36	6.1	4	34	33.4	571.4740	164	45	16.2	1876, Nov. 16.5

No.	Name.	Discovered.			App't Magn.	Mean Distance.	Eccen-tricity.	Peri
		When.	By whom.	Where.				Day
169	Zelia	1876, Sept. 28	Pr. Henry	Paris	11.3	2.360278	0.131402	1324.
170	Maria	1877, Jan. 10	Perrotin	Toulouse	11.7	2.554946	0.063945	1491.
171	Ophelia	1877, Jan. 13	Borrelly	Marseilles	12.1	3.147061	0.117695	2039.
172	Baucis	1877, Feb. 5	Borrelly	Marseilles	10.4	2.379945	0.113301	1341.
173	Ino	1877, Aug. 2	Borrelly	Marseilles	11.0	2.744859	0.204869	1661.
174	Phædra	1877, Sept. 3	Watson	Ann Arbor	11.7	2.863829	0.15[illegible]485	1770.
175	Andromache	1877, Oct. 1	Watson	Ann Arbor	11.2	3.503760	0.348085	2395.
176	Idunna	1877, Oct. 14	Peters	Clinton, N. Y.	12.2	3.190424	0.164324	2081.
177	Irma	1877, Nov. 5	Pa. Henry	Paris	12.1	2.757941	0.232943	1672.
178	Belisana	1877, Nov. 6	Palisa	Pola	11.3	2.459133	0.058152	1408.
179	Clytemnestra	1877, Nov. 12	Watson	Ann Arbor	11.4	2.972847	0.108773	1872.
180	Garumna	1878, Jan. 29	Perrotin	Toulouse	13.3	2.728162	0.170774	1645.
181	Eucharis	1878, Feb. 2	Cottenot	Marseilles	11.1	3.119434	0.219706	2012.
182	Elsa	1878, Feb. 7	Palisa	Pola	11.0	2.417358	0.186180	1372.
183	Istria	1878, Feb. 8	Palisa	Pola	12.6	2.802373	0.353011	1713.
184	Deiopeia	1878, Feb. 28	Palisa	Pola	12.3	3.188269	0.072529	2079.
185	Eunike	1878, March 1	Peters	Clinton, N. Y.	10.3	2.738218	0.126838	1655.
186	Celuta	1878, April 6	Pr. Henry	Paris	11.4	2.362523	0.151299	1326.
187	Lamberta	1878, April 11	Coggia	Marseilles	11.4	2.739820	0.235396	1656.
188	Menippe	1878, June 18	Peters	Clinton, N. Y.	13.0	2.821093	0.217340	1730.
189	Phthia	1878, Sept. 9	Peters	Clinton, N. Y.	11.4	2.450456	0.035583	1401.
190	Ismene	1878, Sept. 22	Peters	Clinton, N. Y.	11.4	3.937835	0.161662	2854.
191	Kolga	1878, Sept. 30	Peters	Clinton, N. Y.	11.7	2.889216	0.081970	1793.
192	Nausicaa	1879, Feb. 17	Palisa	Pola	9.3	2.402551	0.245890	1360.
193	Ambrosia	1879, Feb. 28	Coggia	Marseilles	12.2	2.575805	0.285372	1509.
194	Procne	1879, March 21	Peters	Clinton, N. Y.	10.2	2.619444	0.237328	1548.
195	Eurycleia	1879, April 22	Palisa	Pola	12.3	2.872249	0.092246	1777.
196	Philomela	1879, May 17	Peters	Clinton, N. Y.	10.1	3.088104	0.005413	1982.
197	Arete	1879, May 21	Palisa	Pola	12.7	2.743128	0.164731	1659.
198	Ampella	1879, June 13	Borrelly	Marseilles	11.2	2.454110	0.225396	1404.
199	Byblis	1879, July 9	Peters	Clinton, N. Y.	12.1	3.205760	0.162316	2096.
200	Dynamene	1879, July 27	Peters	Clinton, N. Y.	11.3	2.737726	0.133519	1654.
201	Penelope	1879, Aug. 7	Palisa	Pola	12.2	2.677355	0.182057	1600.
202	Chryseis	1879, Sept. 11	Peters	Clinton, N. Y.	10.7	3.084416	0.097096	1978.
203	Pompeia	1879, Sept. 25	Peters	Clinton, N. Y.	11.0	2.738908	0.059473	1655.
204	Callisto	1879, Oct. 8	Palisa	Pola	12.0	2.672761	0.175511	1596.
205	Martha	1879, Oct. 13	Palisa	Pola	12.5	2.777094	0.034837	1690.
206	Hersilia	1879, Oct. 13	Peters	Clinton, N. Y.				
207	Hedda	1879, Oct. 17	Palisa	Pola	11.5	2.284834	0.029780	1261.
208	Lacrymosa	1879, Oct. 21	Palisa	Pola	12.2	2.871746	0.051381	1777.
209	Dido	1879, Oct. 22	Peters	Clinton, N. Y.	11.3	3.146399	0.064378	2038.
210	Isabella	1879, Nov. 12	Palisa	Pola	12.0	2.745361	0.136104	1661.
211	Isolda	1879, Dec. 10	Palisa	Pola	10.7	3.046439	0.154069	1942.
212	Medea	1880, Feb. 6	Palisa	Pola	11.8	3.115023	0.106268	2008.
213	Lilæa	1880, Feb. 16	Peters	Clinton, N. Y.	12.5	2.756309	0.143701	1671.
214	Aschera	1880, March 1	Palisa	Pola	11.9	2.611334	0.033004	1541.
215	Œnone	1880, April 7	Knorre	Berlin	13.0	2.766285	0.032613	1680.
216	Cleopatra	1880, April 10	Palisa	Pola	10.5	2.795298	0.249222	1707.
217	Eudora	1880, Aug. 30	Coggia	Marseilles	13.8	2.875860	0.306677	1781.
218	Bianca	1880, Sept. 4	Palisa	Pola	11.9	2.667709	0.115370	1591.
219	Thusnelda	1880, Sept. 30	Palisa	Pola	11.6	2.354117	0.224319	1319.
220	Stephania	1881, May 20	Palisa	Vienna	14.8	2.366590	0.265294	1329.
221	Eos	1882, Jan. 18	Palisa	Vienna	10.5	3.010226	0.103516	1907.
222	Lucia	1882, Feb. 9	Palisa	Vienna	12.9	3.115445	0.138241	2008.
223	Rosa	1882, March 9	Palisa	Vienna	13.9	3.094024	0.118557	1987
224	Oceana	1882, March 30	Palisa	Vienna	12.0	2.646534	0.045532	157

No.	Longitude of Perihelion.	Longitude of Ascending Node.	Inclination of Orbit.	Mean daily Motion.	Mean Longitude at Epoch.	Epoch, Berlin mean Time.
	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	
169	326 53 35.9	354 37 2.7	5 30 49.9	978.5025	43 48 42.2	1880, Oct. 30.0
170	95 47 19.5	301 19 32.8	14 22 50.3	868.8279	348 31 42.6	1879, Aug. 22.0
171	143 35 42.2	101 10 11.2	2 33 49.0	635.5487	86 57 28.8	1881, Dec. 9.0
172	328 35 42.0	331 51 30.9	10 1 21.3	966.3982	200 7 49.7	1881, April 13.0
173	13 37 50.0	148 35 50.0	14 14 35.7	780.2369	192 16 22.3	1880, March 29.0
174	253 26 9.1	328 52 14.8	12 10 38.7	732.1255	147 42 37.6	1880, Jan. 29.0
175	293 9 49.8	23 34 7.9	3 46 35.0	541.0099	152 52 11.0	1881, Jan. 3.0
176	20 49 36.8	201 11 49.6	22 31 22.1	622.6360	247 25 14.0	1881, May 23.0
177	25 14 28.6	348 59 55.1	1 26 3.5	774.6923	42 34 7.0	1877, Dec. 7.5
178	268 13 16.5	50 41 8.4	1 56 44.3	920.0970	39 2 46.6	1877, Nov. 7.5
179	354 53 23.4	253 15 44.0	7 47 52.6	692.2257	309 15 7.6	1881, July 22.5
180	126 34 32.2	315 1 13.1	0 53 23.1	787.4120	239 44 36.0	1879, June 23.0
181	95 45 2.4	144 46 32.9	18 35 23.8	644.0102	213 25 42.6	1879, June 3.0
182	54 38 7.9	106 29 32.8	2 0 20.3	944.0487	306 50 30.5	1880, Jan. 0.0
183	44 59 36.2	142 46 2.7	26 30 10.2	756.3767	99 11 17.8	1878, Feb. 10.0
184	169 22 20.4	336 18 30.1	1 12 24.5	623.2669	36 52 54.2	1881, Oct. 30.0
185	15 45 6.6	153 50 8.0	23 15 42.6	783.0772	151 7 34.6	1878, March 5.5
186	327 10 33.9	14 33 56.0	13 10 55.2	977.1085	16 11 51.6	1879, Dec. 18.5
187	213 35 47.0	22 16 34.7	10 42 16.8	782.3914	319 2 52.4	1879, Oct. 1.0
188	309 39 39.7	241 46 7.9	11 21 16.1	748.8250	272 46 42.5	1878, July 5.5
189	6 50 15.2	203 21 57.3	5 9 32.0	924.9882	142 29 37.0	1880, Feb. 18.0
190	105 17 13.3	177 1 54.1	6 7 0.6	454.0674	36 13 55.1	1878, Oct. 26.0
191	16 22 11.8	159 52 47.6	11 32 27.7	722.4983	10 55 45.8	1878, Nov. 15.5
192	10 36 46.9	343 19 13.0	6 52 29.8	952.5933	261 28 57.1	1880, May 8.0
193	70 51 31.1	351 14 32.4	11 38 32.2	858.2960	139 40 6.9	1879, March 25.5
194	319 40 32.5	159 23 19.2	18 24 33.3	836.9383	313 1 21.7	1880, July 14.5
195	106 46 24.4	8 21 51.2	7 15 4.4	728.9100	199 51 55.6	1879, May 24.5
196	352 20 58.9	73 28 29.6	7 16 38.0	653.8370	201 34 43.8	1879, June 13.0
197	324 45 56.6	82 8 5.3	8 47 25.3	780.9746	267 24 49.2	1879, June 27.5
198	354 50 57.5	268 48 34.3	9 17 18.9	922.9325	292 45 9.7	1879, July 11.5
199	260 49 26.0	90 25 4.1	15 18 30.5	618.1730	270 42 42.2	1879, July 25.5
200	46 38 21.0	325 25 49.2	6 55 31.8	783.2609	353 36 5.6	1879, Nov. 7.5
201	334 38 52.1	157 5 55.4	5 43 38.8	809.9320	335 6 50.3	1879, Sept. 12.5
202	127 43 18.2	137 48 23.7	8 47 59.9	655.0080	17 12 46.7	1879, Dec. 7.5
203	43 24 19.8	348 37 57.7	3 12 46.5	782.7813	121 10 53.0	1881, Jan. 27.5
204	257 32 30.0	205 40 14.3	8 19 29.0	812.0185	30 34 18.7	1880, Jan. 9.0
205	21 54 12.7	212 12 18.9	10 39 58.2	766.6919	127 50 15.3	1881, Jan. 22.5
206						
207	217 43 24.7	28 52 9.5	3 49 41.7	1027.3643	172 55 14.7	1881, Feb. 24.5
208	233 13 40.1	7 46 3.7	1 58 2.0	729.1020	37 15 21.5	1879, Nov. 12.5
209	256 18 11.5	1 57 51.1	7 14 32.8	635.7494	334 27 13.6	1884, Sept. 4.0
210	56 42 17.1	32 46 34.9	5 11 43.1	780.0227	52 36 58.4	1879, Dec. 11.5
211	74 12 26.5	265 28 45.5	3 50 52.6	667.2952	98 22 4.5	1880, April 6.5
212	55 30 35.4	315 17 23.3	4 17 44.5	645.3786	15 23 17.9	1883, Sept. 30.0
213	281 4 7.2	122 17 20.9	6 46 44.3	775.3800	116 49 24.1	1884, Jan. 8.0
214	109 27 29.8	342 24 20.6	3 27 13.7	840.8391	142 2 31.6	1884, Feb. 17.0
215	341 18 8.5	25 24 23.5	1 43 19.0	771.1899	206 58 25.8	1880, March 29.0
216	32 14 36.4	215 49 6.7	13 2 17.4	759.2144	17 59 58.1	1882, Sept. 15.0
217	314 54 16.5	164 0 31.0	10 16 11.9	727.5370	332 18 57.5	1880, Sept. 13.5
218	230 14 20.8	170 49 14.5	15 12 35.7	814.3267	294 17 5.5	1884, July 26.0
219	340 21 47.7	200 43 30.0	10 46 40.7	982.3475	263 52 32.7	1883, May 13.0
220	332 52 58.9	258 23 45.3	7 34 52.6	974.5908	267 44 41.1	1881, May 31.5
221	331 6 16.7	142 31 51.9	10 51 6.8	679.3723	313 13 48.7	1884, July 26.0
222	260 13 40.3	80 24 16.7	2 11 12.3	645.2478	174 11 48.5	1882, April 7.5
223	102 48 20.9	48 59 40.6	1 59 20.6	651.9603	158 24 57.7	1882, March 28.5
224	270 50 36.0	353 18 13.8	5 52 25.4	824.1189	216 40 35.8	1882, June 17.5

TABLE XXXVI.

For Sines and Tangents of small Arcs.

Arc.		Log. Sine.	Log. sin. A—log. A″.	Diff. 10″.	Log. Tangent.	Log. tan. A—log. A″.	Diff. 10″.	Arc
°	′							°
o	o	Inf. Neg.	4.6855748,7		Inf. Neg.	4.6855748,7		o
	1	6.4637261	5748,6	0,01	6.4637261	5748,8	0,02	
	2	6.7647561	5748,4	,03	6.7647562	5749,2	,06	
	3	6.9408473	5748,1	,05	6.9408475	5749,8	,10	
	4	7.0657860	5747,7	,07	7.0657863	5750,6	,14	
o	5	7.1626960	4.6855747,1	,09	7.1626964	4.6855751,7	,18	o
	6	7.2418771	5746,5	0,11	7.2418778	5753,1	0,22	
	7	7.3088239	5745,7	,13	7.3088248	5754,7	,27	
	8	7.3668157	5744,7	,15	7.3668169	5756,5	,31	
	9	7.4179681	5743,7	,17	7.4179696	5758,6	,35	
o	10	7.4637255	4.6855742,5	,19	7.4637273	4.6855760,9	,39	o 1
	11	7.5051181	5741,3	0,21	7.5051203	5763,5	0,43	1
	12	7.5429065	5739,8	,23	7.5429091	5766,3	,47	1
	13	7.5776684	5738,3	,26	7.5776715	5769,4	,51	1
	14	7.6098530	5736,7	,28	7.6098566	5772,7	,55	1
o	15	7.6398160	4.6855734,9	,30	7.6398201	4.6855776,2	,59	o 1
	16	7.6678445	5733,0	0,32	7.6678492	5780,0	0,63	1
	17	7.6941733	5731,0	,34	7.6941786	5784,1	,67	1
	18	7.7189966	5728,8	,36	7.7190026	5788,4	,71	1
	19	7.7424775	5726,6	,38	7.7424841	5792,9	,76	1
o	20	7.7647537	4.6855724,2	,40	7.7647610	4.6855797,7	,80	o 2
	21	7.7859427	5721,7	0,42	7.7859508	5802,7	0,84	2
	22	7.8061458	5719,0	,44	7.8061547	5808,0	,88	2
	23	7.8254507	5716,3	,46	7.8254604	5813,5	,92	2
	24	7.8439338	5713,4	,48	7.8439444	5819,2	,96	2
o	25	7.8616623	4.6855710,4	,50	7.8616738	4.6855825,2	1,00	o 2
	26	7.8786953	5707,3	0,52	7.8787077	5831,5	1,04	2
	27	7.8950854	5704,0	,54	7.8950988	5838,0	,08	2
	28	7.9108793	5700,6	,56	7.9108938	5844,7	,12	2
	29	7.9261190	5697,2	,58	7.9261344	5851,7	,16	2
o	30	7.9408419	4.6855693,5	,60	7.9408584	4.6855858,9	,20	o 3
	31	7.9550819	5689,8	0,62	7.9550996	5866,4	1,25	3
	32	7.9688698	5686,0	,64	7.9688886	5874,1	,29	3
	33	7.9822334	5682,0	,66	7.9822534	5882,1	,33	3
	34	7.9951980	5677,9	,68	7.9952192	5890,3	,37	3
o	35	8.0077867	4.6855673,6	,70	8.0078092	4.6855898,7	,41	o 3
	36	8.0200207	5669,3	0,72	8.0200445	5907,4	1,45	3
	37	8.0319195	5664,8	,75	8.0319446	5916,4	,49	3
	38	8.0435009	5660,2	,77	8.0435274	5925,6	,53	3
	39	8.0547814	5655,5	,79	8.0548094	5935,0	,57	3
o	40	8.0657763	4.6855650,7	,81	8.0658057	4.6855944,7	,61	o 4
	41	8.0764997	5645,7	0,83	8.0765306	5954,6	1,65	4
	42	8.0869646	5640,6	,85	8.0869970	5964,8	,69	4
	43	8.0971832	5635,4	,87	8.0972172	5975,2	,74	4
	44	8.1071669	5630,1	,89	8.1072025	5985,8	,78	4
o	45	8.1169262	4.6855624,6	,91	8.1169634	4.6855996,7	,82	o 4
	46	8.1264710	5619,1	0,93	8.1265099	6007,9	1,86	4
	47	8.1358104	5613,4	,95	8.1358510	6019,3	,90	4
	48	8.1449532	5607,6	,97	8.1449956	6030,9	,94	4
	49	8.1539075	5601,6	,99	8.1539516	6042,8	,98	4
o	50	8.1626808	4.6855595,5	1,01	8.1627267	4.6856054,9	2,02	o 5
	51	8.1712804	5589,4	1,03	8.1713282	6067,3	2,06	5
	52	8.1797129	5583,1	,05	8.1797626	6079,9	,10	5
	53	8.1879848	5576,6	,07	8.1880364	6092,8	,14	5
	54	8.1961020	5570,1	,09	8.1961556	6105,9	,18	5
o	55	8.2040703	4.6855563,4	,11	8.2041259	4.6856119,2	,23	o 5
	56	8.2118949	5556,6	1,13	8.2119526	6132,8	2,27	5
	57	8.2195811	5549,7	,15	8 2196408	6146,7	,31	5
	58	8.2271335	5542,6	,17	8.2271953	6160,8	,35	5
	59	8.2345568	5535,5	,19	8.2346208	6175,1	,39	5
o	60	8.2418553	4.6855528,2	,21	8.2419215	4.6856189,7	,43	o 6

For Sines and Tangents of small Arcs.

Arc.		Log. Sine.	Log. sin. A—log. A″.	Diff. 10″.	Log. Tangent.	Log. tan. A—tan. A″.	Diff 10″.	Arc.	
°	′							°	′
1	0	8.2418553	4.6855528,2	1,24	8.2419215	4.6856189,7	2,47	1	0
	1	8.2490332	5520,8	,26	8.2491015	6204,5	,51		1
	2	8.2560943	5513,2	,28	8.2561649	6219,6	,55		2
	3	8.2630424	5505,6	,30	8.2631153	6234,9	,59		3
	4	8.2698810	5497,8	,32	8.2699563	6250,4	,63		4
1	5	8.2766136	4.6855489,9	1,34	8.2766912	4.6856266,2	2,68	1	5
	6	8.2832434	5481,9	,36	8.2833234	6282,3	,72		6
	7	8.2897734	5473,7	,38	8.2898559	6298,6	,76		7
	8	8.2962067	5465,5	,40	8.2962917	6315,1	,80		8
	9	8.3025460	5457,1	,42	8.3026335	6331,9	,84		9
1	10	8.3087941	4.6855448,6	1,44	8.3088842	4.6856348,9	2,88	1	10
	11	8.3149536	5439,9	;46	8.3150462	6366,2	;92		11
	12	8.3210269	5431,2	,48	8.3211221	6383,7	,96		12
	13	8.3270163	5422,3	,50	8.3271143	6401,5	3,00		13
	14	8.3329243	5413,3	,52	8.3330249	6419,5	,04		14
1	15	8.3387529	4.6855404,1	1,54	8.3388563	4.6856437,8	3,08	1	15
	16	8.3445043	5394,9	,56	8.3446105	6456,3	,12		16
	17	8.3501805	5385,5	,58	8.3502895	6475,0	,17		17
	18	8.3557835	5376,0	,60	8.3558953	6494,0	,21		18
	19	8.3613150	5366,4	,62	8.3614297	6513,2	,25		19
1	20	8.3667769	4.6855356,7	1,64	8.3668945	4.6856532,7	3,29	1	20
	21	8.3721710	5346,8	,66	8.3722915	6552,5	,33		21
	22	8.3774988	5336,8	,68	8.3776223	6572,4	,37		22
	23	8.3827620	5326,7	,70	8.3828886	6592,6	,41		23
	24	8.3879622	5316,5	,73	8.3880918	6613,1	,45		24
1	25	8.3931008	4.6855306,1	1,75	8.3932336	4.6856633,8	3,49	1	25
	26	8.3981793	5295,7	,77	8.3983152	6654,8	,53		26
	27	8.4031990	5285,1	,79	8.4033381	6676,0	,57		27
	28	8.4081614	5274,4	,81	8.4083037	6697,4	,61		28
	29	8.4130676	5263,5	,83	8.4132132	6719,1	,66		29
1	30	8.4179190	4.6855252,6	1,85	8.4180679	4.6856741,0	3,70	1	30
	31	8.4227168	5241,5	,87	8.4228690	6763,2	,74		31
	32	8.4274621	5230,3	,89	8.4276176	6785,6	,78		32
	33	8.4321561	5218,9	,91	8.4323150	6808,3	,82		33
	34	8.4367999	5207,5	,93	8.4369622	6831,2	,86		34
1	35	8.4413944	4.6855195,9	1,95	8.4415603	4.6856854,4	3,90	1	35
	36	8.4459409	5184,2	,97	8.4461103	6877,8	;94		36
	37	8.4504402	5172,4	;99	8.4506131	6901,4	,98		37
	38	8.4548934	5160,4	2,01	8.4550699	6925,3	4,02		38
	39	8.4593013	5148,4	,03	8.4594814	6949,5	,06		39
1	40	8.4636649	4.6855136,2	2,05	8.4638486	4.6856973,9	4,11	1	40
	41	8.4679850	5123,9	,07	8.4681725	6998,5	,15		41
	42	8.4722626	5111,4	,09	8.4724538	7023,4	,19		42
	43	8.4764984	5098,9	,11	8.4766933	7048,5	,23		43
	44	8.4806932	5086,2	,13	8.4808920	7073,8	,27		44
1	45	8.4848479	4.6855073,4	2,15	8.4850505	4.6857099,5	4,31	1	45
	46	8.4889632	5060,5	,17	8.4891696	7125,3	,35		46
	47	8.4930398	5047,4	,19	8.4932502	7154,4	,39		47
	48	8.4970784	5034,3	,22	8.4972928	7177,8	,43		48
	49	8.5010798	5021,0	,24	8.5012982	7204,4	,47		49
1	50	8.5050447	4.6855007,6	2,26	8.5052671	4.6857231,2	4,51	1	50
	51	8.5089736	4994,0	,28	8.5092001	7258,3	,55		51
	52	8.5128673	4980,4	,30	8.5130978	7285,6	,60		52
	53	8.5167264	4966,6	,32	8.5169610	7313,2	,64		53
	54	8.5205514	4952,7	,34	8.5207902	7341,0	,68		54
1	55	8.5243430	4.6854938,6	2,36	8.5245860	4.6857369,1	4,72	1	55
	56	8.5281017	4924,5	,38	8.5283490	7397,4	,76		56
	57	8.5318281	4910,2	,40	8.5320797	7425,9	,80		57
	58	8.5355228	4895,8	,42	8.5357787	7454,7	,84		58
	59	8.5391863	4881,3	,44	8.5394466	7483,8	,88		59
1	60	8.5428192	4.6854866,7		8.5430838	4.6857513,1		1	60

Constants.			Logarithms.
Area of a circle to radius 1 Circumference of a circle to diameter 1 Surface of a sphere to diameter 1	$= \pi =$	3.14159,26536	0.4971499
Area of a circle to diameter 1	$= \pi \div 4 =$	0.78539,81634	9.8950899
Capacity of a sphere to diameter 1	$= \pi \div 6 =$	0.52359,87756	9.7189986
Capacity of a sphere to radius 1	$= 4\pi \div 3 =$	4.18879,02048	0.6220886
Diameter of a circle to area 1	$= \sqrt{4 \div \pi} =$	1.12837,91671	0.0524551
Diameter of a sphere to capacity 1	$= \sqrt[3]{6 \div \pi} =$	1.24070,09818	0.0936671
	$\sqrt{\pi} =$	1.77245,38509	0.2485749
	$\pi^2 =$	9.86960,44011	0.9942997
	$1 \div \pi =$	0.31830,98862	9.5028501
	$1 \div \pi^2 =$	0.10132,11836	9.0057003
Base of Naperian logarithms	$= \varepsilon =$	2.71828,18285	0.4342945
Modulus of the common logarithms	$= M =$	0.43429,44819	9.6377843
Naperian logarithm of π	$=$	1.14472,98858	0.0587030
Arc equal to radius expressed in degrees	$=$	57.29577,95130	1.7581226
Arc equal to radius expressed in minutes	$=$	3437.74677,07849	3.5362739
Arc equal to radius expressed in seconds	$=$	206264.80624,70964	5.3144251
Length of one degree in parts of radius	$=$	.01745,32925	8.2418773
Length of one minute in parts of radius	$=$	.00029,08882	6.4637261
Sine of 1 second	$=$	.00000,48481	4.6855749
Sine of 2 seconds	$=$	.00000,96963	4.9866049
Sine of 3 seconds	$=$	.00001,45444	5.1626961
360 degrees expressed in seconds of arc	$=$	1296000	6.1126050
24 hours expressed in seconds of time	$=$	86400	4.9365137
12 hours expressed in seconds of time	$=$	43200	4.6354837
Number of feet in one mile	$=$	5280	3.7226339
Sidereal year in mean solar days	$=$	365.25637,4417	2.5625978
Tropical year, 1850, in mean solar days	$=$	365.24221,6787	2.5625810
Annual variation	$=$	—.00000,00669	
Sidereal rotation of earth in mean solar seconds	$=$	86164.09966,888	4.9353264
Sidereal (*i. e.*, equin.) day in mean solar seconds	$=$	86164.09054,9806	4.9353263
Mean solar day in seconds of sidereal time	$=$	86636.55534,88314	4.9377012
Acceleration of stars in solar day=		236.55535*s.* sidereal time.	2.3739328
		235.90945*s.* solar time.	2.3727453
Compression of the earth=1÷299.152818			7.5241069
Equatorial radius of the earth in English feet	$=$	20923599.98	7.3206364
Polar radius of the earth in English feet	$=$	20853657.16	7.3191823
Degree of latitude at the equator in English feet	$=$	362748.33	5.5596054
Degree of latitude at 45° in English feet	$=$	364571.77	5.5617830
Multipliers for converting French toises into French metres	$=$	1.94903659	Logarithms always additive. 0.2898200
French toises into English yards	$=$	2.13153084	0.3286916
French toises into English feet	$=$	6.39459252	0.8058129
French feet into English feet	$=$	1.06576542	0.0276716
French metres into English yards	$=$	1.093633067	0.0388716
French metres into English feet	$=$	3.280899167	0.5159929
French metres into English inches	$=$	39.37079	1.5951741
British imperial gallon in cubic inches	$=$	277.274	2.4429090
Cubic inch distilled water in grains (B. 30 in. T. 62°)	$=$	252.458	2.4021891
Weight of cubic foot of water in lbs. avoirdupois	$=$	62.32106057	1.7946348
Weight of cubic foot of water in oz. avoirdupois	$=$	997.13696914	2.9987548
Weight of cubic foot of water in lbs. Troy	$=$	75.7374	1.8793104
Weight of cubic foot of water in oz. Troy	$=$	908.8488	2.9584916
Weight of cubic foot of water in grains	$=$	436247.424	5.6397329
Length of seconds pendulum in inches at London	$=$	39.13929	1.5926129
Length of seconds pendulum in inches at Paris	$=$	39.12929	1.5925019
Length of seconds pendulum in inches at New York	$=$	39.1012	1.5921901
Velocity of falling bodies in inches in one second at London	$=$	386.28931	2.5869126
Velocity of falling bodies in inches in one second at Paris	$=$	386.19061	2.5868016
Velocity of falling bodies in inches in one second at New York	$=$	385.91337	2.5864898

EXPLANATION OF THE TABLES.

Table I., page 357, contains the Latitudes and Longitudes of the principal foreign Observatories, taken chiefly from the American Nautical Almanac. In several cases, these numbers differ slightly from those given in the English Nautical Almanac and the Berlin Jahrbuch.

Table II., page 358, contains the Latitudes and Longitudes of various places in the United States. This list is designed to embrace the large cities, the astronomical observatories, and the principal colleges of the country. A few of the determinations are derived from the observations of the United States Coast Survey; others have been derived from the labors of numerous private observers; while many have been taken from maps which are confessedly very imperfect. It is hoped that before many years this Table may be very much improved.

Table III., page 359, serves to convert hours, minutes, and seconds into decimals of a day, and *vice versa.*

Example 1. It is required to convert 14h. 17m. 16.4s. into the decimal of a day.

We find from the Table,

14h. =.5833333
17m. =.0118056
16s. =.0001852
0.4s. =.0000046
Hence 14h. 17m. 16.4s. =.5953287

The equivalent for 0.4s. is derived from the equivalent for 4s. by removing the decimal point one place to the left.

Example 2. Let it be required to convert 0.5953287 day into hours, minutes, and seconds. We find from the Table,

.59 = 14h. 9m. 36s.
.005 = 7 12.0
.0003 = 25.92
.00002 = 1.73
.000008 = .69
.0000007 = .06
Hence .5953287 = 14h. 17m. 16.40s.

The number of seconds corresponding to .00002 is obtained from the little table of proportional parts at the bottom of page 359. Thus, if .0002 is equivalent to 17.28s., .00002 must be equivalent to 1.728s.; and we may proceed in the same manner for other fractions.

TABLE IV., page 360, serves to convert intervals of mean solar time into equivalent intervals of sidereal time.

Example. It is required to find the sidereal interval corresponding to the mean solar interval, 2h. 22m. 25.62s.

2h. 0m. 0s. solar interval equals 2h. 0m. 19.713s. sid. interv.
22 0 “ “ 22 3.614 “
25 “ “ 25.068 “
0.62 “ “ 0.622 “
2h. 22m. 25.62s. solar interval equals 2h. 22m. 49.017s. sid. interv.

The method of converting mean solar time into sidereal time is explained on page 123.

TABLE V., page 361, serves to convert intervals of sidereal time into equivalent intervals of mean solar time.

Example. Find the mean solar interval corresponding to the sidereal interval, 2h. 22m. 49.02s.

2h. 0m. 0s. sid. interval equals 1h. 59m. 40.341s. solar interv.
22 0 “ “ 21 56.396 “
49 “ “ 48.866 “
0.02 “ “ 0.020 “
2h. 22m. 49.02s. sid. interval equals 2h. 22m. 25.623s. solar interv.

The method of converting sidereal time into mean solar time is explained on page 125.

TABLE VI., page 362, serves to convert degrees, minutes, and seconds of space into hours, minutes, and seconds of time. It is founded on the ratio of 15 degrees to 1 hour. The Right Ascensions of the heavenly bodies are sometimes expressed in arc, but generally in time.

Example. The Right Ascension of α Lyræ for January 1, 1855, is 277° 59′ 51″.60. Required its Right Ascension expressed in time.

The Equivalent in time for	277° 0′ 0″	is	18h. 28m. 0s.
" "	59 0	"	3 56
" "	51	"	3.40
" "	0.60	"	0.04
The Right Ascension in time		is	18h. 31m. 59.44s.

In taking out the equivalents for tenths of seconds of space, we may use the units in the seconds column as arguments, taking care to remove the decimal point of the corresponding equivalent one place to the left. Thus the equivalent for 6″ is 0.4s., and for 0″.6 the equivalent is 0.04s.

TABLE VII., page 363, serves to convert hours, minutes, and seconds of sidereal time into degrees, minutes, and seconds of space.

Example. The Right Ascension of α Lyræ for January 1, 1855, is 18h. 31m. 59.44s. Required its Right Ascension expressed in arc.

The Equivalent in arc for	18h. 0m. 0s.	is	270° 0′ 0″
" "	31 0	"	7 45 0
" "	59		14 45
" "	0.44	"	6 .60
The Right Ascension in arc		is	277° 59′ 51″.60

TABLE VIII., pages 364–5, furnishes the amount of atmospheric refraction for all altitudes from the horizon to the zenith.

This Table was constructed by the late Professor Bessel, of Königsberg, and is now more generally used than any other.

It requires, in addition to the observed apparent altitude, an observation of the height of the barometer, upon which depends the factor B; of the thermometer attached to the barometer, upon which depends the factor *t*; and of the temperature of the external air, upon which depends the factor T. If the attached thermometer is not observed, we may assume that its indications are the same as those of the external thermometer.

The refraction may be computed either by natural numbers or by logarithms. The latter method is the most accurate, as the corrections required for small altitudes, indicated by the factors M and N, can be conveniently applied only with logarithms. When the altitude is not very small, and the greatest accuracy is not required, the use of logarithms may be dispensed with.

By natural Numbers.

From the accompanying Table, take the mean refraction corresponding to the observed altitude; take the factor B, corresponding to the height of the barometer; also, take the factor *t*, corresponding to the attached thermometer, and the factor T, corresponding to the external thermometer. Multiply these four numbers together, and you will obtain the true refraction.

Example. The observed apparent altitude of a star was 34° 11′ 15″; the barometer, 28.856 inches; the external and the attached thermometers both stood at +19.6° Fahr. It is required to compute the refraction.

Mean refraction for 34° 11′ 15″ . . . . 1′ 24″.8.
Barometer, 28.856; Factor B, 0.975.
$\text{Thermometer, } 19.6^\circ \left\{ \begin{array}{l} \text{Factor } t,\ 1.001. \\ \text{Factor T, } 1.061. \end{array} \right.$
Product, $0.975 \times 1.001 \times 1.061 = 1.0355$.
True refraction $= 84''.8 \times 1.0355 = 1'\ 27''.8$.

By Logarithms.

Take from the Table the factor log. B, corresponding to the height of the barometer; also the factors log. *t* and log. T, corresponding to the attached and external thermometers. Take also the values of log. A, as also M and N, corresponding to the apparent altitude. Multiply the sum of log. B and log. *t* by M; also, multiply log. T by N. Take the algebraic sum of these products (regard being had to their signs), and add to it log. A, and the logarithmic cotangent of the apparent altitude. The sum will be the logarithm of the refraction expressed in seconds of arc. This rule is expressed more concisely thus:

The logarithm of the refraction is
$=$log. cotangent app. alt.$+$log. A$+$M(log. B$+$log. t)$+$N log. T.

Example 1. The observed apparent altitude of a star was 3° 44′ 40″; the barometer, 30.162 inches; the attached thermometer, 52.2° Fahr.; and the external thermometer, 46.6° Fahr. Required the refraction.

log. factor B, 30.162	+0.00821
log. factor t, 52.2° Fahr.	−0.00078
log. factor T, 46.6° Fahr.	+0.00183
Apparent altitude, 3° 44′ 40″	$\begin{cases} M=1.0187 \\ N=1.1753 \end{cases}$
log. cotang., 3° 44′ 40″	1.18412
log. A,	1.68084
1.0187 × (log. B$+$log. t),	+0.00757
1.1753 × log. T,	+0.00213
log. refraction,	2.87466
Refraction	$=$12′ 29″.3.

Example 2. The observed apparent altitude of a star was 6° 46′ 40″; height of the barometer, 29.772 inches; the attached thermometer, −0.4° Fahr.; and the external thermometer, −2.0° Fahr. Required the refraction.

log. cot. 6° 46′ 40″		0.92500	M=1.0079
log. A		1.73061	N=1.0794
log. B	$=+0.00256$		
log. t	$=+0.00127$		
log. B$+$log. t	$=+0.00383$		
M(log. B$+$log. t)		0.00386	
log. T	$=+0.04545$		
N log. T		0.04906	
	log. refraction,	2.70853	
	The refraction$=$	8′ 31″.13	

TABLE IX., page 366–7, contains the coefficients for computing the corrections required for transit observations at the latitude of Washington Observatory.

The column headed *Azimuth* contains the value of the factor sin. $(\phi-\delta)$ sec. δ, computed for $\phi=38°\ 53'\ 39''$ for all altitudes from the south horizon, corresponding to a north polar distance 140°, to the north horizon, corresponding to north polar distance

$-38°$. The column headed *Level* contains the value of the factor cos. $(\phi-\delta)$ sec. δ, in the same manner for every degree of altitude; and the column headed *Collimation* contains the value of sec. δ. Near the pole, the values of these coefficients change very rapidly, and it is more convenient to compute special tables for such stars as are frequently observed. Page 367 exhibits the form of such tables for Polaris, λ and δ Ursæ Minoris, and 51 Cephei, both for the upper and lower culminations.

The use of this Table has been sufficiently explained on page 73, and several preceding pages.

TABLE X., pages 368–371, furnishes the reduction to the meridian for a star observed a few minutes before or after its meridian passage. It enables us to compute more readily the correction to be applied to the zenith distance observed near the meridian, in order to obtain the true meridional zenith distance. This correction may be put under the following form:

$$x=\mathrm{A}\times\frac{\cos.\ \phi\ \cos.\ \delta}{\sin.\ z}-\mathrm{B}\times\left(\frac{\cos.\ \phi\ \cos.\ \delta}{\sin.\ z}\right)^2\times\cot.\ z,$$

where $$\mathrm{A}=\frac{2\sin.^2\tfrac{1}{2}\mathrm{P}}{\sin.\ 1''},\ \text{and}\ \mathrm{B}=\frac{2\sin.^4\tfrac{1}{2}\mathrm{P}}{\sin.\ 1''}.$$

Part I. shows the value of the factor $\frac{2\sin.^2\frac{1}{2}\mathrm{P}}{\sin.\ 1''}$, and the argument of the table is the distance in time of the sun or star from the meridian. This value (or the sum of those values divided by the number of observations, if more than one observation has been made) must be multiplied by $\frac{\cos.\ \phi\ \cos.\ \delta}{\sin.\ z}$, and the product subtracted from the zenith distance (corrected for refraction, etc.) of the sun or star observed near the meridian. The difference thus obtained will give the true meridional zenith distance of the sun or star as correctly as if it had been observed precisely on the meridian.

When, however, the distance from the meridian is considerable, and when great accuracy is required, this value must be further corrected by the addition of the value of Part SECOND, on page 371, multiplied by $\left(\frac{\cos.\ \phi\ \cos.\ \delta}{\sin.\ z}\right)^2\cot.\ z$.

If the chronometer does not go accurately during the observations, a further correction is required for rate. The last column of page 371 furnishes the logarithm of this correction for a daily loss or gain of the chronometer, varying from 0 to 30 seconds.

An example of the use of this Table will be found on pages 143–5.

TABLE XI., pages 372–3, is for determining the equation of equal altitudes of the sun. If the sun's declination remained the same from the forenoon to the afternoon observations, it is evident that half this interval, added to the time of the first observation, would give the time of apparent noon as shown by the chronometer. But as the sun's declination is continually changing, a correction must be applied on account of this variation. This correction is called the *equation to equal altitudes*, and may be reduced to the form

$$x = -\mathrm{A} \, . \, \mu \, . \, \text{tang.} \, \phi + \mathrm{B} \, . \, \mu \, . \, \text{tang.} \, \delta.$$

Page 372 furnishes the value of $\frac{\mathrm{T}}{30 \text{ sin. } 7\frac{1}{2}\mathrm{T}}$, which is represented by A. This must be multiplied by the hourly variation of the sun's declination (considered as negative when the sun is proceeding toward the south), and also by the tangent of the latitude of the place. Page 373 furnishes the value of $\frac{\mathrm{T}}{30 \text{ tan. } 7\frac{1}{2}\mathrm{T}}$, which is represented by B. This must be multiplied by the hourly variation of the sun's declination, and also by the tangent of the sun's declination at the time of apparent noon on the given day. The sum of these two quantities, taken with their proper signs, is the correction required.

An example of the use of this Table will be found on page 129.

TABLE XII., pages 374–7, furnishes the angle of the vertical, the logarithm of the earth's radius, and the length of a degree of the meridian, and also of a parallel of latitude for every degree of latitude from the equator to the pole. In computing the parallax of the moon, we must employ the *geocentric* latitude, which is equal to the observed latitude *minus* the angle of the vertical; and we must also employ the horizontal parallax belonging to the place, which may be found by adding the logarithm of the horizontal parallax at the equator to the logarithm of the earth's radius, which in Table XII. is set against the given latitude. The use of these numbers is explained on pages 184 and 185.

The length of a degree of the meridian and of a parallel is constantly needed in Geodesy, and will be frequently found useful to the astronomer. This Table is taken from the Berlin Jahrbuch for 1852; but the length of a degree of longitude and latitude, which is there given in toises, has been carefully converted into English feet, and it is hoped will be found correct to the last decimal place.

TABLE XIII., page 378, shows the augmentation of the moon's semi-diameter on account of her apparent altitude, computed from the formula page 200, where its use has also been explained.

TABLE XIV., page 378, shows the quantity by which the moon's equatorial horizontal parallax must be diminished to obtain the horizontal parallax belonging to any other latitude. This reduction is given for three values of the moon's equatorial parallax, viz., 53′, 57′, and 61′; and for any other value, the equatorial parallax may be easily found by interpolation. The use of this Table will be understood from Art. 210, page 186.

TABLE XV., page 379, furnishes the parallax of the sun and planets for all altitudes above the horizon. The horizontal parallax is to be sought for at the top of the page, and the altitude on either the right or left margin. If the given horizontal parallax is not found exactly in the Table, the parallax in altitude may be obtained by interpolating between the numbers given in the Table.

Since parallax always tends to diminish the true altitude of a body, we must *add* the parallax to the observed altitude in order to obtain the true altitude, or we must *deduct* it from the observed zenith distance in order to obtain the true zenith distance.

TABLE XVI., pages 380–3, furnishes the moon's parallax in right ascension, and also in declination for Cambridge Observatory. The form of the Table is somewhat complicated, as it requires three independent arguments, viz., the moon's declination, horizontal parallax, and hour angle. The Table is computed for a declination of 0°, 5°, 10°, 15°, 20°, and 25°; for a horizontal parallax of 53′, 57′, and 61′; and for every five or ten minutes of hour angle from the meridian. For any value of these quantities not contained in the Table, a double or triple interpolation may be required. If, however, we neglect the second differences, this interpolation may be readily performed as follows: Find what number in the Table corresponds most nearly to the given declination, horizontal parallax, and hour angle. Call this number the approximate parallax, and compute the correction which should be added or subtracted on account of the variation of the given arguments from the arguments of the Table. This method will be understood from the following example.

Example. Required the moon's parallax in right ascension and declination for Cambridge Observatory, when the moon's declination is 19° 58′ 9″.4 N., the horizontal parallax of the place 60′ 14″.7, and the moon's hour angle 3h. 47m. 23.50s.

For Right Ascension.

The nearest declination in the Table is 20°; the nearest horizontal parallax is 61′; and the nearest hour angle is 230m. The corresponding parallax on page 381 is 163.47s. The given hour angle is less than 230m. by 2.61m. To find the correction for 2.61m., we form the proportion

10m. : 2.61m. :: 4.62s. : 1.20s.

The given horizontal parallax is less than 61′ by 45″.3. To find the correction for 45″.3, we form the proportion

240″ : 45″.3 :: 10.79s. : 2.03s.

The given declination is less than 20° by 110″.6. To find the correction for 110″.6, we form the proportion

5° or 18000″ : 110″.6 :: 4.47s. : 0.02s.

The required parallax will therefore be

163.47s.
−1.20s.
−2.03s.
− .02s.
160.22s., which corresponds well with the result on page 250.

For Declination.

The approximate parallax, found in a similar manner from page 383, is 1802″.0, which corresponds to Dec. 20° N.; horizontal parallax, 61′; and hour angle, 220m. The corrections for variation of the proposed arguments from the preceding arguments are found by the proportions

$20^{m} : 7.39^{m} :: 67''.8 : 25''.0$, the correction for hour angle.

$240'' : 45''.3 :: 119''.7 : 22''.6$, the correction for horizontal parallax.

$18000'' : 110''.6 :: 193''.3 : 1''.2$, the correction for declination.

The required parallax will therefore be

$$1802''.0 + 25''.0 - 22''.6 + 1''.2 = 1805''.6,$$

which differs less than a second from the result on page 250, and this discrepancy arises from our having neglected second differences in interpolation.

When it is required to compute a long series of occultations and eclipses for a particular place, it is convenient to have a table of parallaxes like the preceding, and then the subsequent computation occupies but a few minutes. With but little additional labor, this Table might be very much expanded, so that the parallaxes for any arguments might be taken out by mere inspection.

TABLE XVII., page 384, contains the angles formed by the intersection of a vertical circle and hour circle for every degree of declination from 29° north to 29° south, and for every ten degrees of hour angle from the meridian to the horizon. A knowledge of these angles is convenient in all observations of the moon, out of the meridian, but especially in observing eclipses and occultations. The method of computing this Table has been explained in Art. 145. Every astronomer will find it convenient to compute a similar table for his own observatory.

TABLE XVIII., page 385, shows the correction to be added to the moon's declination in computing an occultation or eclipse. The reason of this correction has been explained in Art. 228. The declination of the moon is given to every half degree in the first column, and the difference of right ascension between the moon and star in the case of an occultation, or between the moon and sun in the case of a solar eclipse, is given at the top of the page, to every five minutes of arc.

TABLE XIX., pages 386–7, shows the semi-diurnal arc, or the interval of time employed by the sun or a star in passing from the horizon to its point of culmination, and *vice versa*, according to its declination and the latitude of the place. These values have been computed from formula (2), page 114, without considering the effect of refraction, which would increase the duration two or three minutes, and sometimes more than this. The latitude of the place is given at the top of the page, and the declination of the star in the first vertical column. The numbers in this Table are to be subtracted from the time of meridian passage for *rising*, and added to the time of meridian passage for *setting*, as in the following examples:

Example 1. Required the mean time of setting of the planet Venus, July 5, 1855, at New Haven, lat. 41° 18′, the declination of the planet being 13° 30′ N.

Meridian passage July 5, by Nautical Almanac . . . 3h. 9m.
Semi-diurnal arc for lat. 41° 18′, and dec. 13° 30′ N.
(page 387) . . . . 6h. 49m.
Venus sets July 5th, 1855 . . . . 9h. 58m.

Example 2. Required the mean time of rising of the planet Jupiter, July 5, 1855, at New Haven, the declination of the planet being 11° 40′ S.

Meridian passage July 5, by Nautical Almanac . . . 15h. 24m.
Semi-diurnal arc for lat. 41° 18′, and dec. 11° 40′ S.
(page 387) . . . . 5h. 18m.
Jupiter rises July 5th, 1855 . . . . 10h. 6m.

This Table is designed for northern latitudes, but it is equally applicable to southern latitudes by changing the declination of the star from N. to S., and *vice versa*.

TABLE XX., page 388, contains a comparison of French millimeters with English inches, and will be found convenient for reducing French measures into English. It is deduced from the assumption that the French metre at the freezing point is equal to 39.37079 English inches at the temperature of 62° Fahrenheit; the standard temperature of the French scale being 32°

Fahrenheit, and that of the English scale being 62° Fahrenheit. This is the result given by Captain Kater in the Philosophical Transactions for 1818, page 109. The table of proportional parts in the last column gives the value of tenths of a millimeter in English inches, and will serve for hundredths by removing the decimal point one place to the left.

The relation of the metre to the yard adopted by the United States Coast Survey is,

1 metre = 1.0935696 yards, or 39.3685 United States standard inches.

TABLE XXI., page 389, enables us to convert English inches into millimeters, and is derived from the same data as the preceding Table. The table of proportional parts in the last column gives the values of hundredths of an inch in millimeters, and will serve for thousandths by removing the decimal point one place to the left.

TABLE XXII., pages 390–1, is designed for computing the difference in the heights of two places by means of the barometer.

This Table was computed from the formula of Laplace, modified in accordance with the results of more recent determinations.

Suppose that we have observed

At the lower station, $\begin{cases} H, & \text{the height of the barometer.} \\ T, & \text{the temperature of the barometer.} \\ t, & \text{the temperature of the air.} \end{cases}$

At the upper station, $\begin{cases} h', & \text{the height of the barometer.} \\ T', & \text{the temperature of the barometer.} \\ t', & \text{the temperature of the air.} \end{cases}$

Represent by s the height of the lower station above the level of the sea, by L the latitude of the place, and by h the observed height, h', reduced to the temperature T.

The difference of level, x, between the two stations is given by the formula:

$$x = 60158.6 \text{ ft. } \log. \frac{H}{h} \times \left\{ \begin{array}{l} \left(1 + \frac{t + t' - 64}{900}\right) \\ (1 + 0.00265 \cos. 2L) \\ \left(1 + \frac{x + 52251}{20888629} + \frac{s}{10444315}\right) \end{array} \right\}.$$

But h represents the height h', reduced from the temperature T′ to the temperature T. The expansion of mercury for 1° Fahrenheit is 0.0001000; that of the brass, which forms the scale of the barometer, is 0.0000104; the difference is 0.0000896. Hence we have $h=h'\{1+0.0000896(\mathrm{T}-\mathrm{T}')\}$.

Therefore,

$$60158.6\ \text{ft. log.}\ \frac{\mathrm{H}}{h}=60158.6\ \text{ft. log.}\ \frac{\mathrm{H}}{h'}-2.3409\ \text{ft.}\ (\mathrm{T}-\mathrm{T}').$$

Part I. of the Table furnishes in English feet the value of the expression 60158.6 log. H for heights of the barometer from 11 to 31 inches; only they have all been diminished by the constant 27541.5 feet, which does not change the difference,

$$60158.6 \log \mathrm{H}-60158.6 \log h.$$

Part II. furnishes the correction $-2.3409(\mathrm{T}-\mathrm{T}')$, depending upon the difference $\mathrm{T}-\mathrm{T}'$ of the temperatures of the barometers at the two stations. This correction is generally negative. It would be positive if $\mathrm{T}-\mathrm{T}'$ were negative; that is, if the temperature T′ of the barometer at the upper station exceeded the temperature T at the lower station.

Part III. gives the correction $\mathrm{A}\times 0.00265 \cos. 2\mathrm{L}$, to be applied to the approximate altitude A, and which arises from the variation of gravity from the latitude of 45 degrees to the latitude L of the place of observation. This correction has the same sign as cos. 2L; that is, it is positive from the equator to 45 degrees, and negative from 45 degrees to the pole.

Part IV. gives the correction $\mathrm{A}\,.\,\frac{\mathrm{A}+52251}{20888629}$, which is always to be added to the approximate height A, and which is due to the diminution of gravity on the vertical.

Part V. furnishes for the approximate difference of level, A, the small correction $\mathrm{A}\,.\,\frac{s}{10444315}$, corresponding to several values of the height s of the lower station. But in place of s there has been substituted, as the argument of the table, the height H of the barometer at this station.

Method of Computation.

Take from Part I., page 390, the two numbers corresponding to the observed barometric heights H and h'. From their difference subtract the correction $2.3409(T-T')$, found in Part II., with the difference $T-T'$ of the thermometers attached to the barometers. We thus obtain an approximate altitude, a.

We then calculate the correction $a.\frac{t+t'-64}{900}$ for the temperature of the air, by multiplying the nine hundredth part of a by the sum of the temperatures t and t', diminished by 64. This correction is of the same sign as $t+t'-64$. We thus obtain a second approximate altitude, A.

With A and the latitude of the place, L, we seek, in Part III., the correction $A \times 0.00265 \cos. 2L$, arising from the variation of gravity with the latitude.

For the approximate height A, Part IV. gives the correction $A \times \frac{A+52251}{20888629}$, arising from the diminution of gravity on a vertical. This correction is always additive.

Finally, when the height, s, of the lower station is considerable, the small correction $A \times \frac{s}{10444315}$ may be found in Part V. This correction is always additive.

Example 1. M. Humboldt made the following observations on the mountain of Guanaxuato, in Mexico, in latitude 21°, viz.·

	Upper Station.	Lower Station, On the Bank of the Sea.
Thermometer in open air . . .	$t'=70.3$	$t=77.5$
Thermometer to barometer . .	$T'=70.3$	$T=77.5$
Barometer	$h'=23.660$	$H=30.046$

What was the difference in the height of the two stations?

Part I. gives for $H=30.046$ inches	27649.7
Part I. gives for $h'=23.660$ inches	21406.9
Difference . . .	6242.8
Part II. gives for $T-T'=7.2°$	−16.9
Approximate altitude, a	6225.9
$\frac{a}{900}(t+t'-64)=6.918 \times 83.8°$	+579.7
Second approximate altitude, A . .	6805.6
Part III. gives for $A=6806$, and $L=21°$	+ 13.3
Part IV. gives for 6806	+ 19.3
Height above the sea	6838.2 feet.

Example 2. M. Gay Lussac, in his celebrated balloon ascent in 1805, found his barometer to indicate 12.945 English inches, the temperature being 14.9° Fahrenheit. The barometer at Paris at the same time indicated 30.145 English inches, with a temperature of 87.44° Fahrenheit. Required the elevation of the balloon above Paris.

Part I. gives for $H=30.145$ inches	27735.6
for $h'=12.945$ inches	5650.4
Difference . . .	22085.2
Part II. gives for $T-T'=72.54°$	-169.9
Approximate altitude, a	21915.3
$\frac{a}{900}(t+t'-64)=24.35\times 38.34°$	$+933.6$
Second approximate altitude, A .	22848.9
Part III. gives for $A=22848$, and $L=48° 50'$	$-$ 8.2
Part IV. gives for 22848	$+82.1$
Height of balloon above Paris . .	22922.8 feet

TABLE XXIII., pages 392–3, furnishes the coefficients for interpolation by differences. The Table on page 392 contains the values of the coefficients for interpolation by Bessel's formula, given in Art. 223. Column first contains the values of t to each hundredth of unity. Column second contains the values of the factor $t\,.\frac{t-1}{2}$ for each value of t contained in the first column. Column third contains the values of the factor $\frac{t(t-1)\,(t-\frac{1}{2})}{2\,.\,3}$ for each value of t contained in the first column. Column fourth contains the values of the factor $\frac{(t+1)t(t-1)\,(t-2)}{2\,.\,3\,.\,4}$, and column fifth contains the values of the factor $\frac{(t+1)t(t-1)\,(t-2)\,(t-\frac{1}{2})}{2\,.\,3\,.\,4\,.\,5}$ for each value of t contained in the first column.

The coefficients of the second differences are negative; the coefficients of the third differences are positive for values of t less than one half, and negative for values of t greater than one half. The coefficients of the fourth differences are invariably positive; the coefficients of the fifth differences are negative for values of t less than one half, and positive for values of t greater than one half. The mode of using this Table has been explained in Art. 223.

The Table on page 393 contains the values of the coefficients $t, \frac{t(t-1)}{2}, \frac{t(t-1)(t-2)}{2.3}, \frac{t(t-1)(t-2)(t-3)}{2.3.4}$, etc., which, being the same as the coefficients of the binomial formula, are called binomial coefficients, to distinguish them from Bessel's coefficients on page 392. Columns first and second are the same as on page 392. Column third contains the values of the factor $\frac{t(t-1)(t-2)}{2.3}$ for each value of t contained in the first column, and the subsequent columns are constructed in a similar manner. The coefficients for the odd differences are positive, while those for the even differences are negative. The mode of using this Table has been explained in Art. 220.

TABLE XXIV., pages 394–6, contains the logarithms of the coefficients for interpolation by Bessel's formula for every five minutes, the unit of time being supposed to be 12 hours. This Table is from Sawitsch's Practischen Astronomie, and its use has been explained on page 208.

TABLE XXV., page 397, enables us to convert degrees of the centesimal thermometer into degrees of Fahrenheit. It is founded on the equation, x° centesimal $= (32^\circ + \frac{9}{5}x^\circ)$ Fahrenheit.

TABLE XXVI., page 397, enables us to convert degrees of Reaumur's thermometer into degrees of Fahrenheit. It is founded on the equation, x° Reaumur $= (32^\circ + \frac{9}{4}x^\circ)$ Fahrenheit.

TABLE XXVII., page 398, shows the height of the barometer corresponding to temperatures of boiling water from 185° to 214° Fahrenheit. The temperature at which water boils in the open air depends upon the weight of the atmospheric column above it, and under a diminished barometric pressure the water will boil at a lower temperature. Since the weight of the atmosphere decreases with the elevation, it is evident that, in ascending a mountain, the higher the station the lower will be the temperature at which water boils. Hence, if we knew the height of the barometer corresponding to the temperature of boiling water, we could measure the altitude of a mountain by observing the temperature at which water boils. Table XXVII. is derived from a Table by Regnault, published in the Annales de Physique et de Chimie, t. xiv., p. 206. In Regnault's Table the temperature is expressed in centigrade degrees, and the height of the barometer in millimeters. I have deduced from this a new Table, in which the temperature is expressed in degrees of Fahrenheit, and the height of the barometer in English inches.

TABLE XXVIII., page 399, contains the depression of mercury in glass tubes on account of capillarity, according to several different authorities.

TABLE XXIX., page 399, contains the factors by which the difference of readings of the dry-bulb and wet-bulb thermometers must be multiplied in order to produce the difference between the readings of the dry-bulb and dew-point thermometers. These factors are derived from a long series of observations made at the Greenwich Observatory, and enable us to convert observations made with the wet-bulb thermometer into observations made with Daniell's hygrometer.

Example 1. The temperature of the air being 81.3°, and that of the wet-bulb being 68.9°, it is required to determine the dew-point.

The difference between the dry and wet bulb thermometers is 12.4°, which, multiplied by 1.5, gives 18.6°, which is the difference between the dry-bulb and dew-point thermometers. Hence the dew-point was at 62.7°.

Example 2. The temperature of the air being 46.9°, and that of the wet-bulb thermometer 44.2°, it is required to determine the dew-point.

The difference between the dry and wet bulb thermometers is 2.7°, which, multiplied by 2.2, gives 5.9°. Hence the dew-point was at 41.0°.

TABLE XXX., pages 400–459, is a Catalogue of 1500 stars, derived chiefly from the Catalogue of the British Association. This Catalogue contains all the stars of the British Association Catalogue to the fifth magnitude inclusive, and about a dozen stars of the magnitude five and a half, situated within a few degrees of the north pole.

Column first, on the left-hand page, contains the number of the star in this Catalogue; column second contains the equivalent number in the Catalogue of the British Association; column third contains the name of the constellation to which the star belongs, together with Flamsteed's numbers and Bayer's letters, according to the British Association Catalogue; column fourth contains the magnitude of the star according to the same Catalogue; column fifth contains its right ascension on the 1st of January, 1850; column sixth contains the annual variation of the right ascension, and includes proper motion where it exists; column seventh contains the north polar distance on the 1st of January, 1850; and column eighth contains the annual variation of polar distance, including proper motion.

On the right-hand page, column first contains the number of the star repeated from the former page; the next four columns contain the logarithms of the factors a, b, c, and d, for computing the reduction from the mean to the apparent right ascension; while the last four columns contain the logarithms of the factors a', b', c', and d', for computing the reduction from the mean to the apparent polar distance. All the numbers on each page are copied from the British Association Catalogue, with the exception of the right ascensions and polar distances of such of the stars as are contained in the Greenwich twelve-year Catalogue. The places of such stars have been carefully reduced from the years 1840 and 1845 to 1850, and are distinguished from other numbers in the same columns by an asterisk.

In a few cases, in which the Greenwich twelve-year Catalogue differs considerably from the British Association Catalogue, the results of the most recent observations at Greenwich have been combined with former ones, to obtain the mean places which are incorporated in this Table.

The mode of deducing the apparent places of the stars from their mean places has been explained on page 220.

TABLE XXXI., page 460, contains the secular variation of the annual precession in right ascension for the stars of Table XXX. whenever this variation exceeds 0.035s. The annual precession of a star does not remain the same for a long period of time, but undergoes a slight increase or decrease from year to year. As this annual change of the precession is generally small in amount, and constant for a very long period, it is commonly known by the name of the *secular variation;* for, when inserted in tables, as on page 460, it is usually multiplied by 100, for the sake of a convenient arrangement of the figures.

Assuming, therefore, the annual variation of a star in the Catalogue to be denoted by V (which is equal to the sum of the annual precession and the proper motion), the secular variation by S, the change of position in the star (either in right ascension or north polar distance, as the case may be) on January 1st $(1850+y)$, will be expressed by

$$(V+\frac{S}{100}\times\tfrac{1}{2}y)\times y,$$

where y, which denotes the number of years from 1850, must be assumed + *after*, and − *before*, that epoch. And in this manner the mean place of a star should be brought up from the epoch 1850 to the commencement of any other required year before we apply the annual correction for precession, aberration, and nutation. But for most stars, when the period is not very long, the secular variation may be omitted.

Example. It is required to find the mean right ascension of star 46, on page 400, for January 1, 1860.

Here $y=10$ years. Hence $\frac{\frac{1}{2}y\times y}{100}=\frac{1}{2}$.

From page 460, $S=+1.2222$s., and $\frac{S}{2}=+0.61$s.

Hence we have the following results:

Mean right ascension January 1, 1850 .	0h. 49m. 9.55s.
Variation in 10 years	+1m. 7.16s.
Correction for secular variation	+0.61s.
Mean right ascension January 1, 1860 .	0h. 50m. 17.32s.

TABLE XXXII., page 461, contains the secular variation of the annual precession in north polar distance for all stars in Table

XXX. whenever this variation amounts to 0″.43. This Table is to be used in the same manner as the preceding.

Example. It is required to find the mean north polar distance of star 300, page 410, for January 1, 1860.

Here $\frac{\frac{1}{2}y\times y}{100}=\frac{1}{2}$.

From page 461, $S=+1''.367$, and $\frac{S}{2}=+0''.7$.

Hence we have the following results:

Mean north polar distance January 1, 1850 .	10° 57′ 24″.2
Variation in 10 years	−54″.2
Correction for secular variation	+ 0″.7
Mean north polar distance January 1, 1860 .	10° 56′ 30″.7

For most of the stars, the secular variation of precession is inappreciable, except for long intervals of time.

TABLE XXXIII., page 462, contains the principal elements of the planetary system, taken chiefly from Mädler's Populäre Astronomie, vierte Auflage. I have substituted the English denominations for measures of length, in place of the foreign denominations of Mädler, and have substituted more recent elements of Neptune. Several of the numbers in Mädler's Table have been changed in accordance with what were considered to be the best authorities.

TABLE XXXIV., page 463, contains the elements of the satellites of the primary planets. The elements of the moon were derived from "Bailey's Astronomical Tables and Formulæ." Those of Jupiter's satellites were derived from Mädler's Astronomie; those of Saturn's satellites were derived chiefly from Mädler, modified in some instances by comparison with Herschel's Astronomy and Hind's Solar System. The elements of the satellites of Uranus were derived by myself chiefly from the observations of Lassell; and those of the satellite of Neptune were derived from Hind's Solar System.

TABLE XXXV., pages 464–5, contains the elements of 56 asteroids. These elements were mostly derived from the American Nautical Almanac for 1861. Pages 498–9 contain the elements of the asteroids most recently discovered. These elements were derived partly from the Supplement to the English Nautical Almanac for 1868, and partly from recent numbers of Peters's Astronomische Nachrichten.

TABLE XXXVI., pages 466–7, furnishes the constants for obtaining with the greatest accuracy the sines and tangents of arcs not exceeding two degrees. The column headed log. sin. A − log. A″ furnishes the difference between the logarithmic sine of the arc given in the adjacent column, and the logarithm of that arc expressed in seconds. Thus,

The logarithmic sine of 0° 40′ is 8.06577631
The logarithm of 2400″(=40′) is 3.38021124
The difference is 4.68556507

This is the number found on page 466, under the heading log. sin. A − log. A″, opposite 0° 40′; and in a similar manner the other numbers in the Table were obtained. These numbers vary quite slowly for two degrees; and hence, to find the logarithmic sine of an arc not exceeding two degrees, we have but to add the logarithm of the arc expressed in seconds to the appropriate number found in this Table.

Required the logarithmic sine of 0° 24′ 22″.57.

Tabular number from page 466 . . . . 4.6855712
The logarithm of 1462″.57 is 3.1651167
The logarithmic sine of 0° 24′ 22″.57 is 7.8506879

The logarithmic tangent of an arc not exceeding two degrees is found in a similar manner.

The same Table enables us to find the arc corresponding to a given logarithmic sine or tangent. If from the given logarithmic sine, we subtract the corresponding tabular number on page 466, the remainder will be the logarithm of the arc expressed in seconds.

Required the arc corresponding to the logarithmic sine 7.0000000. We find from page 466 that the arc must be nearly 3′; the corresponding tabular number on page 466 is 4.6855748.

The difference is 2.3144252,
which is the logarithm of 206.265.

Hence the required arc is 3′ 26″.265.

In the same manner we may find the arc corresponding to a logarithmic tangent.

The numbers in Table XXXVI. are given to 8 decimal places, in order that we may be sure of getting the seventh figure correct to the nearest decimal; it will be of no use, however, to retain the eighth figure in our computations, unless we employ logarithmic tables of more than seven decimal places.

TABLE XXXVII., page 468, contains a miscellaneous collection of numbers which are most frequently employed in computations. They are derived chiefly from Shortrede's Logarithmic Tables.

CATALOGUE OF ASTRONOMICAL INSTRUMENTS BY SEVERAL DIFFERENT MAKERS, WITH THEIR PRICES.

Telescopes by Alvan Clark and Sons, of Cambridgeport, Mass.

No. 1. Achromatic telescope of 36 inches aperture, designed for the Lick Observatory of California. This telescope is unfinished (1881). The observatory is located on Mount Hamilton, 4250 feet above the level of the sea, and is 13 miles east of San José in an air line.

No. 2. Achromatic telescope of 30 inches aperture, for the Russian Government, and designed for the Pulkova Observatory. Unfinished (1881).

No. 3. Achromatic telescope of 26 inches aperture and 389 inches focal length, with four negative eye-pieces, magnifying 155, 439, 863, and 1360 times; and sixteen positive eye-pieces, magnifying 173, 284, 390, 392, 400, 585, 606, 636, 761, 780, 875, 888, 1103, 1282, 1560, and 1802 times. The hour circle is divided to one minute of time, and is read by two microscopes to one second of time; the declination circle is divided to five minutes of arc, and can be read by two verniers to 12″ of arc. It has two finders, one of which has an aperture of 5 inches and a focal length of 6 feet, with two eye-pieces, magnifying 30 and 75 times; the other finder has an aperture of 2 inches and a focal length of 21 inches, magnifying 12 times. The driving clock is moved by water from the city works, and secures an equable motion in right ascension. This telescope was made for the United States Naval Observatory at Washington, D. C.

Price $38,000.

No. 4. Achromatic telescope of 26 inches aperture, similar to No. 3. This telescope was ordered by L. J. McCormick, of Chicago, and is designed for the observatory at the University of Virginia.

No. 5. Achromatic telescope of 23 inches aperture and 30 feet focal length, designed for the observatory at Princeton, N. J. It is expected to be completed this year (1881).

No. 6. Achromatic telescope of 18½ inches aperture and 23 feet focal length, with four negative eye-pieces, magnifying 135, 225, 450, and 900 times, and five positive eye-pieces, magnifying 120, 190, 287, 385, and 900 times. The hour circle is 22 inches in diameter, divided to one minute of time, and is read by microscopes to one second of time. The declination circle is 30 inches in diameter, divided to five minutes of arc, and reading by two microscopes to 10″ of arc. It has a driving clock, micrometer, etc. The object-glass cost $11,187, and the mounting $7000. This telescope was made for the Dearborn Observatory, Chicago, Ill. With this instrument Mr. Alvan Clark discovered the minute companion of Sirius in January, 1862, for which discovery the Lalande prize was awarded by the Imperial Academy of Sciences at Paris.

No. 7. Achromatic telescope of 16 inches aperture and 22 feet focal length, with magnifying powers from 45 to 2000. Made for the Warner Observatory, Rochester, N. Y.

No. 8. Achromatic telescope of 15½ inches aperture, for the Washburne Observatory, Madison, Wis.

No. 9. Achromatic telescope of 12 inches aperture, for the Lick Observatory of California.

No. 10. Achromatic telescope of 12 inches aperture, for the Wesleyan University, Middletown, Conn.

No. 11. Achromatic telescope of 12 inches aperture, with eye-pieces, magnifying from 50 to 1000 times. Belongs to the Morrison Observatory, Glasgow, Mo., 1876.

No. 12. Achromatic telescope of 12 inches aperture. Made for the Imperial Observatory of Vienna, Austria, 1876.

No. 13. Achromatic telescope of 12 inches aperture. Belongs to Prof. Henry Draper, of Hastings, N. Y., 1876.

No. 14. Achromatic telescope of 12 inches aperture, with eye-pieces, magnifying from 200 to 600 times. Belongs to Vassar College Observatory, Poughkeepsie, N. Y.

No. 15. Achromatic telescope of 12 inches aperture. Belongs to Mr. S. V. White, of Brooklyn, N. Y.

The following are the prices for equatorial telescopes, including circles, driving clock, and micrometer, with heavy stand to be mounted under a dome:

16	inches aperture,	$10,000	8	inches aperture,	$2700
12	" "	6,000	6	" "	1800

The following are the prices for glasses in the cell:

15	inches,	$6400	9¼	inches,	$1600	6¼	inches,	$400
13⅜	"	4800	8⅜	"	1200	5⅜	"	300
12	"	3200	7¼	"	800	5	"	200
10⅜	"	2400	7	"	600	4¼	"	150

Telescopes and Transit Instruments by Fauth and Co., Washington, D. C.

1. Equatorial telescope, object-glass 10 inches clear aperture, with eighteen eye-pieces, large hour circle divided on silver and reading with

two verniers and microscopes; large declination circle on lower end of axis, divided on silver and reading with two verniers and microscopes; improved position micrometer with parallactic eye-piece movement—the position circle is divided on silver and reads by two verniers to minutes; delicate striding level to go over declination axis; driving clock with conical pendulum connected with polar axis; illuminating apparatus to illuminate the wires only, etc. Price $5500.

2. Equatorial telescope, object-glass 9 inches clear aperture, with fifteen eye-pieces. Mounted like No. 1. Price $4000.

3. Equatorial telescope, object-glass 8 inches clear aperture, with twelve eye-pieces. Mounted like No. 1. Price $3000.

4. Equatorial telescope, object-glass 7 inches clear aperture, with ten eye-pieces. Mounted like No. 1. Price $2500.

5. Equatorial telescope, object-glass 6 inches clear aperture, with eight eye-pieces. Mounted like No. 1. Price $1800.

6. Equatorial telescope, object-glass 5 inches clear aperture, with six eye-pieces. Mounted like No. 1. Price $1100.

7. Transit circle, with telescope of 8 feet focal length and 6 inches aperture. The axis carries two circles, each 3 feet in diameter, one of them divided on silver to 5′, and read by four microscopes to single seconds. The other circle serves as a setter. A level of the best quality reading to single seconds is placed over the axis. Six micrometrical eye-pieces and one diagonal eye-piece. Price $3000.

8. Transit circle, with telescope 5 inches aperture and circles 2½ feet in diameter, reading with four microscopes. Four eye-pieces. Price $2200.

9. Transit circle, with telescope 4 inches aperture and circles 20 inches in diameter. In other respects like No. 7. Price $1900.

10. Transit instrument, same as No. 7, without the large circles and reading microscopes, has two 6-inch setting circles attached near the eye-end. Price $1850.

11. Transit instrument, same as No. 8, without the large circles. Price $1500.

12. Transit instrument, same as No. 9, without the large circles. Price $1240.

Telescopes and Transit Instruments by Messrs. T. Cooke and Sons, York, England.

1. Equatorial telescope, object-glass 10 inches aperture, with eight eye-pieces, the two lowest of the Kellner construction, and six Huyghenian, ranging from 50 to 800; six sun-shades; double parallel wire micrometer with six eye-pieces; large position circle at the eye-

end of the tube, graduated on silver, and read with two verniers and microscopes; large declination circle with verniers, graduated on silver, and read by microscopes from the eye-end; hour circle, graduated on silver, with two sets of divisions and verniers and reading microscope; equatorial motion communicated by clock-work, with means for changing sidereal to lunar rate..............................£1200.

2. Equatorial telescope, object-glass 9 inches aperture, complete as above..£1050.

3. Equatorial telescope, object-glass 8 inches aperture, with seven Huyghenian eye-pieces, complete as above.................£790.

4. Equatorial telescope, object-glass 7 inches aperture, with six Huyghenian eye-pieces, complete as above.................£585.

5. Equatorial telescope, object-glass 6 inches aperture, with six Huyghenian eye-pieces, complete as above.................£405.

6. Equatorial telescope, object-glass 5 inches aperture, with five Huyghenian and four micrometer eye-pieces, complete......£275.

7. Equatorial telescope, object-glass 4½ inches aperture, with four Huyghenian and four micrometer eye-pieces, complete......£205.

8. Equatorial telescope, object-glass 4 inches aperture, with four Huyghenian and four micrometer eye-pieces, complete......£150.

Transit Instruments.

1. Transit instrument, object-glass 3 inches aperture, setting circle graduated on silver with delicate level, two verniers and reading microscopes, three micrometer eye-pieces, on cast-iron stand....£86.

Apparatus for lifting and reversing the axis, extra....	18.
Collimating eye-piece, extra......................	2½.
Arrangement for changing the dark lines in a bright field to bright lines in a dark field, extra..........	10.
Hanging level attached to the centre cube of axis, extra....................................	10.
Two setting circles, extra......................	8.
2. Transit instrument, object-glass 2¾ inches aperture.....	68½.
3. Transit instrument, object-glass 2½ inches aperture.....	58½.
4. Transit instrument, object-glass 2¼ inches aperture.....	51½.
5. Transit instrument, object-glass 2 inches aperture......	46½.

Achromatic Object-glasses in Brass Cell.

10 inches	aperture,	£390	6½ inches	aperture,	£100	4¾ inches	aperture,	£37
9 "	"	280	6 "	"	75	4½ "	"	32
8 "	"	190	5½ "	"	55	4¼ "	"	27
7 "	"	125	5 "	"	42½	4 "	"	22

Telescopes by Howard Grubb, Dublin, Ireland.

The following is his price-list of equatorially-mounted refractors:

Diameter of Objectives.	See Specification A.	See Specification B.	Diameter of Objectives.	See Specification A.	See Specification B.
7 inches	£500	£420	18 inches	£2500	£2000
8 "	600	500	21 "	3500	2800
10 "	950	750	24 "	5000	4200
12 "	1200	1000	27 "	7000	6000
15 "	1800	1400			

Specification A includes two right-ascension and one declination circle of as large size as practicable, divided on gold alloy (twelve-carat gold alloyed with silver) to such graduation as the purchaser desires; first right-ascension circle read by two opposite verniers in the usual manner, second right-ascension circle read by one vernier and bent microscope from eye of telescope, declination circle read by two verniers and microscope from eye-end of telescope; dark and bright fields of micrometer illuminated at pleasure from the same fixed lamps which illuminate the right-ascension and declination circles; transparent-position circle also illuminated by the same lamp; clock-work of best construction with Grubb's frictional governor; six negative eye-pieces; improved micrometer with six positive eye-pieces, etc.

Specification B includes one right-ascension and one declination circle divided on silver to such graduation as the purchaser desires; right-ascension circle read by verniers in the usual manner; declination circle read by two opposite verniers, one of which can be viewed from eye-end of telescope by microscope; bright field illumination for micrometer; transparent-position circle; clock-work of best construction; four negative eye-pieces; finders, etc.

The following are his prices of objectives and cells:

3 inches	diameter,	£10	8 inches	diameter,	£100	18 inches	diameter,	£900
4 "	"	18	9 "	"	140	20 "	"	1400
5 "	"	30	10 "	"	200	25 "	"	2000
6 "	"	45	12 "	"	350	27 "	"	2600
7 "	"	70	15 "	"	600			

The foci of these objectives are generally from twelve (for small sizes) to sixteen apertures.

Mr. Grubb furnishes domes for observatories of all sizes up to 45 feet in diameter. The new observatory at Vienna is supplied with three of Mr. Grubb's domes of 27 feet diameter, and one of 45 feet. This last dome is entirely of steel, and is intended for the great 27-inch equatorial recently completed (1881) by Mr. Grubb.

Telescopes by G. and S. Merz, Munich, Bavaria.

1. Refractor of 18 inches=48.7 cm. aperture and 7 m. focus, with an hour circle 46 cm. in diameter, divided into single seconds of time, and a declination circle 65 cm. in diameter, divided to 10″ of arc, with six common astronomical eye-pieces, magnifying from 120 to 1200 times, with a filar micrometer and position circle, nine micrometric eye-pieces, magnifying from 140 to 2000 times, and two ring micrometers. The finder has an aperture of 77 mm.

Price 105,000 marks.

(The value of a mark is 24 cents in United States money.)

2. Refractor of 14 inches=38 cm. aperture and 6 m. focus, with six astronomical eye-pieces, magnifying from 110 to 1000 times, and nine micrometric eye-pieces, magnifying from 120 to 1600 times. Circles and finder same as No. 1. Price 72,000 marks.

3. Refractor of 10½ inches=28.45 cm. aperture and 4½ m. focus, with an hour circle 38 cm. in diameter, divided to 2 seconds of time, and a declination circle 54 cm. in diameter, divided to 10″ of arc, with astronomical eye-pieces, magnifying from 80 to 760 times, and eight micrometric eye-pieces, magnifying from 90 to 1000 times. The finder has an aperture of 68 mm.

Price 45,000 marks.

4. Refractor of 9 inches=24.4 cm. aperture and 4 m. focus, with six astronomical eye-pieces, magnifying from 72 to 680 times, and eight micrometric eye-pieces, magnifying from 81 to 900 times. Finder and circles as in No. 3. Price 35,000 marks.

5. Refractor of 8 inches=21 7 cm. aperture and 3.2 m. focus, with an hour circle 26 cm. in diameter, divided to 4 seconds of time, and a declination circle 41 cm. in diameter, divided to 10″ of arc, with astronomical eye-pieces, magnifying from 60 to 550 times, and six micrometric eye-pieces, magnifying from 100 to 580 times, with one ring micrometer. The finder has an aperture of 48 mm.

Price 19,000 marks.

6. Refractor of 7 inches=18.9 cm. aperture and 3 m. focus, with an hour circle 26 cm. in diameter, divided to 4 seconds of time, and a declination circle 38 cm. in diameter, divided to 10″ of arc, with six astronomical eye-pieces, magnifying from 67 to 630 times, and six micrometric eye-pieces, magnifying from 75 to 880 times.

Price 13,500 marks.

7. Refractor of 6 inches=16.2 cm. aperture and 2.7 m. focus, with an hour circle 24 cm. in diameter, divided to 4 seconds of time, and a declination circle 33 cm. in diameter, divided to 10″ of arc, with six

astronomical eye-pieces, magnifying from 50 to 480 times, and 5 micrometric eye-pieces, magnifying from 120 to 480 times.

Price 11,000 marks.

8. Refractor of 52 lines=117.5 mm. aperture and 2 m. focus, with an hour circle 22 cm. in diameter, divided to 4 minutes of time, and a declination circle 27 cm. in diameter, divided by verniers to 30″ of arc, with six astronomical eye-pieces, magnifying from 32 to 324 times, an annular micrometer, etc., without clock-work.

Price 4800 marks.

9. Heliometer of 7 inches=19 cm. aperture and 3.2 m. focus, with five astronomical eye-pieces, magnifying from 50 to 360 times. Mounted like No. 3. Price 42,000 marks.

10. Heliometer of 6 inches=16.2 cm. aperture and 2.7 m. focus, with five astronomical eye-pieces, magnifying from 40 to 300 times. Mounted like No. 3. Price 35,000 marks.

The following are his prices of objectives:

Aperture.	Focal Length.	Prices.
18 inches=487.0 mm.	877–682 cm.	27,000 marks.
16 " =433.0 "	780–585 "	19,000 "
14 " =379.0 "	682–487 "	13,500 "
12 " =325.0 "	552–422 "	9,000 "
10 " =271.0 "	488–325 "	6,000 "
9 " =244.0 "	422-292 "	4,400 "
8 " =217.0 "	357–260 "	3,200 "
7 " =189.0 "	325–227 "	2,100 "
6 " =162.0 "	259–194 "	1,440 "
5 " =135.0 "	227–162 "	800 "
4½ " =121.8 "	194–146 "	600 "
4 " =108.3 "	160–130 "	540 "

NOTE.—m. stands for metre, cm. for centimetre, and mm. for millimetre.

Meridian Circles and Transit Instruments, by T. Ertel and Son, Munich, Bavaria.

The dimensions are given in old French measure, according to which 1 inch=1.0658 English inches, and 1 foot=12.7892 English inches. The prices are in florins, one florin being equal to 41.7 cents of United States currency.

No. 1. Meridian circle 45 inches in diameter, with a telescope of 9 inches aperture and 13 or 9 feet focal length. At one extremity of the rotation axis is the circle of altitude, reading by four microscopes to one second. At the other extremity of the axis is a circle of the same dimensions, but divided only to single minutes. The instrument has a large level and four astronomical eye-pieces.

Price 16,500 florins.

No. 2. Meridian circle 40 inches in diameter, with a telescope of 8 inches aperture and 11 or 8 feet focal length. Mounted like No. 1.
Price 12,000 florins.

No. 3. Meridian circle 36 inches in diameter, with a telescope of 7 inches aperture and 9½ or 7 feet focal length. Mounted like No. 1.
Price 9000 florins.

No. 4. Meridian circle 36 inches in diameter, with a telescope of 6 inches aperture and 8 or 6 feet focal length. Mounted like No. 1.
Price 7200 florins.

No. 5. Meridian circle 33 inches in diameter, with a telescope of 4 inches aperture and 5 or 4 feet focal length. Mounted like No. 1.
Price 5800 florins.

No. 6. Meridian circle 28 inches in diameter, with a telescope of 3 inches aperture and 3½ or 3 feet focal length. Mounted like No. 1.
Price 3500 florins.

No. 7. Transit instrument, with an object-glass of 9 inches aperture and 13 or 9 feet focal length. Construction like that of a meridian circle. Price 12,000 florins.

No. 8. Transit instrument, with an object-glass of 8 inches aperture and 11 or 8 feet focal length. Mounted like No. 7.
Price 8500 florins.

No. 9. Transit instrument, with an object-glass of 7 inches aperture and 9½ or 7 feet focal length. Mounted like No. 7.
Price 6000 florins.

No. 10. Transit instrument, with an object-glass of 6 inches aperture and 8 or 6 feet focal length. Mounted like No. 7. Price 4000 florins.

No. 11. Transit instrument, with an object-glass of 4 inches aperture and 5 or 4 feet focal length. Mounted like No. 7.
Price 2500 florins.

No. 12. Transit instrument, with an object-glass of 3 inches aperture and 3½ or 3 feet focal length. Mounted like No. 7.
Price 1700 florins.

No. 13. Prime vertical transit instrument, with an object-glass of 7 inches aperture and 9½ or 7 feet focal length. The telescope is at the end of the horizontal axis, with an arrangement for rapid reversal.
Price 7500 florins.

No. 14. Prime vertical transit instrument, with an object-glass of 6 inches aperture and 8 or 6 feet focal length. Mounted like No. 13.
Price 6000 florins.

No. 15. Prime vertical transit instrument, with an object-glass of 5 inches aperture and 6½ or 5 feet focal length. Mounted like No. 13.
Price 4300 florins.

No. 16. Prime vertical transit instrument, with an object-glass of 4 inches aperture and 5 or 4 feet focal length. Mounted like No. 13.
Price 3000 florins.

No. 17. Prime vertical transit instrument, with an object-glass of 3 inches aperture and 3½ or 3 feet focal length. Mounted like No. 13.
Price 1800 florins.

The firm of Pistor and Martins, of Berlin, has become extinct; but Carl Bamberg, of Berlin, advertises to make astronomical instruments in the same style as they were formerly made by Pistor and Martins.

Astronomical instruments of the first class are made by several other manufacturers in Europe, particularly by A. Repsold and Son, of Hamburg, Germany; but the latter firm does not publish a catalogue, and it is necessary to make a special contract for such instruments as may be required.

LOOMIS'S SERIES OF TEXT-BOOKS.

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